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Teaching guide

for Seeing through arithmetic 5

This is the Teaching Guide for *Seeing Through Arithmetic 5*, the fifth book of The Basic Mathematics Program, which is a unit of THE CURRICULUM FOUNDATION SERIES ®, published by Scott, Foresman and Company.

The authors of this book are Maurice L. Hartung, Professor of Education, University of Chicago; Henry Van Engen, Head of the Department of Mathematics, Iowa State Teachers College; and Lois Knowles, Associate Professor of Education, and Elementary Supervisor, Laboratory School, University of Missouri.

A collaborator in the preparation of this book is R. A. Rennie, Associate Professor of Psychology, University of Saskatchewan.

This book was prepared by the editoral staff under the direction of George E. Russell, Directing Editor of The Basic Mathematics Program.

This book was designed by William Nicoll and illustrated by Janet LaSalle.

Teachin

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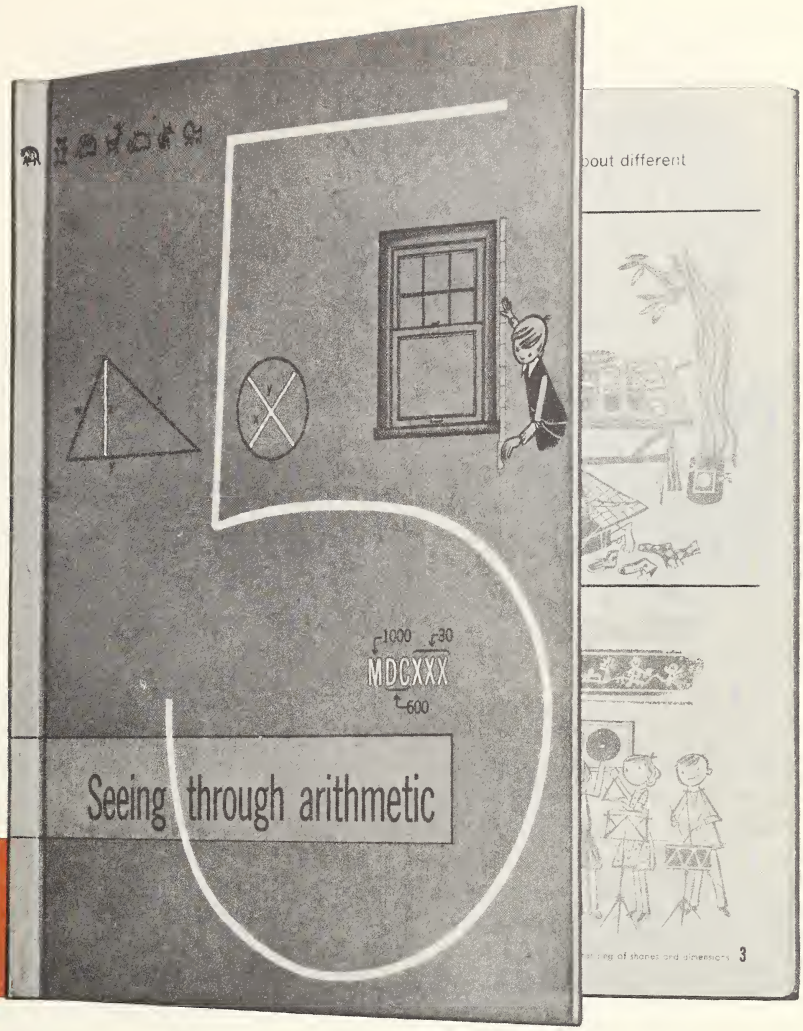

guide

Scott, Foresman and Company

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Getting acquainted with this book

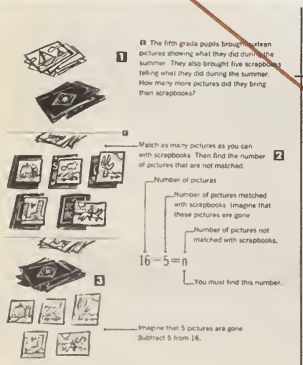
In this *Teaching Guide* you will find several different kinds of help in planning your work.

The various kinds of help are so organized as to meet the varied needs of teachers. For some, the brief notes keyed to the reproduced pages from the pupils' book may be sufficient. Others may want to use the Expanded Notes and activity suggestions. You can decide for yourself how much use to make of these various materials in the *Teaching Guide*.

A section called

Lesson briefs on pages 19-241

provides, through reduced reproductions of pupils' lesson pages, accompanied by short, keyed teaching suggestions, a quick overview of how to teach the lesson pages in each block of work. Also provided for you are objectives, a list of new vocabulary, brief comments on the block of work in general, and an answer key for all exercises (except those concerned with the basic facts) and problems.


1
The fifth grade pupils brought sixteen pictures showing what they did during the summer. They also brought five scrapbooks telling what they did during the summer. How many more pictures did they bring than scrapbooks?
2
Match as many pictures as you can with scrapbooks. Then find the number of pictures that are not matched.
Number of pictures
Number of pictures matched with scrapbooks. Imagine that these pictures are gone
Number of pictures not matched with scrapbooks.
16 − 5 = n
You must find this number.
3
Imagine that 5 pictures are gone
Subtract 5 from 16.
n

21

1 Have the pupils read the problem and identify the scrapbooks and pictures shown in the first scene.

2 Ask the pupils to read this paragraph and look at the picture. Have them note that 5 pictures are matched with 5 scrapbooks. They should see that at the top of this scene there is a pile of pictures that are unmatched. Point out that when they remove 5 pictures from the total 16, some are left. Have the children explain the equation in terms of the illustration.

3 The pupils should see in this scene that the 5 pictures that are dimmed are to be imagined as gone. This should suggest subtraction to them. Have someone show the computation on the chalkboard.

23 Thinking straight; Keeping skillful

Expanded Notes are not considered necessary for these lessons.

Objectives

The child reviews the rounding off of large numbers to the nearer ten thousand, hundred thousand, and million. He practices multiplication and division basic facts and solving the equation types just studied.

Vocabulary

New words page 23 passenger*, United States*, approximate, most*

Comments

The children must understand that numbers are only rounded off if there is a good reason for doing so. Discuss the fact that "round numbers" are easier to remember, but you might mention that such numbers as telephone numbers and car license numbers must not be rounded off. Whether or not a number is rounded off, then, is determined by the use to be made of it. The exercises in this lesson require rounding off in both upward and downward directions.

Answers

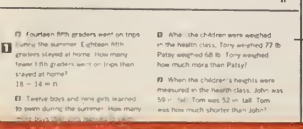
Thinking straight:

A Exact; because the numerol is not r
B Yes; 948 is close to 1000.
C The nearer hundred thousand
D 6,000,000; because 6,116,948 is close than to 7 million
E 6,116,950; 6,120,000; 6,116,900
F 6,117,000. It is easy to work with a off only 52.

	Nearer 1000	Nearer 100,000	N
G	1,739,000	1,700,000	
H	16,085,000	16,100,000	
I	36,026,000	36,000,000	
J	421,762,000	421,800,000	
K	704,551,000	704,600,000	

Keeping skillful:

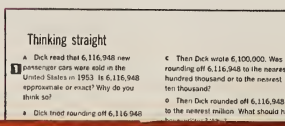
(block 2)			(block 3)		
A 34	D 128	G 143	A 38	D 7	
B 98	E 165	H 734	B 225	E 3	
C 392	F 1245	I 519	C 243	F 2	


1
Fourteen fifth graders went on trips during the summer. Eighteen fifth graders stayed at home. How many fewer fifth graders went on trips than stayed at home?
18 − 14 = n
Twelve boys and nine girls learned to swim during the summer. How many more boys than girls learned to swim?
When the children were weighed in the health class, Tony weighed 77 lb Patsy weighed 68 lb. Tony weighed how much more than Patsy?
When the children's heights were measured in the health class, John was 59 in. tall. Tom was 52 in. tall. Tom was how much shorter than John?

22

1 Discuss Problems D to F and the accompanying equations with the class. Have pupils compute on the chalkboard to find the answers.

2 Assign Problems G to N as written work. Tell the children to write an equation to describe


Thinking straight
1
a Dick read that 6,116,948 new passenger cars were sold in the United States in 1953. Is 6,116,948 approximate or exact? Why do you think so?
b Dick tried rounding off 6,116,948
c Then Dick wrote 6,100,000. Was he rounding off 6,116,948 to the nearest hundred thousand or to the nearest ten thousand?
d Then Dick rounded off 6,116,948 to the nearest million. What should he

23

1 Handle Exercises A to F orally. dren read the exercises and ons tions. Discuss each one. To hel round off to the nearer thousa thousand, call attention to the

A section called

Expanded notes

on pages 243-350

gives detailed suggestions for teaching each lesson. The Expanded Notes for each block of work are organized to provide for individual differences. First there are teaching suggestions for the class as a whole, then special suggestions for able pupils, and finally, suggestions for the slow learners. Sections called "Charting the Course" are interspersed throughout the Expanded Notes to introduce a block of work or related blocks of work.They describe how the block of lessons in the pupils' book fits into the arithmetic program. They may be read all at one time or individually as the work with the book progresses. You should, however, have these general principles in mind as you examine the more detailed suggestions that are provided for teaching each lesson. You will be in the best possible position to plan your lessons if you study both this section and the section entitled Lesson Briefs.

A section called

Activities

on pages 355-381

is a real source book of games and activities useful in arithmetic. References to them are made in the Expanded Notes, but they can be used at other times.

A section called

Reference materials

on pages 383-400

provides you with a useful bibliography, a description of the problem types, a grade-placement chart for the program, a suggested time schedule, alphabetical word list, table of mathematical content, and an index. You will find it profitable to read the brief overview of the major features of the *Seeing Through Arithmetic* program described

in the section

Guiding principles

that follows.

Explained there are the features of this program, all of which have been designed to help you and your pupils discover that learning arithmetic can be an interesting and satisfying experience.

Guiding principles

Arithmetic is a fundamental subject.
Children should learn to use it with ease,
with confidence,
and with enjoyment.

The best way to reach this goal is to help children gain insight into arithmetic.

The objective: insight

"Insight" is a term that has come into prominence in modern psychology. Insight depends upon the recognition of relationships. When insight occurs, the parts that make up a total situation are seen in relation to each other and to the whole situation. The number system, for example, involves many relationships. When children have gained insight as to these relationships, a great many facts that would otherwise remain isolated are recognized as elements of a few general principles. The processes of arithmetic, for example, become understandable, and not just a set of arbitrary rules.

Seeing Through Arithmetic 5 provides for the development of insight in many ways. Among these are:

1. Fundamental mathematical ideas, and in particular those underlying fractions, rates, and comparisons, are clarified as they have never been in the past.
2. Problem solving is clarified by use of principles and techniques that help pupils learn to analyze problem situations.
3. The learning of arithmetic is simplified by the way the work is organized.
4. New and improved methods of presentation are systematically used to make the learning of arithmetic easier and more enjoyable than it has been in the past.

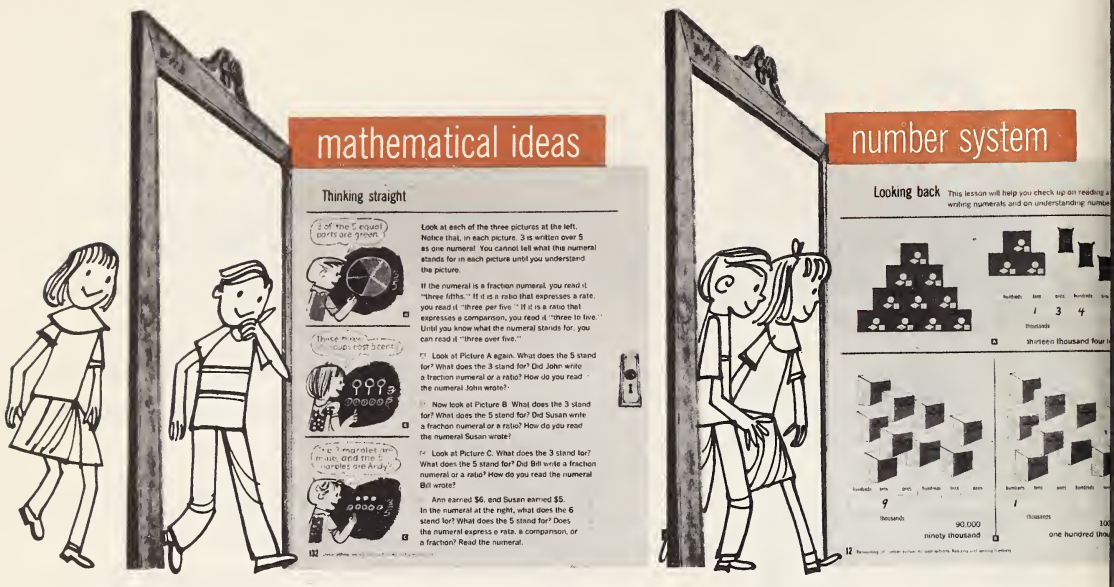
These features and the psychological backgrounds for them are explained in the following sections.

1 Developing fundamental mathematical ideas

A modern arithmetic program must emphasize fundamental ideas. These ideas should stand out above the many less important details, and should not be obscured by premature attention to computational processes with symbols used to represent them.

Among the most fundamental of all mathematical ideas are those suggested by the terms *rate, comparison,* and *fraction. Seeing Through Arithmetic 5* develops these ideas by methods that are new and distinctive.

Fractions have always been considered a difficult part of arithmetic. An understanding of fractions involves several ideas, all of which must be carefully taught. In the past, pupils have too frequently been expected to compute with fractions—to reduce to lower terms, to add, subtract, multiply, and divide—before they learned what the fraction numerals stood



Doors to success—understanding fundamental mathematical ideas and the number system; seeing ideas as wholes; visualizing

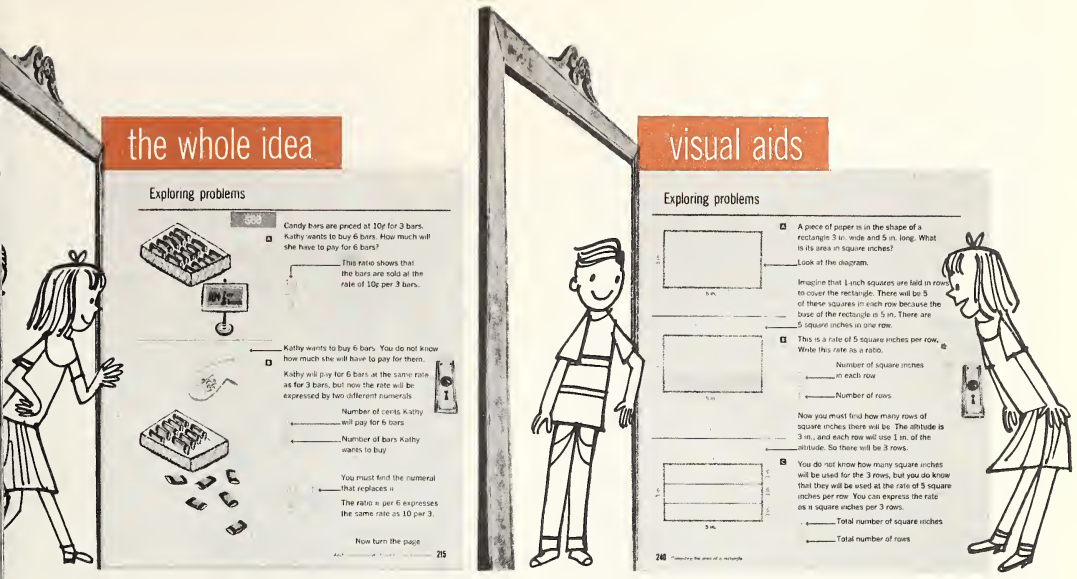
for. Because insight and understanding were lacking, it was necessary for teacher and pupil to rely on a fairly complex set of rules which, if learned at all, were usually learned by drill methods. Under these circumstances, it was inevitable that many pupils would not achieve the goals set for them.

In modern arithmetic programs, work on fractions begins in earnest in Grade 5. Work in previous grades has been aimed primarily at introducing the *concept* of fraction, with little if any emphasis upon computation with fractions. A major objective of Grade 5 is to increase the pupils' understanding of fractions and to teach the addition and subtraction of fractions and mixed numbers. Modern learning theory and research make clear the paramount importance of developing understanding *before* effort is devoted to learning rules of procedure and acquiring skill in computation. *Seeing Through Arithmetic 5* provides materials designed with these principles always in mind. The learning activities focus attention on the ideas involved in the fraction concept and clarify them as they have never been clarified before. Using these insights, the pupils will better acquire understanding of and skill in computation with fractions.

An understanding of fractions depends upon recognizing the distinction between a fraction numeral, or symbol for a fraction, and the idea it stands for, the fraction itself. For example, suppose a pupil has partitioned a bar of candy into 4 equal parts and eaten one of them, so that only 3 parts are left. He can describe how much of his candy remains by saying "three fourths" or by writing the fraction numeral $\frac{3}{4}$. There are, however, many other fraction numerals (for example, $\frac{6}{8}$, $\frac{9}{12}$, $\frac{75}{100}$) that may be used to describe the same fraction or part of the original bar. Thus $\frac{3}{4}$ and $\frac{6}{8}$ are not to be thought of as two different fractions. Instead, they are two different fraction numerals which represent the same fraction.

Moreover, the same symbols may be used to express other situations which are quite different. For example, suppose 3 pieces of a certain kind of candy cost 4 cents. Then 6 pieces will cost 8 cents, 9 pieces will cost 12 cents, and so on. This situation involves the concept of *rate,* and in this example the rate is "3 pieces for 4 cents" or, more briefly, "3 for 4." This idea is simple and familiar to children.

However, in this situation the 3 pieces of candy are clearly not a fraction of, or "part of," the 4 cents.


the whole idea
Exploring problems
Candy bars are priced at 10¢ for 3 bars. Kathy wants to buy 6 bars. How much will she have to pay for 6 bars?
This ratio shows that the bars are sold at the rate of 10¢ per 3 bars.
Kathy wants to buy 6 bars. You do not know how much she will have to pay for them.
Kathy will pay for 6 bars at the same rate as for 3 bars, but now the rate will be expressed by two different numerals.
Number of cents Kathy will pay for 6 bars
Number of bars Kathy wants to buy
You must find the numeral that replaces n
The ratio n per 6 expresses the same rate as 10 per 3.
Now turn the page
215
visual aids
Exploring problems
A piece of paper is in the shape of a rectangle 3 in. wide and 5 in. long. What is its area in square inches?
Look at the diagram.
Imagine that 1-inch squares are laid in rows to cover the rectangle. There will be 5 of these squares in each row because the base of the rectangle is 5 in. There are 5 square inches in one row.
This is a rate of 5 square inches per row. Write this rate as a ratio.
Number of square inches in each row
Number of rows
Now you must find how many rows of square inches there will be. The altitude is 3 in., and each row will use 1 in. of the altitude. So there will be 3 rows.
You do not know how many square inches will be used for the 3 rows, but you do know that they will be used at the rate of 5 square inches per row. You can express the rate as n square inches per 3 rows.
Total number of square inches
Total number of rows
240

Rate situations like this one are expressed by using *ratios*—for example, the ratio "3 to 4," and the ratio is often written in the form ¾. Since this looks like a fraction numeral, it takes some clear thinking to see why this form is justified. Eventually, when children have developed a clear understanding of *both* fractions and rates, they may see that this rate may also be expressed by the ratio "¾ to 1." This ratio briefly expresses the same rate in terms of the situation "¾ of one piece of candy for 1 cent." Such an understanding rarely develops spontaneously in children. It must be carefully developed over a period of time by activities specifically planned for this purpose.

There is another type of everyday situation which is described by using ratios. This situation occurs when two quantities are compared. For example, suppose one girl has a collection of 6 dolls and her friend has 8 dolls. The ratio "6 to 8," or the equal ratio "3 to 4," or the symbol ¾, can all be used to express a comparison of one of the sets of dolls to the other. This is, however, neither a fraction situation nor a rate situation.

A careful analysis of situations of these types shows that although they are quite different, when they are expressed mathematically there are some basic similarities in the way the symbols are treated. *There are also some basic differences.* In the past these situations have not been adequately explained to children, and the result has been not only lack of understanding but actual confusion and the abandonment of efforts to understand.

In *Seeing Through Arithmetic 5* a beginning is made toward the development of those basic understandings. These ideas are not difficult for children who meet them initially under proper guidance and who learn through carefully planned activities. To older children and adults the ideas sometimes seem more difficult than they really are, in part because some of these persons have more or less successfully developed formal ways of dealing with the situations without understanding them. In *Seeing Through Arithmetic 5* a foundation is laid for the development of broader and richer understandings of these concepts in later grades. The possession of these understandings by the pupil will have far-reaching consequences in problem solving and in his later ability to grow in mathematical insight. Since in the past these ideas have never been adequately clarified, failure to under-

stand fractions and these related basic ideas is natural and not surprising. Instructional programs in arithmetic have been so eager to rush on to computation with fractions that they have neglected these basic insights. In contrast, *Seeing Through Arithmetic 5* provides learning materials that develop, by visual and other methods, understanding and insight concerning these ideas. (See pages 112-123, and also 126-133.) Then, and not until then, instruction in computation is provided. These learning materials on computation also emphasize the development of insight, by visual and other methods, as an essential element in understanding and remembering the various operations (reducing, adding, etc.) with fractions and mixed numbers.

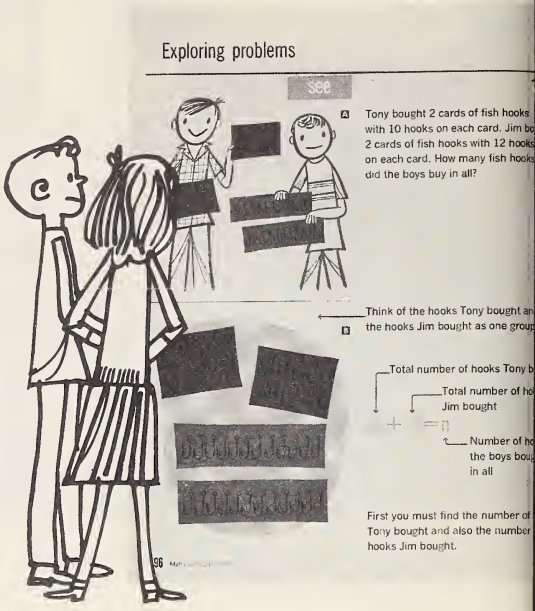
In this brief introduction, it is not possible to discuss at length these and other similar features of *Seeing Through Arithmetic 5*. A careful reading of the pupils' textbook and of the more extended discussions in later sections (for example, notes for pages 112-131, 215-230) of this *Teaching Guide* will, however, reveal the care that has been taken to provide the best possible materials for learning to understand fractions, rates, and comparisons and the way symbols for them are used to solve problem situations. The distinction between the ratio (as a symbol for rate and comparison) and the fraction numeral is important.

2 Problem solving

In *Seeing Through Arithmetic 5* insight into problem solving is fostered by the systematic use of distinctive methods of teaching. First, the pupil is helped to recognize what is happening in the problem situation. Second, he is taught to describe the situation by using arithmetic symbols. This symbolizing includes the use of very simple equations. Third, the pupil is shown how to select the proper arithmetic process to solve the equations.

Analyzing the problem situation

In the past, little has been done to help pupils develop skill in analyzing problem situations. The instructional methods have relied heavily upon "cue" words. (Examples: How many *in all?* How many are *left?*) They also have relied on advice which is of no real help to the pupil. (Examples: Read the problem


Exploring problems
See
A
Tony bought 2 cards of fish hooks with 10 hooks on each card. Jim bo 2 cards of fish hooks with 12 hooks on each card. How many fish hooks did the boys buy in all?
Think of the hooks Tony bought an the hooks Jim bought as one grou
B
Total number of hooks Tony b
Total number of ho Jim bought
Number of ho the boys bou in all
First you must find the number of Tony bought and also the number hooks Jim bought.
96

carefully. Think what to do with the numbers.) The difficulty of many problems lay in the pupil's inability to analyze them.

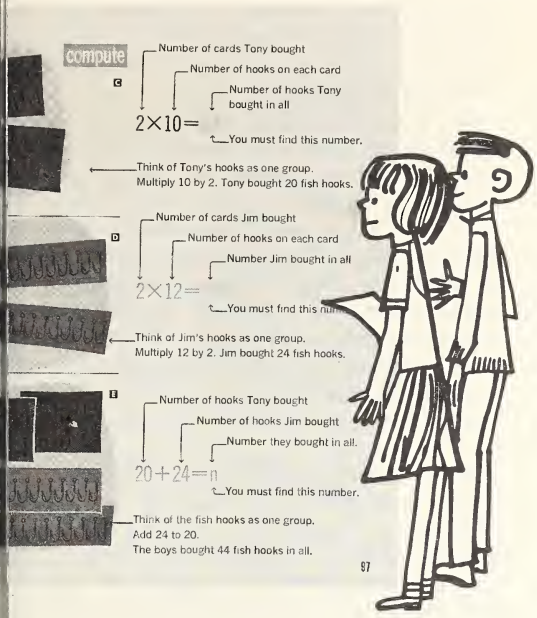
To be successful at problem solving, the pupil must be able to recognize the type of action that takes place in a particular problem. Moreover, he should know that many seemingly different situations all embody the same type of action. For example, each of the following situations suggests the combining of two groups.

(a) Mary had 9 apples. Tom gave her 3 apples. How many apples did Mary have then?

(b) John has 26 cents. He wants a kite which costs 37 cents. How much more money must he get before he can buy the kite?

(c) Sam had some marbles. A friend gave him 18 more marbles as a birthday gift. After accepting them, Sam said, "Now I have 29 marbles." How many marbles did Sam have before he received the gift?

These situations, and many similar situations, all involve the same type of action—they are structurally similar. The action in example (a) may be symbol-


compute
Number of cards Tony bought
Number of hooks on each card
Number of hooks Tony bought in all
2×10=
You must find this number.
Think of Tony's hooks as one group.
Multiply 10 by 2. Tony bought 20 fish hooks.
Number of cards Jim bought
Number of hooks on each card
Number Jim bought in all
2×12=
You must find this number.
Think of Jim's hooks as one group.
Multiply 12 by 2. Jim bought 24 fish hooks.
Number of hooks Tony bought
Number of hooks Jim bought
Number they bought in all.
20+24=n
You must find this number.
Think of the fish hooks as one group.
Add 24 to 20.
The boys bought 44 fish hooks in all.
97

ized by the equation $9+3=n$. The action in Example (b) may be symbolized by the equation $26+n=37$. The action in Example (c) may be symbolized by the equation $n+18=29$. To understand problem solving, the pupil must not only recognize the basic structural nature of such situations as those described above—they are all additive—but also be able to express them in equation form and learn the appropriate computational procedure in each case.

In other situations a single group is seen at first. If part of this group is taken away, a *subtractive* action occurs. These situations are all structurally similar and can be symbolized by equations of the following types: $26-11=n$, $38-n=11$, and $n-42=66$.

In still other situations, actions occur that are *multiplicative* or *divisive*. Such actions, real or imagined, are always important aspects of the total situation. The child, learning arithmetic with insight, learns to recognize these four classes of situations (additive, subtractive, multiplicative, and divisive). In each of the four classes, the situations are similar in structure and each can be represented by an appropriate equation.

The teaching of problem solving in *Seeing Through Arithmetic 5* is based on clear distinctions among: (1) the recognition of the structural nature of the situations, (2) the use of symbols to show what is happening, and (3) the processing of the symbols, i.e., adding, subtracting, multiplying, and dividing. These distinctions are important because the process used is not always what the action of the situation seems to suggest. For example, there are problems in which the child is usually taught to subtract, but which, as he sees them, are additive situations. Thus, consider the following situation: "John has 25 cents, and a toy truck that he wants to buy costs 45 cents. How much more money does he need?" There is evidence in the research on arithmetic that children recognize the structure of the situation as additive and tend to think of this as an additive problem.* This tendency is not at all surprising. Some money must be combined with the 25 cents to make the needed 45 cents. All the child's previous experiences have taught him to think of the combining of two groups as an additive situation. Too often this difficulty is not recognized, and the child is told only that this is a subtraction problem. Confusion and lack of understanding naturally result.

In *Seeing Through Arithmetic 5* the child is not only permitted to think of the above situation as additive—he is taught to think this way. Although the *situation* is admitted to be additive, he learns that the answer is obtained by the process of *subtraction*. He is given help in analyzing the situation and gains insight concerning the nature of the solution. In short, he learns how to see it and how to think about it. Similar principles apply to the analysis of all problem situations.

In the *Seeing Through Arithmetic* program, problems of this type were introduced in Grade 3 and retaught in Grade 4. Only a brief review is necessary in Grade 5 (see page 20 of the pupils' book).

Teachers often say the difficulty in problem solving is that "the pupils cannot read the problems." The pupils usually recognize the individual words; what

*See, for example: Gibb, E. Glenadine, "Children's Thinking in the Process of Subtraction." Unpublished doctoral dissertation, University of Wisconsin, Madison, Wisconsin. Lazerte, M. E., *The Development of Problem Solving Ability in Arithmetic*. Clarke Irwin and Co., Ltd., Toronto, 1933. Crumley, Richard D. Unpublished doctoral dissertation, University of Chicago, Chicago, Illinois, 1956.

the teacher means is that the pupils cannot see the structure of the *situations* and symbolize them appropriately. Dependence on "cue" words or other superficial techniques leaves the pupils in a weak and helpless position. They must learn to base their decisions about which process to use on an understanding of the total situation. The development of such an understanding on the part of the child has been of primary concern to the authors of this text.

The basic types of problems introduced in each grade are selected from a complete list of such types. A table of these types appears on pages 388-389 of this *Teaching Guide*. This list, which is the result of an analysis by the authors, provides a guide for a consistent and comprehensive treatment of problem solving. The eight simplest types of problems were presented in *Seeing Through Arithmetic 3*. These were reviewed and three new types were introduced in *Seeing Through Arithmetic 4*. These are all reviewed or retaught, and several new types are introduced in *Seeing Through Arithmetic 5*.

The reader should not infer from this organization for teaching purposes that the pupil is expected to recognize each problem by its type and follow a routine procedure for that type. Some problems can be correctly viewed in more than one way. What is essential is that the pupil who adopts a possible view knows what to do with the situation as he sees it.

Describing the situation by using symbols

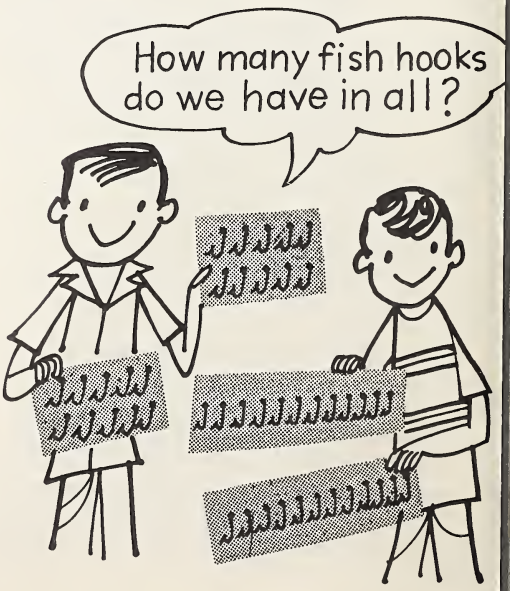
As a second major aspect of problem solving, the child learns how words and other symbols are used to say or write what happens. In particular, he uses number symbols (15, 31, 5, etc.) to tell how many are in a group of objects. Other symbols ($+$, $-$, $\times$, $\div$) are used to express the actions. Thus the equation $27+46=\square$ shows that a group of 46 objects is joining a group of 27 objects and that the number of the combined group is wanted. In every case the expressions derive their meaning from the situations out of which they are seen to arise.

In earlier grades of the *Seeing Through Arithmetic* program a "screen" ($\square$) has been used as a symbol for an unknown number. This "screen" could always be read "what number." In *Seeing Through Arithmetic 5* the symbol n for an unknown number is introduced, and it also can be read "what number." The use of these symbols introduces no difficulties that are not also encountered when a blank such as in $5+\_\_=8$, or a question mark, as in $5+?=8$, or any similar device is used. Such devices have long been commonplace in arithmetic.

In *Seeing Through Arithmetic 5*, lessons entitled "Exploring problems" show the pupil how to write simple equations to describe the problem situations. This part of the lesson is labeled "See." The lessons lead him to *analyze the situation* instead of rushing on to a computational process that may or may not be appropriate. The equation focuses attention on understanding the situation and serves as a record of how the pupil sees and thinks about the situation. If the pupil cannot analyze the situation, any attempt to do the computational step is a blind and often futile procedure.

Processing the arithmetic symbols

As the third major aspect of problem solving, the child learns to go through the appropriate process with the number symbols. In the simplest situations this involves the direct use of the addition, subtraction, multiplication, or division process as indicated by the equation. In other situations, as was noted above, the process is not the one suggested by the



action in the situation. In these cases the pupil must be helped to see why another process can be used. In *Seeing Through Arithmetic 5,* he is shown how to think about the situation in a way that reveals the appropriate process. He is then ready to compute with insight into the total situation. This part of the problem-solving lesson is labeled "Compute."

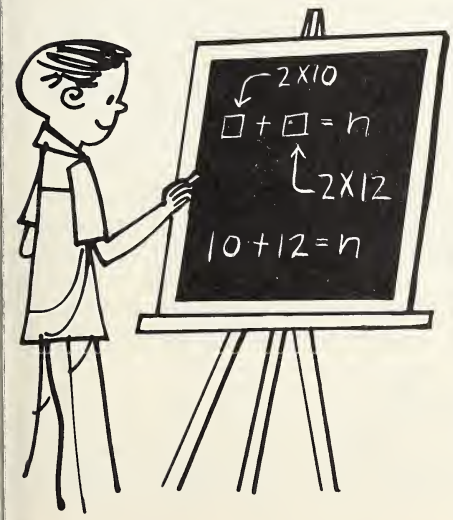
Solving more complex situations

Certain other features of the way problem solving is taught are also noteworthy. One of these is that the pupil is frequently expected to use data that were given (or found by computation) in preceding problems. This resembles the situation in real life, where problems do not always occur in isolation. Instead, several related problems are often involved in a total situation.

Another important feature is the manner in which multiple-step problems are analyzed. The pupil is encouraged to see the situation as a whole from the outset. For example, suppose the problem reads as follows: "Tony bought 2 cards of fish hooks with 10 hooks on each card. Jim bought 2 cards of fish hooks with 12 hooks on each card. How many fish hooks did the boys buy in all?" The dominant aspect of this problem is that the total of two groups of unknown size is to be found—the situation is basically additive. Another and secondary aspect of the situation is that each of the groups can be viewed as formed by multiplicative action. The pupil who sees the situation this way knows all the steps he is going to follow *before* he begins to compute. This is in contrast to the pupil who starts to compute more or less blindly with no clear idea as to what the subsequent steps are to be.

In Book 5 the screen (▤) is used in the equation to hold a place for a number that will be found by computation. In the equation for the problem cited above, one screen is used in the equation for Tony's hooks, and a second screen is used for Jim's hooks. The numbers that replace these screens are found by computation. Then the equation is rewritten with the numerals for the numbers inserted.

In *Seeing Through Arithmetic 5* pupils are taught to analyze multiple-step problems by methods that emphasize the total situation and then fit the details of the separate steps into this larger picture. Equations are used to symbolize all problem situations. This procedure becomes increasingly meaningful and useful as problems involving rates, comparisons, and fractions are all seen to involve similar equations.



3 Simplified organization

Organization of the learning situation is of great importance in fostering insight. For several decades the trend in arithmetic was toward increasing complexity in the organization of topics. This came about because the stimulus-response theory of learning dominated instruction in arithmetic. With this theory as a basis, drill played a dominant rôle. Every desired response was specifically identified and connected to a stimulus for it.

The process of multiplication, for example, was usually separated into many—a dozen or more—different "skills." Multiplication of two-figure numbers *with* carrying was delayed for weeks or even months after multiplication *without* carrying had been taught.

Because of these traditional organizations, children often failed to understand the nature of the task as a whole. For example, when multiplication without carrying comes first, the pupil cannot understand why he is told to begin multiplying with the "ones." He

can find the right answers by working with the "tens" first. He can work the examples either way he pleases and often forms bad habits that escape notice by the busy teacher. In the long run, lack of insight makes the situation more difficult for both pupil and teacher.

In *Seeing Through Arithmetic 5,* the multiplication of general two-figure numbers is introduced with an example that involves carrying.* When the child understands the way ones in excess of 10 must be reorganized to make tens—the major difficulty in multiplication of two-figure numbers—he has the main idea. He sees the "whole" situation. Learning to multiply by working many examples without any carrying before meeting one that does require carrying develops an incomplete concept of the process. It is like learning to drive on a street where, by special arrangement, the traffic control lights are *all* green and cease to be observed by the driver.

The method of teaching long division is especially noteworthy. It enables pupils to gain a general understanding of *the nature of the process* before they are faced with some of the difficulties inherent in it. In particular, the determination at each step of the "correct" quotient figure is not necessary. Even mature and skilled computers do not always find the "right" quotient figures immediately. The long division method used in *Seeing Through Arithmetic 5* enables pupils to compute correct answers with understanding, but without unnecessary stumbling over quotient difficulties. Pupils are then shown gradually how to improve their estimates of the quotient figure.

In brief, learning tasks are so organized as to develop *insight* and to enable the pupil to use it as a means to further learning. The effect is then cumulative: better understanding at each stage contributes to better understanding at following stages. The result is that fewer treatments of a topic are needed, and the organization of the subject as a whole is simplified. These are only a few of the many examples of reorganization suggested by psychological principles concerned with the development of insight. The simplification in organization discussed above is feasible in this program because new visual methods of presentation and provision of a good understanding of the number system are used to open the door to insight.

*See *Seeing Through Arithmetic 5,* pages 48-52.

4 Effective methods

In *Seeing Through Arithmetic 5,* the learning of arithmetic is made efficient and enjoyable for most children by a variety of new methods of presentation. These methods include extensive creative use of the pictorial and typographic arts and many other features of presentation, emphasis, and organization. Introduction of these features is based on accepted principles of learning theory. These methods are used systematically, not erratically or superficially. A complete discussion of all of these methods would unduly extend these introductory comments, but some general remarks will be helpful here.

Presentation of processes

The "four-step teaching method" is an important feature of *Seeing Through Arithmetic 5.* This method is used in sections called "Learning how," in which the processes are taught. (See, for example, pages 48-52 of the pupils' textbook.) In Step 1, called "See," the pupil is shown in detail what to do. Pictorial representations show what happens to the objects involved. The process with the symbols is shown and is closely connected with the visual representation. Verbal explanations are included, but they are simple and brief. In Step 2, called "Think," another example is worked out and the pupil is asked some questions about important details. He is encouraged and helped to *think* about the example. In Step 3, called "Try," other completely worked out examples are given, but the pupil is expected to work these out for himself and then compare his work with the completed examples in his book. In Step 4, called "Do," examples for practice are given.

The "four-step method" has proved its effectiveness through years of use. In *Seeing Through Arithmetic 5* it has been adapted to new visual techniques and modern principles of learning theory. It guides the pupil so that, literally, he can see, think, try, and really do a process on his own and with confidence.

Visual aids

In *Seeing Through Arithmetic 5,* all pictures are learning aids. No picture is presented for its purely decorative or presumed interest-generating value. In many cases the pictures show both the basic data and

Learning how In this lesson you will learn how to subtract fractions.

See

A Mrs. Price has $\frac{5}{6}$ of an apple pie. She plans to use $\frac{1}{2}$ of the whole pie for lunch. What part of a pie will be left?

$\frac{5}{6} - \frac{1}{2} = \square$ ← You must find this number.

← Take $\frac{1}{2}$ of a pie from $\frac{5}{6}$ of a pie.

← You do not know what fraction of a whole pie will be left.

You will subtract $\frac{1}{2}$ from $\frac{5}{6}$. Change the fraction numerals before you subtract.

← Think of the whole pie as divided into sixths. Then there will be 4 pieces, or 4 sixths, in the part Mrs. Price has.

There will be 3 pieces, or 3 sixths, in the part she plans to use for lunch.

Before you subtract, you change the fraction numerals so that they will have a common denominator. 6 is the smallest common denominator.

$\frac{5}{6} - \frac{1}{2} = \frac{5}{6} - \frac{3}{6}$

← Take 3 sixths of the pie from the 4 sixths. One sixth will be left.

Subtract the numerators to find how many sixths are left. One sixth is left.

$\frac{5}{6} - \frac{3}{6} = \frac{1}{6}$ ← Write 1 as the numerator of the answer. You are subtracting sixths. So write 6 as the denominator.

Mrs. Price will have $\frac{1}{6}$ of a pie left.

$\frac{5}{6} - \frac{1}{2} = \frac{1}{6}$

think

Mrs. White has $\frac{5}{6}$ of a cake. She plans to put $\frac{1}{2}$ of the whole cake in Tony's lunch box. How much of a whole cake will she have left?

$\frac{5}{6} - \frac{1}{2} = \square$ ← You must find this number.

What must you do with the fraction numerals before you subtract?

Why do you think of the whole cake as divided into twelfths? How many twelfths are there in the part that Mrs. White has?

In the part she is going to put in Tony's lunch box there are [6] twelfths.

Why can you use 12 as the common denominator when you subtract?

$\frac{5}{6} - \frac{1}{2} = \frac{5}{6} - \frac{6}{12}$

Now turn the page. ☞

← Take [6] twelfths of the cake away from the twelfths. [6] twelfths will be left.

$\frac{5}{6} - \frac{1}{2} = \frac{5}{12} - \frac{6}{12} = \frac{\square}{12}$

What do you do with the numerators?

What do you write as the numerator of the answer? Why do you write 12 as the denominator?

Mrs. White will have $\frac{\square}{12}$ of a whole cake left.

$\frac{5}{6} - \frac{1}{2} = \frac{\square}{12}$

try

A $\frac{5}{6} - \frac{1}{3} = n$

Use 24 for the denominator when you subtract. Why?

$\frac{5}{6} - \frac{1}{3} = n$

$\frac{\square}{24} - \frac{\square}{24} = \frac{\square}{24}$

You may write your work this way if you want to.

$\frac{5}{6} = \frac{\square}{24}$

$\frac{1}{3} = \frac{\square}{24}$

$\frac{\square}{24} - \frac{\square}{24} = \frac{\square}{24}$

B $\frac{11}{12} - \frac{2}{24} = n$

Why do you [illegible] common numerals?

[illegible] do you k[illegible] for $\frac{8}{12}$?

[illegible] simplest

Your work may be written this way.

$\frac{11}{12} = \frac{\square}{24}$

$\frac{2}{24} = \frac{\square}{24}$

$\frac{\square}{24} - \frac{\square}{24} = \frac{\square}{24}$

C $\frac{3}{4} - \frac{1}{8} = n$

What [illegible] you use?

[illegible] you use?

You may write your work in this way.

$\frac{3}{4} = \frac{\square}{8}$

$\frac{1}{8} = \frac{\square}{8}$

$\frac{\square}{8} - \frac{\square}{8} = \frac{\square}{8}$

D $\frac{11}{12} - \frac{1}{6} = n$

What common denominator do you use? Why do you not change $\frac{11}{12}$?

$\frac{11}{12} - \frac{1}{6} = n$

$\frac{11}{12} - \frac{\square}{12} = \frac{\square}{12}$ or $\square$

Your work may be written this way.

$\frac{11}{12} = \frac{\square}{12}$

$\frac{1}{6} = \frac{\square}{12}$

$\frac{\square}{12} - \frac{\square}{12} = \frac{\square}{12}$ or $\square$

do

A Jim bought $\frac{3}{4}$ lb. of peanuts. He took $\frac{1}{2}$ lb. to the park to feed the squirrels. He had what fraction of a pound of peanuts left?

$\frac{3}{4} - \frac{1}{2} = n$

B Susan had $\frac{7}{8}$ yd. of blue cloth. She used $\frac{3}{4}$ yd. to make a dress for her doll. How much blue cloth did Susan have left?

$\frac{7}{8} - \frac{3}{4} = n$

Show your answers in simplest form.

E	$\frac{1}{2} - \frac{1}{3} = n$	F	$\frac{5}{6} - \frac{2}{3} = n$	G	$\frac{7}{16} - \frac{1}{8} = n$	H	$\frac{3}{16} - \frac{1}{4} = n$	I	$\frac{7}{10} - \frac{2}{5} = n$
J	$\frac{3}{5} - \frac{1}{2} = n$	K	$\frac{11}{12} - \frac{1}{2} = n$	L	$\frac{1}{2} - \frac{1}{10} = n$	M	$\frac{1}{2} - \frac{1}{6} = n$	N	$\frac{11}{16} - \frac{3}{8} = n$
O	$\frac{7}{10} - \frac{3}{5} = n$	P	$\frac{1}{4} - \frac{1}{5} = n$	Q	$\frac{1}{10} - \frac{1}{15} = n$	R	$\frac{5}{8} - \frac{1}{2} = n$	S	$\frac{7}{8} - \frac{3}{8} = n$
T	$\frac{5}{8} - \frac{1}{4} = n$	U	$\frac{3}{5} - \frac{1}{5} = n$	V	$\frac{7}{10} - \frac{2}{5} = n$	W	$\frac{7}{10} - \frac{1}{2} = n$	X	$\frac{5}{6} - \frac{1}{2} = n$
Y	$\frac{5}{6} - \frac{2}{3} = n$	Z	$\frac{13}{16} - \frac{3}{4} = n$	AA	$\frac{1}{2} - \frac{2}{5} = n$	BB	$\frac{5}{6} - \frac{1}{3} = n$	CC	$\frac{7}{8} - \frac{3}{4} = n$
DD	$\frac{3}{4} - \frac{1}{4} = n$	EE	$\frac{3}{5} - \frac{1}{5} = n$	FF	$\frac{3}{5} - \frac{1}{10} = n$	GG	$\frac{1}{2} - \frac{1}{4} = n$	HH	$\frac{13}{16} - \frac{1}{4} = n$

Now you should be able to subtract one fraction from another.

Keeping skillful

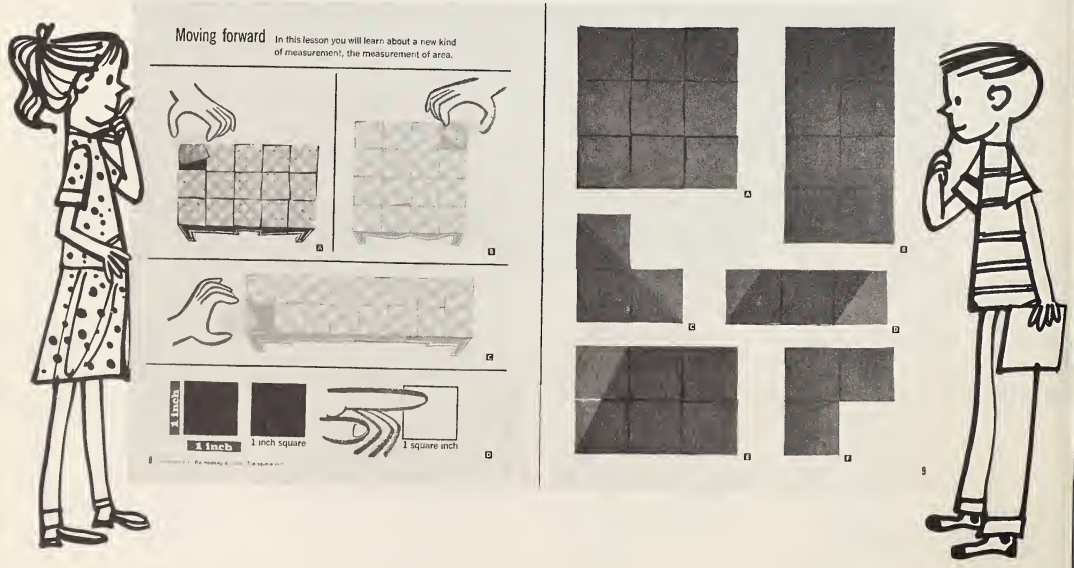
A	$\frac{3}{4} + \frac{5}{8} + 1\frac{1}{4} = n$	A	$n - 4\frac{1}{2} = 6\frac{1}{4}$	H	$8\frac{1}{12} + 5\frac{1}{12} = n$				$= n$
B	$2\frac{1}{6} + 5\frac{1}{2} = n$	B	$11\frac{11}{12} + 4\frac{1}{2} = n$	I	$n - \frac{1}{2} = \frac{1}{16}$				$= 2863$
C	$\frac{1}{4} + \frac{1}{2} + \frac{3}{8} = n$	C	$n - 1\frac{1}{8} = 2\frac{3}{4}$	J	$4\frac{1}{2} + 2\frac{1}{4} = n$				$96 = n$
D	$8\frac{1}{2} + 7\frac{1}{2} = n$	D	$\frac{11}{12} + \frac{1}{2} + \frac{1}{3} = n$	K	$\frac{7}{8} + 1\frac{3}{8} = n$				437
E	$\frac{7}{16} + \frac{3}{8} + \frac{1}{2} = n$	E	$n - \frac{11}{16} = 3\frac{1}{2}$	L	$n - \frac{1}{2} = \frac{3}{4}$				401
F	$4\frac{1}{4} + 6\frac{3}{4} = n$	F	$n - \frac{1}{6} = 4\frac{11}{12}$	M	$\frac{1}{4} + 1 + \frac{1}{16} = n$				
G	$3\frac{1}{2} + 1\frac{7}{16} = n$	G	$\frac{1}{4} + 1 + \frac{1}{16} = n$	N	$4\frac{1}{4} + 8\frac{1}{4} = n$				

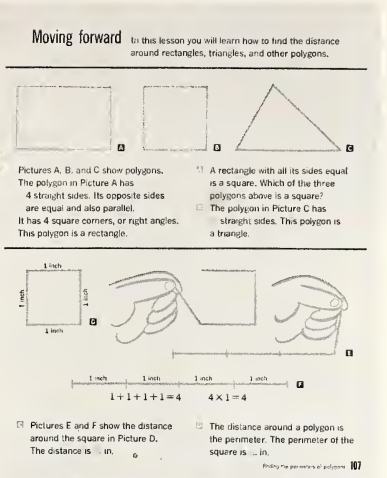
the action—that is, the grouping and regrouping involved in the problem situation. The child can actually see what is happening. Moreover, because many lessons employ a sequence of related scenes, or "movies," they serve as a guide to teachers and pupils who wish to use actual objects to gain additional experience with the ideas. The Expanded Notes, which begin on page 243, contain many suggestions for this kind of enrichment.

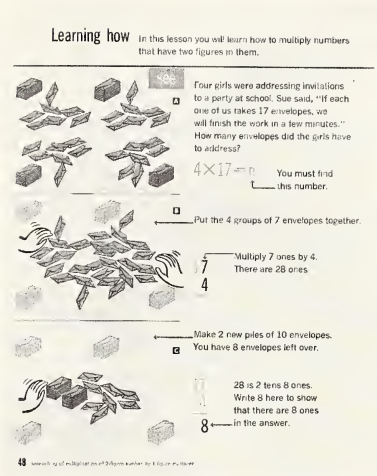
In similar fashion, the use of colors and of many special printing devices is the result of careful study. Color is used not only to add general attractiveness or to call attention to some special point, but also to help teacher and pupils communicate with each other. For instance, the teacher may say, "Now we will do the examples beside the red letters." Special printing devices are used to "dim out" of pictures everything that is not to be in the focus of attention. The design of every page is worked out with teaching values as a dominant criterion. The goal is to help the child see—visually and intellectually—what he is asked to learn.

The value of visual learning aids is now well established. Film strips and moving pictures are increasingly popular. The visual aids in this book have a distinct advantage, however, since teachers and pupils can study them as long and as often as necessary. Arithmetic requires study. Pictures and other visual devices, when they are properly planned, make this study possible and fruitful. In his study time, each pupil can take as much time as he needs on each scene. This contrasts sharply with visual aids which move from scene to scene at a speed supposedly suitable for the average pupil, but which is certainly too slow for the able pupil and much too fast for the slow pupil.

Illustrations of the kind used in this program are an important reading aid. Much of the difficulty encountered in reading arithmetic materials results from the pupil's inability to visualize. Pictures carefully related to the text aid in visualizing. Moreover, since the visual aids reduce the reading load and aid the comprehension of the pupils, they can later return to earlier lessons for review and restudy and use the materials effectively for this purpose. The value of an ordinary illustration is soon used up, but the value of carefully worked out visual devices can continue long after they are first used.


Moving forward
In this lesson you will learn about a new kind of measurement, the measurement of area.
1 inch
1 inch
1 inch square
1 square inch


Moving forward In this lesson you will learn how to find the distance around rectangles, triangles, and other polygons.
Pictures A, B, and C show polygons. The polygon in Picture A has 4 straight sides. Its opposite sides are equal and also parallel. It has 4 square corners, or right angles. This polygon is a rectangle.
A rectangle with all its sides equal is a square. Which of the three polygons above is a square?
The polygon in Picture C has straight sides. This polygon is a triangle.
1 inch
1 inch
1 inch
1 inch
1 inch
1 inch
1 inch
1 inch
1+1+1+1=4
4×1=4
Pictures E and F show the distance around the square in Picture D. The distance is in.
The distance around a polygon is the perimeter. The perimeter of the square is in.
107


Learning how In this lesson you will learn how to multiply numbers that have two figures in them.
Four girls were addressing invitations to a party at school. Sue said, "If each one of us takes 17 envelopes, we will finish the work in a few minutes." How many envelopes did the girls have to address?
4×17=n
You must find this number.
Put the 4 groups of 7 envelopes together.
Multiply 7 ones by 4. There are 28 ones.
7
4
Make 2 new piles of 10 envelopes. You have 8 envelopes left over.
28 is 2 tens 8 ones. Write 8 here to show that there are 8 ones in the answer.
8
49

The number system

The *Seeing Through Arithmetic* program provides an unusually careful development of the number system. Many pictures showing the grouping and regrouping of objects in tens, in hundreds, and so on, are accompanied by corresponding arrangements of numerals in terms of place-value ideas. (See, for example, pages 12 to 15 of the pupils' textbook.) Seeing numbers as members of a system involves seeing *relations,* and thus requires much more than knowing the names of the "places" in written numerals. There is much talk nowadays about teaching the number system. *Seeing Through Arithmetic* teaches it thoroughly and distinctively.

In *Seeing Through Arithmetic 5,* the major extension of the number system involves common fractions. Books for preceding grades have included work designed to provide readiness for the more comprehensive study of fractions in Grade 5. As stated above, the emphasis is upon the development of genuine understanding of fractions before computational work with them is demanded. In the book for Grade 6, decimal fractions will be taught as an extension of the number system.

Vocabulary control

The purpose of studying arithmetic is to learn how to solve quantitative problems. A vocabulary and general reading abilities are of course necessary. However, since the task of learning arithmetic is sufficiently difficult for the child, it is foolish to add extra burdens in the form of vocabulary and reading difficulties. For this reason the vocabulary of *Seeing Through Arithmetic 5* is carefully controlled.

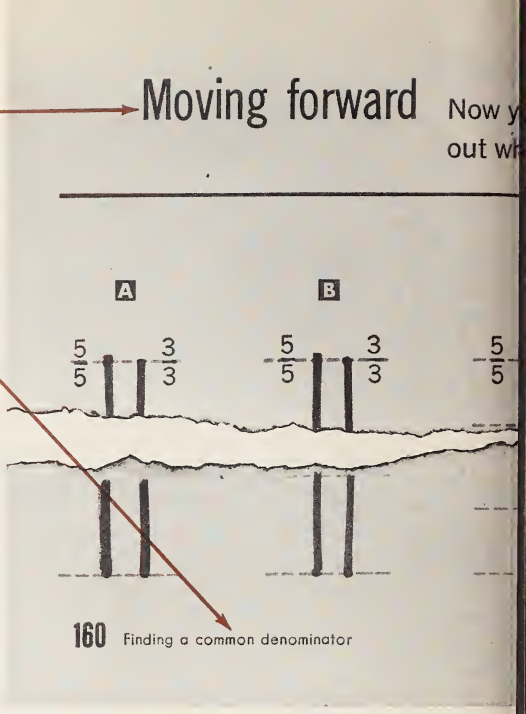
In *Seeing Through Arithmetic 5,* it is assumed that the children know only the 658 reading words (both arithmetical and general) that appeared in the preceding books of the *Seeing Through Arithmetic* program. These words are listed alphabetically on pages 362-364 of the *Teaching Guide* for *Seeing Through Arithmetic 4.* All other words are considered new words and are listed, for special attention by the teacher, at the beginning of the Lesson Briefs for each block of work. New words are introduced at a cautious rate. The total vocabulary of *Seeing Through Arithmetic 5* is 979 words. See pages 392-395 of the *Teaching Guide* for a complete alphabetical list of the words used and for detailed statistics on the vocabulary.

Lesson headings

Seeing Through Arithmetic 5 uses headings that describe the nature of the various types of lessons. They serve to alert the child immediately as to the nature of the lesson—learning a new computational skill, practicing problem solving, maintaining skills, and so forth. In connection with these headings, there is, whenever necessary, a statement to the teacher (in small type opposite the page number) that gives a technical description of the content of the lesson. With some of these lessons, too, there is a statement opposite the heading giving the child more information about the nature of the lesson.

These various types of lessons provide a wealth of material for the teaching of concepts, for practice, for maintenance, for review, and for testing achievement. Teachers who want suggestions on the varied uses of these materials will find them in the Lesson Briefs (pages 19-241) and in the Expanded Notes, which begin on page 243, for each section. These materials provide the basis for effective lesson planning by teachers and effective learning by pupils.

All material in *Seeing Through Arithmetic 5* is organized under nine headings:



Learning how	lessons using the four-step teaching method to develop or review computational skills
Exploring problems	lessons developing specific problem-solving techniques
Moving forward	lessons developing other new material, including use of ratios to express rates and comparisons, fractions, and measurement
Using arithmetic	maintenance and practice in solving verbal problems
Keeping skillful	maintenance and practice in computation with whole numbers and fractions
Checking up	tests of various types: inventory, achievement, diagnostic (end-of-block), and problem-solving
Thinking straight	lessons involving special aspects of quantitative thinking
Looking back	lessons involving reteaching or review of previously taught skills
Side trip	lessons on various topics related to the arithmetic program: Roman numerals and time zones

Lesson briefs

This section of the *Teaching Guide* contains the aids that every teacher will need in using *Seeing through Arithmetic 5*. Collected together in one place for your convenience you will find:

Objectives

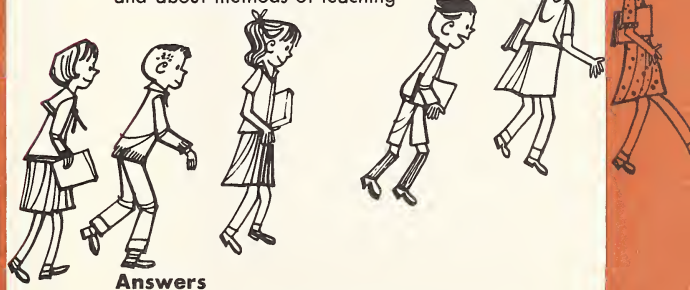
A very brief statement of the purpose of the lesson to help you see its relation to the program

Vocabulary

For each lesson a list of all words not used in preceding books of the program

Comments

Helpful information about the arithmetic itself and about methods of teaching



Answers

Answers for all exercises except basic facts and very simple items

Reproduced pages

A reproduction of each page from the pupil's book for your easy reference

Keyed notes

Brief directions for teaching each page, keyed by numbers to the reproduced page

Lesson briefs

A productive teaching situation includes interested children, an inspired teacher, and good materials. In using the materials of this program, do not forget the Expanded Notes that begin on page 243.

3-7 Moving forward

Expanded Notes for this lesson are on pages 245-251.

Objectives

The child learns to identify circles and many polygons. He learns how circles and some of the polygons studied are measured. He also learns to recognize base and altitude in triangles and parallelograms.

Vocabulary

A vocabulary list like the one below is given in this *Teaching Guide* for each lesson in *Seeing Through Arithmetic 5*. The list consists of words that are introduced in the book and are, therefore, new to this arithmetic program. Words marked with asterisks are already familiar to the children who have completed The New Basic Reading Program through *More Times and Places* (4/2). On pages 392-395 you will find a complete alphabetical list of all new words in this book.

The word *polygon* is not used in the book until page 107, but it should be used orally in this lesson. Use the word *polygon* rather than *figure*, since the latter word may confuse the pupils because of its many meanings. The words *right angle, rectangle, square,* and *triangle* also should be used orally in this lesson.

New words page 7 altitude, diameter, meant*, width

Comments

If you have not already read the introductory pages in this *Teaching Guide,* you will want to do so before proceeding with these comments and the keyed notes. To use the *Teaching Guide* effectively, you should be familiar with the general aims and principles underlying the *Seeing Through Arithmetic* program. See pages 6-18.

For each lesson entitled "Moving forward" and "Learning how" there is a statement beside the heading that tells the children what they are going to learn. At the bottom of the first page of each new lesson there is a description of the contents of the lesson for use of the teacher.

In the lesson on pages 3-7 the children are introduced to various types of geometric shapes found in their everyday surroundings. Through discussion they learn that these familiar shapes have names, and through observation they note several distinguishing features. They learn that rectangles have four sides and square corners; that squares are rectangles that have four sides of the same length; that some four-sided figures are not rectangles because they have opposite sides of different lengths or do not have square corners; that a pentagon has five sides, a hexagon has six sides, and an octagon has eight sides; that all points on a circle are the same distance from a point within, called the center, and all diameters in a particular circle are the same length.

Through discussion and comparisons, help the children to discover various kinds of four-sided polygons and triangles, to see the difference between the circle and the ellipse, and to distinguish between hexagons that have sides the same length and those that do not (see top of page 4). Some children in the class will already be acquainted with words like *pentagon* and *hexagon.* Others will be curious to know what the polygons are called. Feel free to use these terms with the children when they ask about them, but you should not insist that the name of each kind of polygon be learned.

After the children are able to recognize and name the circle, square, rectangle, and triangle, develop the idea that each geometric shape can be measured, that each has dimensions. Let the children begin with the length and width of rectangles and squares. They can easily see in these two shapes that the altitude meets the bottom, or base, at right angles (makes "square corners"). Help the children see that, when measuring polygons other than squares and rectangles, the altitude is measured on a straight line that lies at right angles to the base and passes through the point of the polygon farthest away from the base.

You will discover that height and altitude are confusing terms, as are length and width. Normally, length is the term for the longer dimension of a rectangle, no matter in what position it is placed. To illustrate this, draw on the board several rectangles representing the same physical thing, but draw them in different positions to show how the longer dimension, or length, may be placed in several different positions. Emphasize that length is the longer dimension of a rectangle and width the shorter. Base is, usually, one of the sides that comes nearer to being horizontal on the paper, and altitude is either one of the two sides perpendicular to the base.

Since circles have no straight sides, the children at first may be in doubt about what measurements can be made pertaining to circles. Help the pupils to see that the width of a circle is measured on a straight line that passes through the center. Show the class that this line, or diameter, may be drawn in various directions through the center of the circle and will always be the same length.

Large cardboard shapes which the pupils can examine and measure should be provided. Also, let the children measure familiar objects found in the classroom.

Answers

Page 7:

A	base u, altitude r or t	H	base y, altitude z
B	base y, altitude z	I	base t, altitude u
C	base z, altitude w or y	J	base y, altitude z
D	base t, altitude u	K	diameter x
E	base t, altitude s	L	diameter s
F	base z, altitude w or y	M	diameter w
G	base u, altitude r or t	N	diameter t or u

3

1 For each "Moving forward" lesson, an opening statement such as this one tells the children what they are going to learn. Have them read and discuss this statement.

2 Before working with this page tell the children that objects in the world about them are made up of many different shapes. Discuss various geometric shapes and provide cardboard models in the shape of circles and polygons for the children to examine.

3 Have the children take turns pointing to and identifying the circles, ellipses, and polygons that they see in this picture. Encourage them to talk about the various circles and polygons in terms of number of sides, differences, and likenesses.

4 Follow the same procedure for this picture.

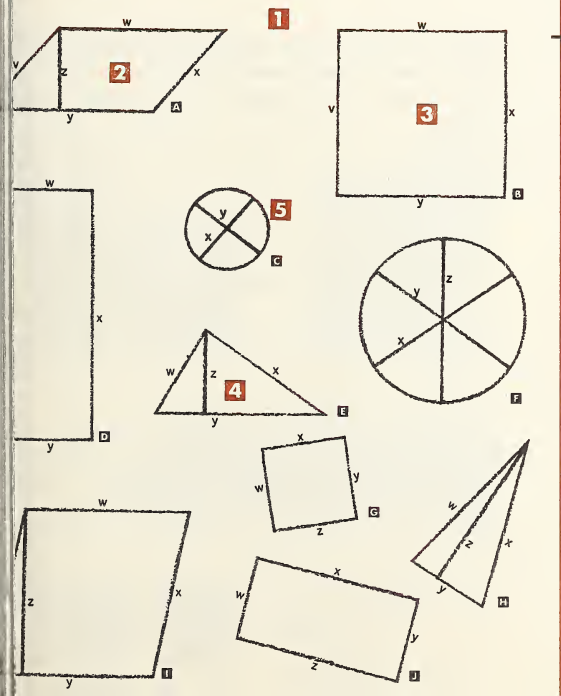
4

1 Compare these examples of polygons for likenesses and differences. Let the children select polygons and show that all of the sides of some of them are equal. They should use rulers or a piece of paper marked off in inches on the edge. Also let them show that some of the polygons have sides that are not equal. Discuss Pictures G and H. Get the children to observe that points on the circle are all the same distance from the center, but that this is not true of the ellipse. Then draw examples of each on the chalkboard. Arrive at conclusions concerning the differences in G and H. Use the word *circle* for G. Permit the pupils to call H an oval.

2 Let the children match the circles and polygons in each of these pictures with a corresponding one above (A to K).

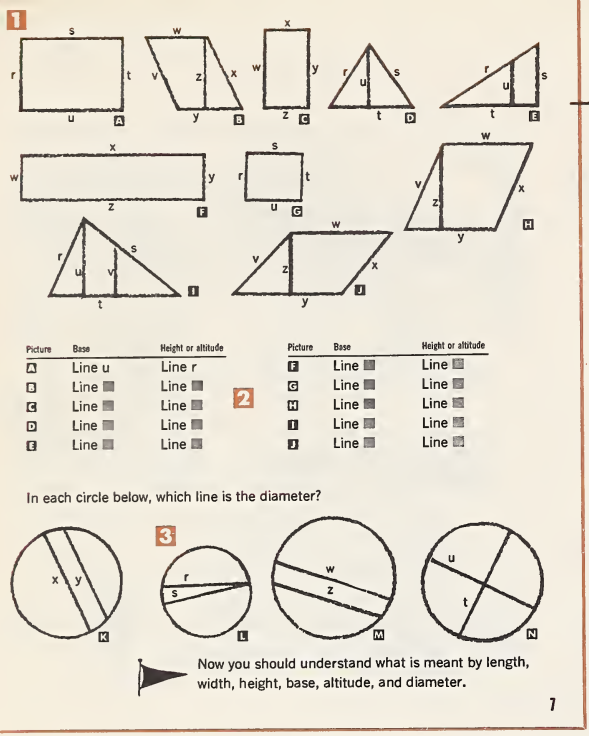
5

1. Explain that the length and width of surfaces that are rectangles are measured along the sides and that these lines must form square corners. Emphasize that the word *length* is used for the longer side and the word *width* for the shorter side. Let the children practice measuring length and width of rectangular surfaces.
2. Explain that the height of the window may be thought of as the altitude of a rectangle. Ask how the length of the base can be found.
3. Have the pupils tell what kind of polygon the children are measuring in this picture.
4. Use a circular piece of cardboard to show what the girl is doing. Use the word *diameter*.
5. Point out that *base* and *altitude* are the correct terms to use with a triangle like this flower bed.
6. Bring out that the dotted line suggests the altitude of a triangle. Help the children find the triangle.



6

1. First discuss polygons in terms of sides and angles. Explain the term *right angle*. Have pupils explain how to recognize a circle.
2. Ask if the sides of this polygon are parallel and if the angles are right angles. Bring out that Line z, which is the altitude, shows how far apart Lines w and y are. Ask the children to compare this with Polygon I and tell how it differs [all sides are equal].
3. Help the children conclude that this rectangle is a square. Ask what Line v or Line x represents [altitude]. Have them find another square [G]. Ask how Polygons D and J are different from Polygons B and G.
4. Help the pupils explain why Line z is the altitude. Help them discover how Triangle H is like Triangle E and how it is different [base shorter, etc.].
5. Ask how many lines are shown for Circle C and for Circle F on which the diameter can be measured.



7

1 Have the pupils identify each polygon (A to J) as a rectangle, square, triangle, or as a polygon with four sides that is not a rectangle and to explain why they think so.

2 Discuss the answers for Exercises A to J. If these exercises are used for written work, have the children put their work in three columns. In the first column have them write the capital letter that identifies the picture, in the second column the letter that indicates the base, and in the third column the letter or letters that indicate the height or altitude.

3 Ask for reasons for identifying Pictures K to N as circles.

8-11 Moving forward

Expanded Notes for this lesson are on pages 251-253.

Objectives

The child learns what is meant by the measurement of area, to use the inch square as a measuring unit, and how to estimate (by counting) the number of square inches in small flat surfaces.

Vocabulary

New words page 8 area; page 11 between*, flat*

Comments

In previous grades the children have learned about the measurement of distance, weight, time, temperature, liquids. They have become aware of the need for standard units of measure and what some of these standard units are. In this lesson the measurement of area is introduced. The work is confined to measuring area by the square inch.

To comprehend the concept of area, pupils must understand the need for a standard unit for measuring amount of surface. Practical situations in which the measurement of area is needed, such as buying paint for the walls of a room, should be discussed. No attempt should be made in this lesson to introduce any computational procedures for finding area or to define the word *surface*. For the present, the study of surface measurement is meant to develop only the meaning of such measurement. Computational procedures will be taught later (pages 240-250 of the text).

In this lesson the only unit of measure introduced is the square inch. The children should have opportunity to estimate small areas in terms of square inches, and to check their estimates by using the inch square.

Inch squares made of tissue paper, cardboard, construction paper, oaktag, etc., can be cut out by the children to use with the work in this lesson. The inch squares can be placed on various small surfaces to measure their area. Surfaces to be measured should not exceed 25 or 30 square inches. Because rectangles and squares are easy to work with, begin with these.

Have many polygons and circles of different sizes available for the children to estimate and measure with their inch squares.

To extend the idea of the square inch, cut an inch square into two parts. Help the children see how the pieces can be put together to form polygons of various shapes but whose area is, of course, a square inch. Apply this idea when estimating the area of pictures like Picture D on page 9 and Picture J on page 10.

When the children have developed an idea of how big a square inch is, let them make judgments as to how many square inches they think are in a particular polygon or circle and make comparisons between one or more polygons or circles.

Do not mention the square foot and the square yard unless the children do. Then discuss these square units, but do not have the children work with them at this time. These units of square measure should not be mentioned at this time because they bring up the problem of changing from one unit to another. The children are not ready to do this.

Answers

Page 11:

A Between 12 and 20 square inches, about 16 square inches

B 24 square inches

C Between 3 and 6 square inches or, better, between 4 and 5 square inches

D 12 square inches

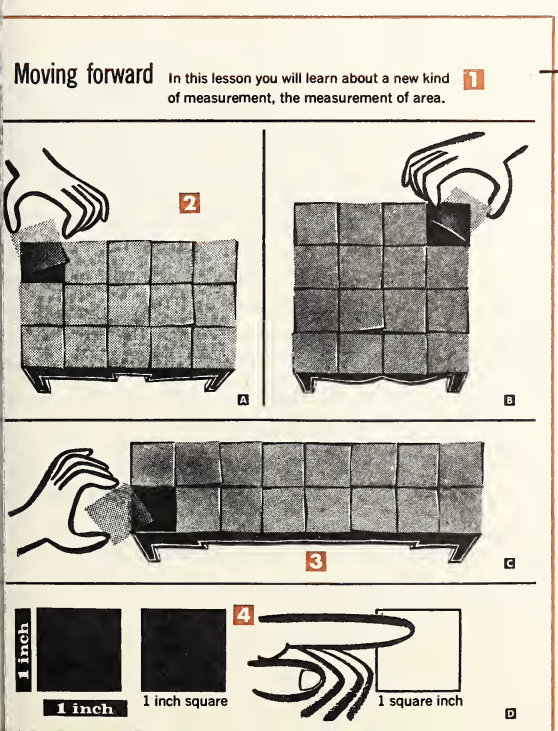
E Between 12 and 16 square inches, about 14 square inches

F 14 square inches

G Between 16 and 17 square inches

H Between 16 and 20 square inches, about 18 square inches

I 12 square inches



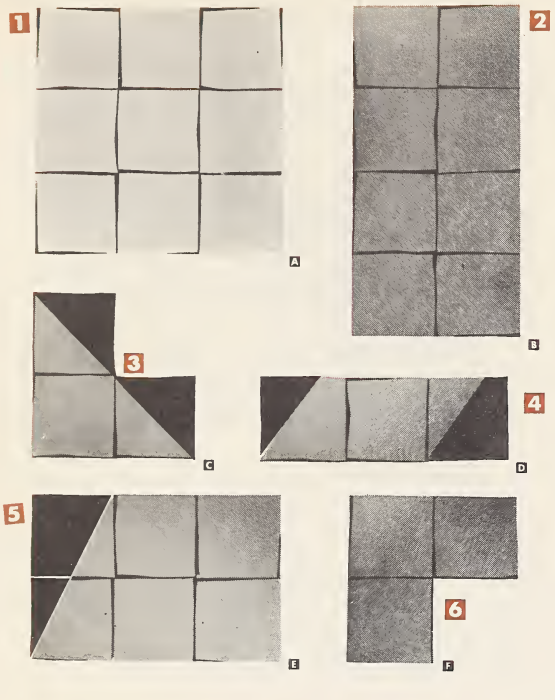
8

1 Discuss the heading and the objective of the lesson. Review briefly the use of linear measurement in determining length.

2 Discuss the difference in the shape of the tops of the doll tables in Pictures A and B. Ask the children to count the squares of tissue on the tables. Ask, "Which table top is larger?" Use the answers to develop the idea of area.

3 Now have the pupils count the squares on this table top. Have them tell how it compares in area with the table top in Picture B. Make sure they understand that shapes may be different and still be equal in area.

4 Point out that each side of a one-inch square is 1 inch long. Let them make one-inch squares of paper or oaktag. Explain that the square inch will be used to measure area.



9

9

1 Make clear that the small squares are square inches. Let the children count the square inches in Picture A. Then have them lay their square inches over it to show that it contains 9 square inches.

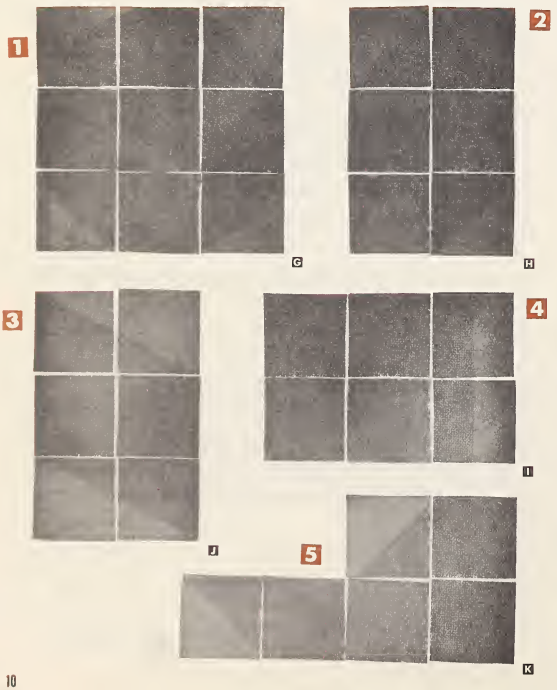
2 Have the children proceed as for Picture A. Compare the sizes and shapes of Pictures A and B.

3 Tell the children that the surface to be measured is indicated by the dotted background. Have them estimate the area. Ask them to cut an inch square diagonally from corner to corner and place the pieces and one full square on the triangle. This will enable them to see that the area of the triangle is 2 square inches.

4 Again let the children estimate the area. Then let them cut one square and proceed as for Picture C.

5 Proceed as for Pictures C and D.

6 Proceed as for Pictures A and B.



10

10

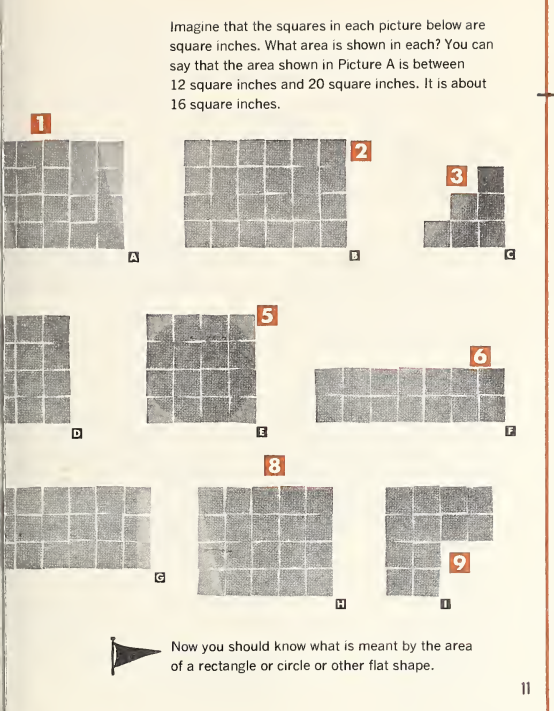
1 Encourage the children to use ingenuity in estimating the area of shapes other than squares and rectangles. Let them place inch squares of paper on the circle to help them estimate that the area is about 7 square inches.

2 Have the children cut inch squares diagonally and place the pieces on the polygon along with 2 complete squares. Help them see that the area is about 4 square inches.

3 Have the children cut inch squares diagonally to fit the polygon. Have them use these pieces, along with 2 whole inch squares, to find what the area is [4 square inches].

4 Let the pupils cut one inch square in half and then place the pieces on it beside the 4 whole inch squares. Ask them what the area of this polygon is [5 square inches].

5 Follow the same procedure as above.


Imagine that the squares in each picture below are square inches. What area is shown in each? You can say that the area shown in Picture A is between 12 square inches and 20 square inches. It is about 16 square inches.
1
2
3
A
B
C
5
6
D
E
F
8
9
G
H
I
Now you should know what is meant by the area of a rectangle or circle or other flat shape.
11

11

1 Explain that each small square represents one inch square or one square inch. Let the children estimate the number of square inches in this polygon [16].
2 Let them estimate the number of square inches this rectangle represents.
3 Ask what the approximate area of the triangle is.
4 Follow the procedure for Picture B.
5 Help the children decide that the circle contains about 14 square inches.
6 Follow the procedure for Picture B.
7 Follow the procedure for Picture A. [Between 16 and 17 square inches]
8 Lead the children to decide that this polygon contains between 16 and 20, or about 18 square inches.
9 Follow the procedure for Picture B.

12-15 Looking back

Expanded Notes for this lesson are on pages 254-258.

Objectives

The child reviews what he has learned about place value in the number system as it relates to regrouping or carrying and borrowing.

Vocabulary

New words page 15 system

Comments

Children in the fifth grade can hardly escape large numbers in their everyday reading of magazines and newspapers, and in various books used in their school work. They need to be able to read large numbers and to understand their meaning.

There are two basic ideas that the child must grasp to understand the number system as a *system*. One is that the same symbol—any one-figure numeral—may be used to represent ones, tens, hundreds, thousands, etc. The other idea is that we know what a symbol represents by its position with respect to other symbols in the numeral.

In the books *Numbers We See*, *Numbers in Action*, and *Seeing Through Arithmetic 3* of this program, children were helped to obtain these basic ideas through work with objects and pictures of objects in groups. They assembled (or saw assembled) ones to make groups of tens, tens to make hundreds, and hundreds to make thousands. Then they learned to use tally marks to represent hundreds, tens, and ones and to indicate what the tallies represented by the places in which they were written. Finally they learned to use numerals in place of the tally marks.

In Book 4 of this program, the children extended their knowledge of the number system to 9 places. Here they worked with, or saw pictures of, boxes labeled "100," "1000," "10 thousand," and "100 thousand" and represented the collection of objects these boxes stood for with number symbols written in the proper places. Just as they had discovered that, for a

numeral of three figures, such as 135, they must say something for each of the figures—"one hundred" for 1, "thirty" for 3, and "five" for 5—they learned that when they see a numeral which has three more of the same figures they still must read the three figures as "one hundred thirty-five" and say the word "thousand." If they saw 135,135, for example, they learned to read it "one hundred thirty-five thousand, one hundred thirty-five." Then they learned that if they have three more figures in the numeral, they read it "one hundred thirty-five *million*, one hundred thirty-five *thousand*, one hundred thirty-five." In this book, children review these ideas as they work with numbers in the thousands and millions.

As you proceed with this lesson, stress the fact that each group of three figures is read in the same way and then the name of the group is given (except for the first group, counting groups toward the left). Point out that the group of three figures second from the right has the name "thousands," and the next group to the left has the name "millions." You might mention that this goes on and on—that each additional group of three figures has a special name.

Since the word "and" is to be reserved for the decimal point in reading numerals, we do not say "and" between the "one hundred" and the "thirty-five" in the numeral 135. Encourage the children to read it "one hundred thirty-five."

On pages 12 and 13, the children work with numerical representations of objects grouped in hundreds, thousands, 10 thousands, and 100 thousands. On page 13 they also practice reading and writing large numbers.

Pages 14 and 15 provide practice in the regrouping basic to an understanding of carrying and borrowing in computation. All too often children have difficulty in working with the processes because they do not understand the principles of the number system underlying carrying and borrowing. The regrouping involved in these procedures is retaught here in connection with the number system so that it can be applied when the processing of larger numbers is studied.

If you find that some of the children have trouble understanding the regrouping procedures used with these larger numbers, review the grouping of objects in tens and ones, the tallying, and changing to number symbols as taught in *Seeing Through Arithmetic 3* and *4*. You will want to give these children plenty of opportunities to make these groupings and regroupings themselves with real objects.

Answers

The practice material in *Seeing Through Arithmetic 5* is arranged on the pages in blocks, which are referred to as Block 1, Block 2, etc., in the answer sections of this *Teaching Guide*. In the text the exercises in each block are identified by distinctively colored letters. On page 13, for instance, Block 1 is labeled with blue letters, Block 2 with black letters, Block 3 with white letters on a black background, Block 4 with white letters on a blue background.

Page 13:

(block 4)

A	3019	E	64,100,004	I	71,030,000
B	620,401	F	7506	J	1,000,900
C	2,080,007	G	408,000,070	K	563,005
D	98,090	H	29,800,000		

Page 14:

(block 1)

C	14	D	6	E	3

Page 15:

(block 1)

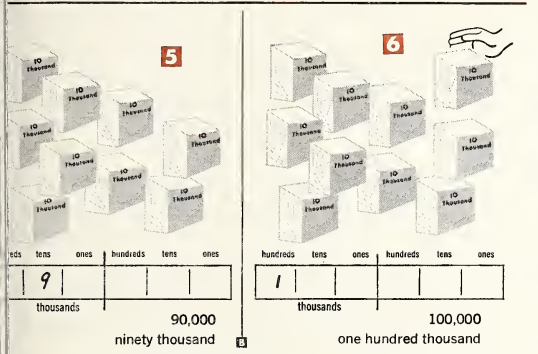
F	12	H	1	J	599
G	6	I	10	K	41

(block 2)

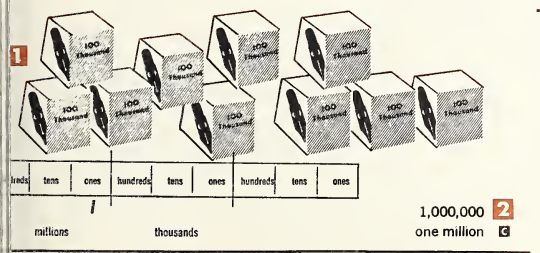
A	ten thousands'	G	15 thousands
B	thousands'	H	2 thousands
C	hundred thousands'	I	14,438
D	9140	J	85,590
E	56,839	K	8 thousands, 6 tens
F	11 hundreds, 16 ones	L	6 thousands, 10 tens

Looking back

This lesson will help you check up on reading and writing numerals and on understanding numbers.



oaching of number system through millions. Reading and writing numbers



Say million. Say thousand. **3**

5,262,000
41,785,439
8,027,050
376,000,000
90,001,002
223,500,010

Read. **4**

A 15,402,716
B 3,850,300
C 961,700,000
D 1,001,001
E 60,090,040
F 778,200,509

te with numerals.

three thousand nineteen
six hundred twenty thousand four hundred one
two million eighty thousand seven
ninety-eight thousand ninety
sixty-four million one hundred thousand four
seven thousand five hundred six
four hundred eight million seventy
twenty-nine million eight hundred thousand
seventy-one million thirty thousand
one million nine hundred
five hundred sixty-three thousand five

Read. **4**

A 38,020
B 4,480,601
C 761,000
D 924,000,775
E 50,082,146
F 23,008
G 40,070,010
H 17,209
I 6,000,000
J 600,000
K 430,500,008
L 219,003
M 7460
N 8,004,002
O 113,030
P 72,900,404
Q 5,298,743
R 90,010,200

12

1 Tell the pupils that the boxes and bags contain wooden beads. Call attention to the pile of 10 boxes of 1000 each, the pile of 3 boxes of 1000 each, and the 4 bags of 100 each.

2 Discuss where tally marks for each pile would be put in the tally space and why each numeral appears where it does.

3 Help the pupils discover that "1" and "3" are in the group to be read as "thousand" and that "400" is in the group read without a group name.

4 Explain that the zeros in 13,400 keep the other numerals in their places just as the tally spaces do. Explain the use of the comma.

5 Help the pupils discover that there are 9 ten thousands, or 90 thousand sticks. Discuss the position of 9 in the tally space.

6 Bring out that now there are 10 ten thousands and this equals 1 hundred thousand. Discuss the position of 1 in the tally space and the use of the zeros in 100,000.

13

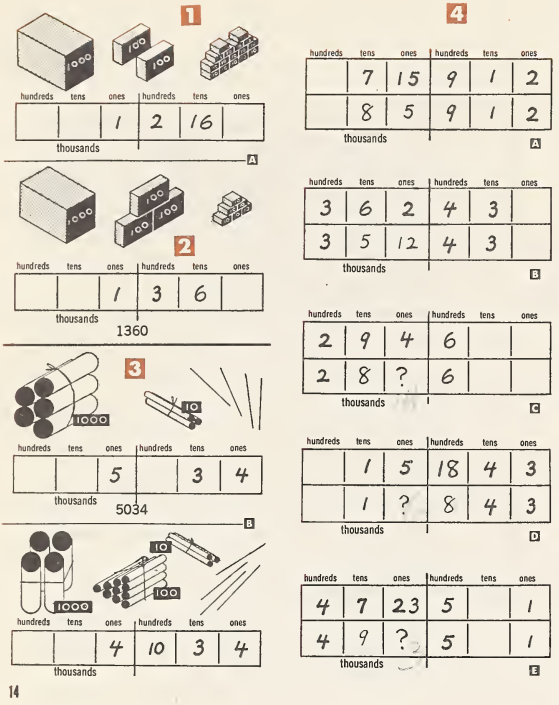
1 Have the pupils count the boxes of pens. Then tell them that 10 groups of 100 thousand are called 1 million. Have them note that in the tally space another section of three places has been included for millions and that we have put 1 in ones' place because we have 1 million pens. Ask pupils to explain why there are no figures in the two sections of places at the right. Be sure they understand that 1 million is equal to 10 hundred thousand.

2 Have them read this numeral and explain the zeros.

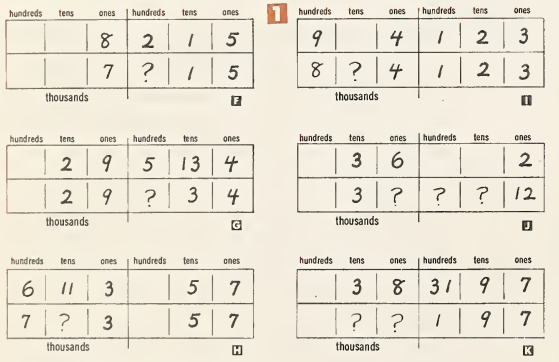
3 Explain to the pupils the relation of the numerals in black and those in blue to the tally space in Picture C. Be sure they understand what to say for each group of three numerals. Let each child have a turn in reading.

4 Let the pupils take turns in reading these exercises aloud.

5 Assign Exercises A to K as written work.



14



2

A In 16,472, the 1 is in what place?
B In 739,568, the 9 is in what place?
C In 208,411, the 2 is in what place?
D 7 thousands 21 hundreds 4 tens = ■
E 4 ten thousands 16 thousands 7 hundreds 13 tens 9 ones = ■
F 5126 = 4 thousands ■ hundreds 1 ten ■ ones
G 35,948 = 2 ten thousands ■ thousands 9 hundreds 4 tens 8 ones
H 83,015 = 8 ten thousands ■ thousands 10 hundreds 1 ten 5 ones
I 1 ten thousand 3 thousands 14 hundreds 2 tens 18 ones = ■
J 6 ten thousands 25 thousands 4 hundreds 19 tens = ■
K 9671 = ■ thousands 16 hundreds ■ tens 11 ones
L 46,309 = 4 ten thousands ■ thousands 2 hundreds ■ tens 9 ones

By now you should have a good understanding of the number system.

14

1 Ask the pupils how many boxes of 1000, 100 and 10 there are. Have them explain why each of the figures representing the boxes has been put where it is in the tally space. Ask what has to be done with the 16 tens. Help them recall that 10 tens equal 100 and get them to see that the 16 boxes of ten will have to be rearranged into 1 box of 100 and 6 boxes of 10.

2 Now have the pupils note that the numerals have been changed to agree with the rearrangement of the boxes. Then ask a pupil to read the complete numeral. Bring out that this kind of regrouping of figures is used in addition.

3 Relate the picture in the first scene to the numerals in the tally space. Then have the pupils look at the second scene and observe what has happened. Help them see that this kind of regrouping is used in subtraction.

4 Discuss the changes indicated by the blue numerals and question marks.

15

1 Continue discussing changes indicated by blue numerals and question marks. Pupils should compare the numerals in the second row of tally spaces with the numerals in the first row and decide what numerals should be placed where the question marks are. Help the pupils determine whether the changes made would be used in adding or subtracting.

2 Use these exercises orally first. Let the pupils take turns in giving answers. Then assign the entire exercise as written work for the class. Direct the pupils to write only the missing numerals for Exercises D to L. For F, they would write "F 11 hundreds, 16 ones."

16-17 Side trip

Expanded Notes for this lesson are on page 258.

Objectives

The child reviews reading and writing Roman numerals through XXXIX and extends his knowledge of the Roman numeral system through the thousands.

Vocabulary

New words **page 16** alike*; **page 17** Arabic, column, position

Comments

Roman numerals occupy a relatively unimportant part in the child's world of numbers. But children *do* encounter them from time to time as the volume numbers of books and magazines, in chapter numbers, in dates of movies, and in dates on cornerstones. They also use Roman numerals in making and reading outlines. So they must know how to read and write these numerals.

The material in this lesson shows children how to read and write Roman numerals through the thousands. Comparison of them with our Hindu-Arabic system should quickly show the advantages of our system.

Only a few basic principles need to be learned in working with Roman numerals. The chart on page 16 illustrates them. In the second vertical column are shown the seven letters that are used in writing all the Roman numerals. The first column shows the numerals that are written by writing the symbol for a lesser number at the left of the symbol of a greater number. The third column shows that all other numerals are combinations of those in Columns 1 and 2, made by building up the numeral by annexing symbols at the right. When the children have familiarized themselves with the numerals in the first and second columns, they can make any Roman numeral.

Answers

Page 16:

[I, V, X, L, C, D, M]

Each of them has a letter for a lesser number written at the left of a letter for a greater number. Each of them is made by combining letters from Columns 1 and 2.

[XX, XXI, XXII, XXIII, XXIV, XXV, XXVI, XXVII, XXVIII, XXIX, XXX]

Page 17:

(block 1)

E [XL, XLI, XLII, XLIII, XLIV, XLV, XLVI, XLVII, XLVIII, XLIX, L]

F [XC, XCI, XCII, XCIII, XCIV, XCV, XCVI, XCVII, XCVIII, XCIX, C]

G [CCL, CCLI, CCLII, CCLIII, CCLIV, CCLV, CCLVI, CCLVII, CCLVIII, CCLIX, CCLX]

H [MDCXXXI, MDCXXXII, MDCXXXIII, MDCXXXIV, MDCXXXV, MDCXXXVI, MDCXXXVII, MDCXXXVIII, MDCXXXIX, MDCXL]

(block 2)

A	17	E	103	I	40	M	409	Q	89	U	1940
B	32	F	156	J	24	N	440	R	909	V	1444
C	55	G	320	K	19	O	919	S	1214	W	1956
D	61	H	1008	L	94	P	990	T	1519	X	1899

(block 3)

A	[VIII]	H	[LXXXV]	O	[CCCXLIV]
B	[XVI]	I	[CI]	P	[DXIX]
C	[XXXVII]	J	[CXXV]	Q	[MCCXV]
D	[LIII]	K	[CXLIX]	R	[MDCCCLXXV]
E	[LXXII]	L	[CCCX]	S	[MCMLXIX]
F	[LXXVIII]	M	[CCCXVIII]	T	[MDXLVIII]
G	[LXXXI]	N	[CCCXXXIX]	U	[MDCCLXXVI]

(block 4)

A [D]; no

B No; A small number often has more letters than a large number. It may take several letters to show a number in the hundreds, but only one to show 1000. Also, numbers like 40 or 900 take more places than numbers like 50 or 1000.

C 27,652; yes

D 27 and 72; The figures in any number with more than 1 figure can be rearranged to make a larger or smaller number.

E [VII, LXX, DCC]; no; None of the letters used to write 7 is used for 70 and 700.

Side trip

	I	II	III				
2	1	2	3				
IV	V	VI	VII	VIII			
4	5	6	7	8			
IX	X	XI	XII	XIII	XIV	XV	XIX
9	10	11	12	13	14	15	19
	1	XX	XXVI	XXX	XXXV	XXXVII	4
		20	26	30	35	37	
XL	L	XLIV	XLVIII	LIII	LX	LXX	LXXX
40	50	44	48	53	60	70	80
XC	C	XCIX	CXX	CC	CCC	CCCXCVIII	
90	100	99	120	200	300	398	
CD	D	CDLX	DXLI	DC	DCC	DCCCII	
400	500	460	541	600	700	802	
CM	M	CMLXXIV	MD	MDC	MCMXIV		
900	1000	974	1500	1600	1914		

A Only seven letters are used to write all the Roman numerals. What are they?

B Look at the Roman numerals for 4, 9, 40, 90, 400, and 900. How are they made? In what way are they alike?

C Look at the Roman numerals for 2, 6, 11, 20, and 44. How are they made? In what way are they alike?

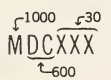
D Write the Roman numerals for the numbers from 20 to 30.

E Write the Roman numerals for the numbers 40 to 50. 1

F Write the Roman numerals for the numbers 90 to 100.

G Write the Roman numerals for the numbers 250 to 260.

H Write 1630 in Roman numerals this way.



Now write the Roman numerals for the numbers 1631 to 1640.

2 Read these Roman numerals.

A	XVII	I	XL	Q	LXXXIX
B	XXXII	J	XXIV	R	CMIX
C	LV	K	XIX	S	MCCXIV
D	LXI	L	XCIV	T	MDXIX
E	CIII	M	CDIX	U	MCMXL
F	CLVI	N	CDXL	V	MCDXLIV
G	CCCXX	O	CMXIX	W	MCMLVI
H	MVIII	P	CMXC	X	MDCCCXCIX

3 Write as Roman numerals.

A	8	H	85	O	344
B	16	I	101	P	519
C	37	J	125	Q	1215
D	53	K	149	R	1875
E	72	L	310	S	1969
F	78	M	318	T	1548
G	81	N	339	U	1776

4 A Our numerals are called Arabic numerals. Below is the number 2529, written with both Arabic and Roman numerals. We know that the 5 in 2529 shows hundreds because it is the third figure from the right. Which letter in MMDXXIX shows hundreds? Is it in the same position as the 5 in 2529?

B Can you tell how large a Roman numeral is by the number of letters it has? Why or why not?

C Which Arabic numeral means more, 3489 or 27,652? Does the number of figures help tell you which is larger?

D Write two Arabic numerals in which the number of figures does not tell you which numeral is larger.

E Write the Roman numerals for 7, 70, and 700. Does knowing the Roman numeral for 7 help you write the numeral for 70 or for 700? Why or why not?

F Write the Roman numerals for 39, 55, and 108 in a column and try to add them. What happens?

16

1 Have the pupils note carefully the letters in this column as they answer Exercise A. Tell them that all other Roman numerals are made by combining these letters.

2 As the children observe this column in answering Exercise B, they should see that the letters on the right side are the same as those in the second column, but that an I, X, or C at the left reduces the number by 1, 10, or 100. Tell them that these numerals are the only ones made in this way.

3 Have the pupils note the first five Roman numerals in this column as they answer the questions in Exercise C. Then have them look at the Roman numerals for 11 to 15. Call on pupils to go to the board to write the Roman numerals for 16 to 19.

4 To help the pupils write the numerals asked for in Exercise D, they may refer to this row. Then have them study the rest of the chart.

17

1 For Exercises E to H, let the pupils refer to the chart on page 16.

2 Have the pupils take turns in reading these numerals. Note that A to H are numerals formed without the use of those shown in Column 1 of the chart on page 16, that I to P use the symbols from Column 1, and that Q to X involve symbols from both columns. It may help the children to mark off recognizable combinations of letters as they work. Refer to the diagram above for 1630.

3 After all of the children have written these Roman numerals on paper, have one or two children write them on the board so the pupils can check their work.

4 Bring out in discussion of Exercises A to F the advantages of the Hindu-Arabic system as compared with the Roman system.

18 Checking up

Expanded Notes for this lesson are on pages 258-260.

Objectives

The child tests his knowledge of addition, subtraction, the multiplication basic facts with products of 36 or less, and the division basic facts with dividends of 36 or less.

Vocabulary

There are no new words.

Comments

In this book the sections called "Checking up" are tests. Page 18 provides three inventory tests on addition. Test 1 is on addition of one-figure numbers; Test 2 is on addition of two-figure numbers; and Test 3 is on addition of three-figure numbers. There are two subtraction tests. Test 4 involves subtraction of two-figure numbers; Test 5, subtraction of three-figure and four-figure numbers. Test 6 is a test involving both addition and subtraction of numbers representing money. Test 7 is on the multiplication basic facts with products of 36 or less, and Test 8 is on the division basic facts with dividends of 36 or less.

You will notice that all the examples appear in equation form. The children must place the numerals in Tests 2 to 6 in the correct computational form to find the answers. For Tests 7 and 8, they should write the answers only. The equation form is used throughout the book in tests and exercises. This has been done because the authors believe that children need practice in arranging numerals in computational form, since it will be necessary for them to do so with problems in real life. Notice also that a screen (▤) has been used to indicate the answer that must be found in each equation. Children who have learned arithmetic through the use of earlier books in this program will be familiar with this device and with the equation form as well.

The arrangement of the tests by specific areas should help reveal the weaknesses of each child and should help you decide what areas need reteaching, either on a class or individual basis. The tests are not speed tests, and the children should be given ample time to complete their work.

The abilities covered by these tests were first taught in *Seeing Through Arithmetic 3* and completely retaught in *Seeing Through Arithmetic 4*. In this book they are inventoried only. To help you in any reteaching that is necessary, page references to these topics as retaught in the pupils' text of *Seeing Through Arithmetic 4* are given below. You will find it advantageous to have a few copies of Book 4 in your classroom.

Remember that a child's success in arithmetic depends upon his knowing *all* the basic facts. If there is any evidence that he does not know all of them, special steps should be taken to help him. Suggestions are given under the heading "Helping the slow learner" in the Expanded Notes. Much can be learned about the child's proficiency by noting his work habits and the procedures he uses to get his answers. Of particular significance are speed and assurance, signs of frustration and hesitation, counting on fingers or tallying, and erasures that show faulty knowledge of the basic facts or a change of mind about the procedure. All of these things should be taken into account in evaluating the results of these tests, and appropriate remedial work should be planned.

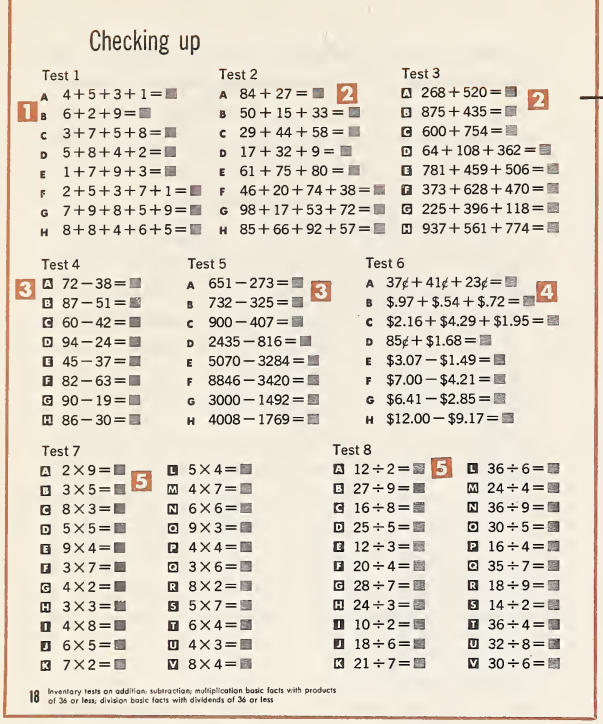
Reteaching chart

Test	Pages (Book 4)	Test	Pages (Book 4)
1	18-19	5	36-41
2	20-23	6	80-81; 82-83
3	24-27	7	46-48; 49-51
4	32-35	8	55-59; 60-65

Answers

Since only basic facts are involved in Tests 7 and 8, answers are not provided for these tests.

Test 1		Test 2		Test 3	
A 13	E 20	A 111	E 216	A 788	E 1746
B 17	F 18	B 98	F 178	B 1310	F 1471
C 23	G 38	C 131	G 240	C 1354	G 739
D 19	H 31	D 58	H 300	D 534	H 2272
Test 4		**Test 5**		**Test 6**	
A 34	E 8	A 378	E 1786	A $1.01	E $1.58
B 36	F 19	B 407	F 5426	B $2.23	F $2.79
C 18	G 71	C 493	G 1508	C $8.40	G $3.56
D 70	H 56	D 1619	H 2239	D $2.53	H $2.83



Checking up

Test 1

A 4+5+3+1=
B 6+2+9=
C 3+7+5+8=
D 5+8+4+2=
E 1+7+9+3=
F 2+5+3+7+1=
G 7+9+8+5+9=
H 8+8+4+6+5=

Test 2

A 84+27=
B 50+15+33=
C 29+44+58=
D 17+32+9=
E 61+75+80=
F 46+20+74+38=
G 98+17+53+72=
H 85+66+92+57=

Test 3

A 268+520=
B 875+435=
C 600+754=
D 64+108+362=
E 781+459+506=
F 373+628+470=
G 225+396+118=
H 937+561+774=

Test 4

A 72−38=
B 87−51=
C 60−42=
D 94−24=
E 45−37=
F 82−63=
G 90−19=
H 86−30=

Test 5

A 651−273=
B 732−325=
C 900−407=
D 2435−816=
E 5070−3284=
F 8846−3420=
G 3000−1492=
H 4008−1769=

Test 6

A 37¢+41¢+23¢=
B $.97+$.54+$.72=
C $2.16+$4.29+$1.95=
D 85¢+$1.68=
E $3.07−$1.49=
F $7.00−$4.21=
G $6.41−$2.85=
H $12.00−$9.17=

Test 7

A 2×9=
B 3×5=
C 8×3=
D 5×5=
E 9×4=
F 3×7=
G 4×2=
H 3×3=
I 4×8=
J 6×5=
K 7×2=
L 5×4=
M 4×7=
N 6×6=
O 9×3=
P 4×4=
Q 3×6=
R 8×2=
S 5×7=
T 6×4=
U 4×3=
V 8×4=

Test 8

A 12÷2=
B 27÷9=
C 16÷8=
D 25÷5=
E 12÷3=
F 20÷4=
G 28÷7=
H 24÷3=
I 10÷2=
J 18÷6=
K 21÷7=
L 36÷6=
M 24÷4=
N 36÷9=
O 30÷5=
P 16÷4=
Q 35÷7=
R 18÷9=
S 14÷2=
T 36÷4=
U 32÷8=
V 30÷6=

18 Inventory tests on addition; subtraction; multiplication basic facts with products of 36 or less; division basic facts with dividends of 36 or less

18

1 Tell the pupils to add "in their heads" and write only the answers, or, if they need to, they may write the numerals in a column and add. Have them label each example with its identifying letter.

2 Tell the pupils to write the numerals in computational form and add. Have them label each example with its identifying letter.

3 Ask the children to write the numerals in computational form and subtract.

4 Have the pupils put these numerals in computational form and add or subtract as indicated. Insist upon the proper use of the dollar sign and the point.

5 Tell pupils to write the answers only.

19-22 Exploring problems

Expanded Notes for this lesson are on pages 261-263.

Objectives

The child tests his ability to solve five basic types of problems. He learns to use the letter *n* to stand for the unknown number in the equation he writes to tell the story of a problem situation.

Vocabulary

New words page 19 gym, shirt*, pupils*, Wednesday; page 20 arithmetic*, page 22 stayed*, drove*, drive*, sewing*, health, Patsy, studied*

Comments

Most of the children in the fifth grade who have learned arithmetic by means of the *Seeing Through Arithmetic* program have acquired a good working knowledge of how to solve problems. By the completion of Grade 4 they have studied èleven types of problem situations. They have learned to analyze problem situations in terms of the groups of objects involved and the action observed taking place or imagined to be taking place. They have learned to symbolize the action with numerals and signs and to write equations. And they have learned how to compute to find the number that *n* represents in each situation.

In this lesson five types of problem situations are reviewed: (1) finding the total when one group is combined with another; (2) finding the remainder when part of a group is removed; (3) finding how many more were added or are needed; (4) finding the number to begin with when the total and the number added are known; and (5) finding how many more or fewer there are in one group than in another. All these problem situations require either addition or subtraction to find the unknown number.

For each problem type, a representative problem is worked in the book. The equation that tells the story of the problem is shown and explained, and the process used in computing to find the numeral to replace *n* (the answer) is suggested.

Two problem types (labeled 1 and 2 above) are illustrated on page 19. In connection with the simple additive type problem in which one group is combined with another, it should be clear that sometimes the problem implies the definite action of putting one group with another; in other cases the groups must be imagined as being put together in order to find the total. For example, a situation like this—John has 3 model cars, and then his mother gives him 5 more for his birthday—suggests real action. If the pupils understand the problem, an accurately descriptive equation for this problem would be 3+5=n. However, suppose there are 10 trees in one orchard and 35 in another, and you want to know how many trees there are in all. Then the problem situation suggests no real combining action, and the action must be imagined. The equation can be either 10+35=n or 35+10=n. The pupils should know, of course, that in either case the order of the addends does not affect the answer.

There are also some subtractive situations (see Problem D, page 19) that do not involve the actual separation of one group from another. In these situations such action is implied and must be imagined as taking place.

Two problem types (labeled 3 and 4 above) are reviewed on page 20. For each, two-scene pictures illustrate the problem situation and the solution. The first scene shows the problem situation and helps the children make the equation that describes the problem. Here they see that the unknown number does not always stand alone on one side of the equals sign. The second scene shows what action must be imagined as taking place to find the answer and suggests the process to use in computation.

The fifth problem type is illustrated on page 21. Two scenes illustrate the problem situation and show that it is a comparison of groups. The third scene suggests, by dimming off part of one group, that subtraction is the process used to solve the problem.

For the children who have not previously studied the problem-solving method outlined in this lesson (and used throughout this program), some time may need to be spent in familiarizing them with equation-making as a means of telling the story in problem situations. These children will have to be taught to *think* about the situation expressed in a problem before they begin to compute. The equation which each child should be required to make for each problem is a written indication of his thinking, and it will be of service to you in appraising the child's learning. To help these children, it is suggested that you provide objects to use in showing the groupings and the action indicated in the picture sequences for the various problem types.

The only new thing for children who have learned this problem-solving method is the introduction of the use of the letter *n* to stand for the unknown number in the equation. In earlier grades the screen (▒) was used for this purpose. A discussion of the picture on page 19 should make clear how *n* is used. Point out to the children that using *n* in place of the screen is a kind of growing-up in arithmetic.

How to compute with numbers that stand for money and the use of data to be found in a previous problem are also reviewed in this lesson.

Answers

The brackets represent the place where the letter *n* has been written in the equation. The numeral within the brackets is the answer for the problem. Two equations are given for problems involving additive action that must be imagined.

Page 19:

A $3.89+$.98=[$4.87] or $.98+$3.89=[$4.87]
B $5.00−$3.89=[$1.11]
C 15+16=[31]
D 32−17=[15]
E 50¢−36¢=[14¢]
F $1.78+$.39=[$2.17] or $.39+$1.78=[$2.17]
G 58+*67*=[125]

Pages 20-22:

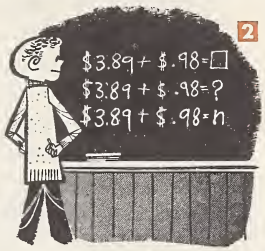
A 24+[8]=32
B [7]+16=23
C 16−5=[11]
D 18−14=[4]
E 12−9=[3]
F [248]+87=335
G 21−9=[12]
H $7.50+[$6.85]=$14.35
I 77−68=[9]
J 59−52=[7]
K [2867]+528=3395
L 283+97=[380] or 97+283=[380]
M 528−380=[148]
N [8]+6=14

Exploring problems

1 A Each boy in the fifth grade needed $3.89 for gym shoes and $.98 for a gym shirt. How much money did each boy need for these two things?

Amount needed for gym shoes.
Amount needed for gym shirt.
3 Total amount needed.

$3.89+$.98=n You must find this number.



2

B Each girl needed $3.89 for gym shoes, too. Jane paid for her shoes with a five-dollar bill. How much should the clerk have given her in change?

4 Amount Jane had at first.
Amount she spent.
Amount left.

$5.00−$3.89=n You must find this number.

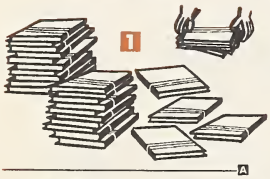
C Peggy brought 15 flowers to school on the first day. Sue brought 16 flowers to school. Together, the girls brought how many flowers?
5 15+16=n

D There were 32 pupils in the fifth grade. Seventeen of them were girls. How many boys were in the fifth grade?
32−17=n

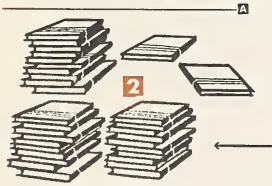
E Ann went to the school store with 50¢. She spent 36¢ for a ruler and some crayons. How much money did she have left?

F Tom spent $1.78 for a lunch box and $.39 for a pencil box. How much did he spend for these two things?

G The clerk at the school store said that she sold 58 boxes of crayons on Tuesday and 67 boxes on Wednesday. She sold how many boxes of crayons on the two days?


1

A


2

A The fifth-grade class had 24 new arithmetic books. There were 32 pupils in the class. How many more arithmetic books were needed?

Number of books the class had.
Number of books that must be added.
Total number of books needed.

24+n=32
You must find this number.

Imagine that 24 books are gone. Subtract 24 from 32.

3

B

4

B Nancy had some crayons. Then she bought a new box of 16 crayons. Now she has 23 crayons. How many crayons did she have at first?

Number of crayons Nancy had at first.
Number she bought.
Number she has now.

n+16=23
You must find this number.

Imagine that 16 crayons are gone. Subtract 16 from 23.

19

1 Have the pupils read the problem only. Discuss the fact that they must combine the two amounts of money to find the total.

2 Point out that the same equation is shown in three ways. Call attention to the fact that the first equation shows the form used in the third- and fourth-grade books (a screen or a square). Tell the children that now they are going to use the letter *n* to hold a place in the equation for a number they must find.

3 Discuss this equation. Make sure pupils see that the equation tells the story. Ask them if the answer would be the same if the equation were written $.98+$3.89=n. Then have a pupil go to the board, write the numerals in computational form, and add.

4 Discuss this problem and the equation. Adapt suggestions in Note 3 to this problem.

5 Call on pupils to write the equations and computation for Problems C to G on the board.

20

1 Have the pupils discuss this picture in terms of the problem and the equation. Ask them to point out the 24 books the class had and tell what the books in the hands show. Have them tell why the plus sign is used in the equation and why the letter *n* appears where it does.

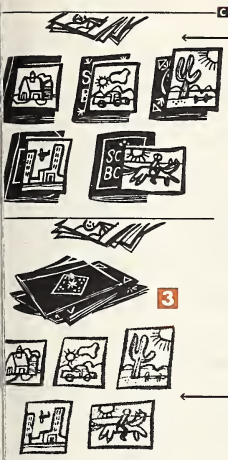
2 Get the pupils to see that this picture shows the 32 books needed by the class. After reading the statements beside the arrow, the pupils should see that 24 of the books are dimmed to show that this group is to be removed. This action suggests subtraction. Have a child compute at the chalkboard.

3 Discuss this picture in terms of the problem and the equation. Have the children tell why the equation tells the story of the problem.

4 Help the pupils see that this picture shows the 23 crayons Nancy has in all, that 16 are dimmed to suggest they are gone. Let a pupil do the subtraction at the chalkboard.

1

C The fifth-grade pupils brought sixteen pictures showing what they did during the summer. They also brought five scrapbooks telling what they did during the summer. How many more pictures did they bring than scrapbooks?



C

Match as many pictures as you can with scrapbooks. Then find the number of pictures that are not matched. 2

Number of pictures.

Number of pictures matched with scrapbooks. Imagine that these pictures are gone.

Number of pictures not matched with scrapbooks.

$16 - 5 = n$

You must find this number.

3

Imagine that 5 pictures are gone. Subtract 5 from 16.

21

D Fourteen fifth graders went on trips during the summer. Eighteen fifth graders stayed at home. How many fewer fifth graders went on trips than stayed at home?
$18 - 14 = n$

E Twelve boys and nine girls learned to swim during the summer. How many more boys than girls learned to swim? How many fewer girls than boys learned to swim?
$12 - 9 = n$

F Ellen and her family drove to a town 335 miles away. After the first day, they still had 87 miles to drive. How many miles had they driven on the first day?
$n + 87 = 335$

G The 21 Brownies in Sue's troop had sewing and cooking classes. Nine of the girls were in the sewing class. The others were in the cooking class. How many girls were in the cooking class?

H Don saved $7.50 during the summer to buy a record player. The record player he wanted to buy cost $14.35. How much more money did he need to buy the record player?

I When the children were weighed in the health class, Tony weighed 77 lb. Patsy weighed 68 lb. Tony weighed how much more than Patsy?

J When the children's heights were measured in the health class, John was 59 in. tall. Tom was 52 in. tall. Tom was how much shorter than John?

K This year the school library bought 528 new books. There are now 3395 books in the library. How many books did the library have before the new books were bought?

L 283 of the new books are story books. 97 of them are science books. How many of the new books are story books and science books?

M How many of the 528 new books are not science books or story books?

THINK In some problems you must use an answer from another problem. Where will you get a number you need to find the answer to Problem M?

N Peggy lives 14 blocks from school. Every morning her father drives her part of the way. Then she still has 6 blocks to walk. How many blocks does her father drive her? 3

Now you should be able to make equations for the addition and subtraction problems you have studied.

21

1 Have the pupils read the problem and identify the scrapbooks and pictures shown in the first scene.

2 Ask the pupils to read this paragraph and look at the picture. Have them note that 5 pictures are matched with 5 scrapbooks. They should see that at the top of this scene there is a pile of pictures that are unmatched. Point out that when they remove 5 pictures from the total 16, some are left. Have the children explain the equation in terms of the illustration.

3 The pupils should see in this scene that the 5 pictures that are dimmed are to be imagined as gone. This should suggest subtraction to them. Have someone show the computation on the chalkboard.

22

1 Discuss Problems D to F and the accompanying equations with the class. Have pupils compute on the chalkboard to find the answers.

2 Assign Problems G to N as written work. Tell the children to write an equation to describe each problem situation. Explain that they should write an *n* to hold a place for the number they are to find. Then tell them to compute to find the answer. When they have finished the computation, have them rewrite the equation with the answer in place of *n*.

3 Provide answers for the children to use in verifying their work. Discuss the problems that caused the most trouble.

23 Thinking straight; Keeping skillful

Expanded Notes are not considered necessary for these lessons.

Objectives

The child reviews the rounding off of large numbers to the nearer ten thousand, hundred thousand, and million. He practices multiplication and division basic facts and solving the equation types just studied.

Vocabulary

New words page 23 passenger*, United States*, approximate, most*

Comments

The children must understand that numbers are only rounded off if there is a good reason for doing so. Discuss the fact that "round numbers" are easier to remember, but you might mention that such numbers as telephone numbers and car license numbers must not be rounded off. Whether or not a number is rounded off, then, is determined by the use to be made of it. The exercises in this lesson require rounding off in both upward and downward directions.

Answers

Thinking straight:

A Exact; because the numeral is not rounded off

B Yes; 948 is close to 1000.

C The nearer hundred thousand

D 6,000,000; because 6,116,948 is closer to 6 million than to 7 million

E 6,116,950; 6,120,000; 6,116,900

F 6,117,000; It is easy to work with and is rounded off only 52.

	Nearer 1000	Nearer 100,000	Nearer million
G	1,739,000	1,700,000	2,000,000
H	16,085,000	16,100,000	16,000,000
I	36,026,000	36,000,000	36,000,000
J	421,762,000	421,800,000	422,000,000
K	704,551,000	704,600,000	705,000,000

Keeping skillful:

(block 2)			(block 3)		
A 34	D 128	G 143	A 38	D 73	G 982
B 98	E 165	H 734	B 225	E 344	H 1741
C 392	F 1245	I 519	C 243	F 262	I 533

Thinking straight

1 A Dick read that 6,116,948 new passenger cars were sold in the United States in 1953. Is 6,116,948 approximate or exact? Why do you think so?

B Dick tried rounding off 6,116,948 in different ways to make the number easy to remember. First he rounded of the number to the nearest thousand. He wrote 6,117,000. Was he right? How do you know?

C Then Dick wrote 6,100,000. Was he rounding off 6,116,948 to the nearest hundred thousand or to the nearest ten thousand?

D Then Dick rounded off 6,116,948 to the nearest million. What should he have written? Why?

E What other ways can you think of to round off 6,116,948?

F Which of these ways of rounding off 6,116,948 do you think is most useful? Why?

Round off the numbers below.

2

	Nearest thousand	Nearest hundred thousand	Nearest million
G 1,738,669	1,739,000	1,700,000	■
H 16,084,742	■	■	■
I 36,025,888	■	■	■
J 421,762,471	■	■	■
K 704,551,099	■	■	■

Keeping skillful

3

A $7 \times 3 = n$	J $12 \div 6 = n$	
B $24 \div 8 = n$	K $18 \div 2 = n$	
C $14 \div 2 = n$	L $35 \div 5 = n$	
D $5 \times 2 = n$	M $7 \times 4 = n$	
E $15 \div 5 = n$	N $24 \div 6 = n$	
F $5 \times 6 = n$	O $4 \times 5 = n$	
G $3 \times 9 = n$	P $3 \times 8 = n$	
H $20 \div 5 = n$	Q $18 \div 3 = n$	
I $32 \div 4 = n$	R $28 \div 4 = n$	

4

A $37 + n = 71$	
B $41 + 57 = n$	
C $n + 208 = 600$	
D $578 + n = 706$	
E $n + 357 = 522$	
F $836 + 409 = n$	
G $66 + 18 + 59 = n$	
H $n + 278 = 1012$	
I $318 + n = 837$	

5

A $60 - 22 = n$	
B $401 - 176 = n$	
C $732 - 489 = n$	
D $630 - 557 = n$	
E $987 - 643 = n$	
F $800 - 538 = n$	
G $2073 - 1091 = n$	
H $4000 - 2259 = n$	
I $1008 - 475 = n$	

23

1 Handle Exercises A to F orally. Have the children read the exercises and answer the questions. Discuss each one. To help the children round off to the nearer thousand or hundred thousand, call attention to the examples in Exercise G.

2 Assign Exercises G to K as written work. Tell the children to write the three headings *Nearer thousand*, *Nearer hundred thousand*, and *Nearer million* on their papers. Under these headings they should write their answers after the appropriate letters.

3 Have the pupils write the answers to these basic facts after the appropriate letters.

4 Tell the pupils that these addition equations are of the types used in the problem situations they have just studied. Ask them to put the numerals in the proper computational forms and find the answers.

5 Direct pupils to subtract and find the answers.

24-26 Exploring problems

Expanded Notes for this lesson are on pages 263-264.

Objectives

The child reviews simple problem situations that are solved by multiplication and division.

Vocabulary

New words **page 24** Kathy, polish, teaspoons*; **page 25** cases*, bowls*; **page 26** ironing*, handkerchiefs*, towels, shopping*, price*

Comments

This lesson continues the review of problem solving by illustrating and explaining three types of problem situations: finding the size of equal groups when the total number and the number of equal groups are known (partitive division); finding the number of groups when the total number and the size of the equal groups are known (quotitive division); and finding the total when the size and number of equal groups are known.

For each type of problem situation there is a representative problem. The pictures and the equation that describe the problem situation are provided to indicate the process to use to find the answer. This procedure is followed even though only basic facts are involved and all the problems can be solved without pencil.

By insisting that the children make the proper equation for each of the problems they must solve on page 26, you will be helping them build the habit of analyzing the problem situation before leaping into hit-or-miss computation. Careful attention should be given to the distinction between the equations that describe partitive division situations (for example, $16 \div n = 2$) and those that describe quotitive division situations ($16 \div 8 = n$).

A partitive division situation is described on page 24. In this "sharing" type of division, the number of equal groups is known. The number of things in each group is to be found. In the quotitive division situation described on page 25, the number of things in each of the equal groups is known. The number of groups is to be found. For these two types of divisive situations the computation is the same—whether you are finding the size of the equal groups or the number of equal groups.

In Problem A on page 24, the numbers are so small that we know immediately that the answer is 8. However, when the numbers are larger, it is not so easy to know the answer. For example, suppose 174 marbles are to be shared equally by 3 boys—that is, divided into 3 equal groups. The equation is $174 \div n = 3$. We do not know without computing how many marbles should be put into each group to make 3 equal groups.

The method of solving this type of problem with objects is indirect. In this example, if each boy takes 1 marble, a group of 3 marbles is removed from the original whole group. Suppose that each boy takes another marble and that they keep repeating this procedure. Each boy accumulates a pile of marbles, and after 58 rounds the marbles will be equally divided. There will be 58 marbles in each group. Notice that the children can solve this problem by using objects even if they do not know how to divide 174 by 3.

To solve this problem by arithmetic, we notice that 3 marbles are taken each time. We then ask: How many groups of 3 are there in 174? This is the "measurement," or quotitive, type of division problem, and we can divide 174 by 3. The answer is 58, and so we have learned indirectly by arithmetic that each boy will get 1 marble a total of 58 times, or each boy will get a group of 58 marbles.

Similarly, in the easier example on page 24, where 16 teaspoons are to be shared by 2 girls, or $16 \div n = 2$, one round of sharing between 2 girls amounts to forming a group of 2 teaspoons. Indirectly the question becomes: 16 teaspoons will make how many groups of 2 teaspoons? The answer, 8, tells the number of "rounds" and also how many teaspoons each girl will get.

The quotitive division illustrated on page 25 is the kind used in studying the basic facts and should cause no special difficulty.

Answers

A $16 \div [8] = 2$
B $3 \times 8 \div [24]$
C $12 \div 3 = [4]$
D $3 \times 7 = [21]$
E $15 \div [3] = 5$
F $45 \div 5 = [9]$
G $30 \div 6 = [5]$
H $36 \div 9 = [4]$
I $8 \times 3¢ = [24¢]$
J $3 \times 6 = [18]$
K $36¢ \div [6¢] = 6$
L $10 \div 5 = [2]$
M $4 \times 6 = [24]$
N $3 \times 9¢ = [27¢]$
O $32¢ \div [4¢] = 8$
P $12 \div 4 = [3]$

A Kathy and Jane Fisher are going to polish teaspoons for their mother. There are 16 teaspoons. If the girls share the work equally, how many teaspoons should each girl polish?

1

Total number of spoons.
Number of spoons in each group.
Number of equal groups of spoons.

$16 \div n = 2$ 2

To find this number, you divide 16 by what number?

Each time the girls polish 1 spoon apiece, they polish a group of 2 spoons. How many groups of 2 spoons must they polish to finish the spoons?

3

4

Each girl has polished ■ spoons in all.

B The girls have 3 cases of forks to polish, too. There are 8 forks in each case. How many forks do the girls have to polish?

1

Number of groups, or cases, of forks.
Number of forks in each group.
Total number of forks.

$3 \times 8 = n$ 2

You must find this number.

The girls have to polish ■ forks. 3

C Mrs. Fisher has asked the girls to plant 12 flower bulbs in bowls. They are to put 3 bulbs in each bowl. How many bowls will the girls need?

4

5

Total number of bulbs. 6
Number of bulbs in each group.
Number of groups, or bowls, of 3 bulbs.

$12 \div 3 = n$

You must find this number.

The girls will need ■ bowls.

7

24

1 Have the pupils read this problem. Help them see that they must separate 16 spoons into 2 groups of equal size (2 groups because there are 2 girls).

2 Help the pupils relate this equation to the problem and the picture. Point out that they know the total number of spoons and the number of equal groups, but they do not know the size of the groups. Therefore, the letter *n* must come immediately after the division sign.

3 Call attention to this picture and its accompanying text. Help the pupils see that each time the 2 girls polished 1 spoon, a group of 2 spoons was taken from the total group. So the problem becomes: "How many groups of 2 are there in 16?" In this kind of division, the pupils know that they can divide 16 by 2 to get the answer. Or they can use objects and remove groups of 2 to find the answer.

4 Let the pupils count the spoons in front of each girl to verify the answer.

25

1 Have the pupils read this problem and note in the picture the 3 groups of 8 that are to be combined.

2 Call on a pupil to read aloud this equation and the explanations. Be sure the pupils can see why a "times sign" must be used.

3 Since a basic fact is involved, the pupils should know the answer. Let them verify it by counting the forks.

4 Ask the pupils to read this problem.

5 Direct attention to the 12 bulbs. Get the pupils to see that they cannot tell how many bowls will be needed.

6 Let the pupils study this equation. They should be able to explain that a division equation should be used because they are to find how many groups of 3 there are in 12.

7 After they find that there are 4 groups of 3, they can match bowls (objects) to groups of bulbs, as shown in the picture, to verify the answer.

1

D Tom Fisher was helping his father make a bookcase. The bookcase had 3 shelves, and each shelf was 7 ft. long. How many feet of boards in all did they need for the three shelves?

E Jane helped her mother with the ironing. She ironed 5 handkerchiefs in 15 minutes. If she spent the same amount of time on each handkerchief, how long did it take her to iron one handkerchief?

F Kathy ironed some dish towels. It took her about 5 minutes to iron each towel. About how many towels could she iron in 45 minutes?

G Tom was putting 30 packages of strawberries in the freezer for his mother. He could carry 6 packages each trip. How many trips did he have to make to the freezer?

H Jane helped her mother cut some ribbon. She needed pieces of ribbon 9 in. long. How many pieces of this length could Jane cut from 36 inches of ribbon?

I Tom took 8 empty bottles to the store. He got 3¢ apiece for them. How much money did he get for the 8 bottles?

J Mrs. Fisher went shopping with the girls. She bought 3 packages of rolls. There were 6 rolls in each package. How many rolls did she buy altogether?

K Jane bought 36¢ worth of candy bars at the grocery store. She bought 6 candy bars and paid the same price for each one. How much did each candy bar cost?

L Mrs. Fisher was buying ice cream. She needed enough ice cream for 10 people. She knew she could get 5 dishes from one quart. How many quarts of ice cream should she buy?

M Mrs. Fisher bought 4 cartons of orange drink. There were 6 bottles in each carton. How many bottles of orange drink did Mrs. Fisher buy?

N Mrs. Fisher also bought 3 small cans of baked beans at 9¢ a can. How much did she pay for the baked beans?

O Kathy bought 8 lollipops of the same kind at the store. She paid 32¢ for them. How much did each cost?

P Mrs. Fisher asked Kathy to buy 12 tomatoes. The tomatoes were sold in packages of 4. How many packages did Kathy need to buy?

2

Now you should be able to make equations for the multiplication and division problems you have studied.

26

1 Tell the pupils that in this set of problems there are the three types of problem situations they have just studied. Ask them to read each problem carefully, copy its identifying letter on their papers, and write the proper equation with *n* representing the unknown number. Then have them find the answer and rewrite the equation, putting the answer in place of the letter *n*.

2 Provide answers so that the children may correct their own papers. Then discuss the problems that caused the most trouble. Be sure the children can explain the equations they made for each problem.

27-30 Looking back

Exponded Notes for this lesson are on poges 265-266.

Objectives

The child reviews the meaning of multiplication and the multiplication basic facts with products of 40 to 81.

Vocabulary

New words page 27 masks; page 28 pumpkins*; page 29 noisemakers*

Comments

Since the multiplication basic facts with products of 40 to 81 were taught in *Seeing Through Arithmetic 4* (pages 102-104 and 110-113), you may want to use the first block of exercises (A to S) on page 30 of Book 5 to inventory what the children remember from the previous grade before starting the work on page 27. Such a survey will help you decide which of these facts need to be retaught. Also, the Expanded Notes for this lesson (pages 265-266) include activities and suggestions from which you may choose those best suited to your group.

In reviewing the multiplication basic facts, you will want to be sure that children understand what makes a situation multiplicative. Otherwise, when they begin to solve problems, they may tend to think of multiplicative situations as additive situations. Emphasize the idea that, in the multiplicative situation, two or more *equal* groups are all joined to make one larger group. Be sure the children observe not only the equality and size of the groups, but also the number of groups that are being combined. In additive situations one group is thought of as being put with another group, but the two groups are not necessarily equal.

On pages 27 and 28, the multiplication concept is reviewed with problems and pictures illustrating 5×8, 6×7, 5×9, 6×8, 7×7, and 6×9. Each picture, except the one illustrating 7×7, is used to illustrate two multiplication basic facts. It is possible to illustrate two facts with one picture because the objects are arranged in rows that may be viewed either horizontally or vertically. Thus the basic fact $5\times8=40$ is

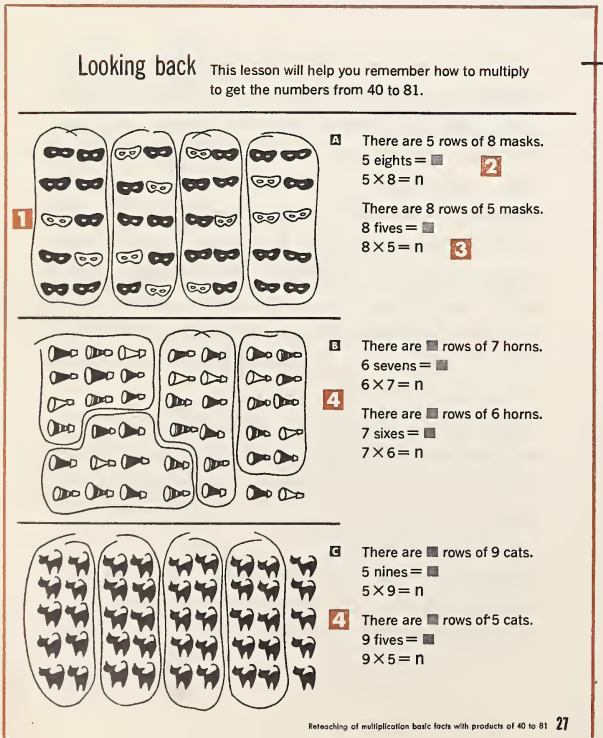
illustrated by five rows of eight objects each that are viewed first as five separate rows and then as one large group. The basic fact $8 \times 5 = 40$ is seen by viewing the same picture as eight rows of five objects each that combine into one large group of 40. This arrangement in the illustration enables the child to see quickly that either order of the factors (5×8 or 8×5) gives the same answer.

Examples for practice are given in equation form at the bottom of page 28. The computational form of multiplication is unnecessary at this time, since only basic facts are encountered. The first block of equations is both an inventory test and a practice exercise for all the basic facts reviewed on pages 27 and 28. In the second block of equations, the basic facts just reviewed are mixed with other multiplication basic facts previously studied.

The device of circling or ringing a group of ten objects within a larger group is used to help children determine how many there are in all without counting by ones. They find the total number by counting the tens and any ones left over. This technique of illustrating objects in rows, with ten objects ringed, is used on pages 29 and 30 to complete the review of the multiplication basic facts for 7×8, 7×9, 8×8, 8×9, and 9×9. Each basic fact and its reverse are taught together, except for the 64 and 81 groups (Pictures I and K), which have only one multiplication basic fact for each group.

To develop the meaning of multiplication more thoroughly, it might be well to supplement the work on these pages with activities with real objects. Let the children demonstrate with objects (buttons, stones, paper clips, etc.) the horizontal and vertical rows for each basic fact, and then make groups of ten to find the total number.

Since all the required answers involve basic facts, no answers are included for this lesson.


Looking back
This lesson will help you remember how to multiply to get the numbers from 40 to 81.
1
A
There are 5 rows of 8 masks.
5 eights = ■
5 × 8 = n
2
There are 8 rows of 5 masks.
8 fives = ■
8 × 5 = n
3
B
There are ■ rows of 7 horns.
6 sevens = ■
6 × 7 = n
4
There are ■ rows of 6 horns.
7 sixes = ■
7 × 6 = n
C
There are ■ rows of 9 cats.
5 nines = ■
5 × 9 = n
4
There are ■ rows of 5 cats.
9 fives = ■
9 × 5 = n
Reteaching of multiplication basic facts with products of 40 to 81
27

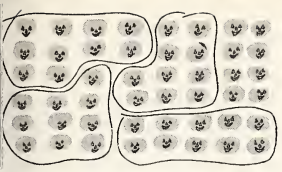
27

1 Show the children the first horizontal row of masks. Ask: "How many masks are there in this row? How many rows like this are there? How many groups of 8 masks are there? How many masks are there in all?" Be sure the children understand that each ring encloses 10 masks, and that there are no masks outside the four rings.

2 Have one child read the first line of text aloud. Direct attention to the picture. The statement on the next line may be presented to the children as a question: "5 eights equal how many?" Have them refer back to the picture. Have another child read the second and third lines aloud and supply the missing numbers.

3 Proceed in the same way for the 8 vertical rows of 5 masks and the accompanying text.

4 Use the method described in Notes 1, 2, and 3. Draw attention to the objects not ringed when finding the total number.



D There are ■ rows of 8 pumpkins.
6 eights = ■
6 × 8 = n

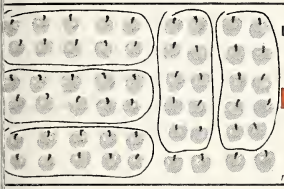
2

There are ■ rows of 6 pumpkins.
8 sixes = ■
8 × 6 = n

3

E There are ■ rows of 7 candies.
7 sevens = ■
7 × 7 = n

4



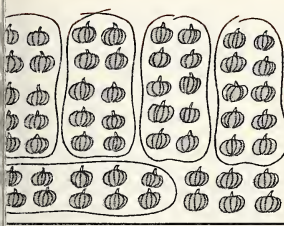
F There are ■ rows of 9 apples.
6 nines = ■
6 × 9 = n

5 There are ■ rows of 6 apples.
9 sixes = ■
9 × 6 = n

6

A	8 × 6 = n	G	7 × 6 = n	A	3 × 4 = n	G	2 × 6 = n
B	5 × 9 = n	H	6 × 8 = n	B	5 × 5 = n	H	7 × 5 = n
C	6 × 7 = n	I	9 × 5 = n	C	8 × 3 = n	I	4 × 5 = n
D	8 × 5 = n	J	5 × 8 = n	D	3 × 3 = n	J	3 × 8 = n
E	7 × 7 = n	K	9 × 6 = n	E	7 × 3 = n	K	6 × 3 = n
F	6 × 9 = n			F	6 × 4 = n	L	4 × 4 = n

7



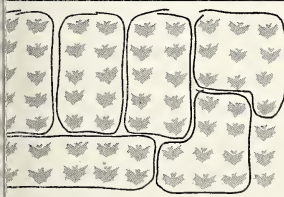
G There are ■ rows of 8 pumpkins.
7 eights = ■
7 × 8 = n

2

There are ■ rows of 7 pumpkins.
8 sevens = ■
8 × 7 = n

3

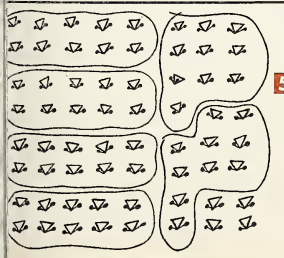
1



H There are ■ rows of 9 bats.
7 nines = ■
7 × 9 = n

There are ■ rows of 7 bats.
9 sevens = ■
9 × 7 = n

4



I There are ■ rows of 8 noisemakers.
8 eights = ■
8 × 8 = n

5

28

1 Start with the first picture and ask the children how many pumpkins there are in the first row across. Then ask how many rows of 8 pumpkins there are. Make certain the class understands that each ring indicates a group of 10. Draw attention to the pumpkins not ringed. Then ask the class how many pumpkins there are altogether.

2 Refer to the picture again. This time have several children read aloud the accompanying three lines of text. They should say the missing number in each statement as they read.

3 Follow the same procedure for the vertical rows of pumpkins and the accompanying text.

4 Use the method described in Notes 1 and 2.

5 For Picture F use Notes 1, 2, and 3.

6 All the basic facts reviewed so far are included in Exercises A to K. These exercises may be answered first orally, then used as written work.

7 Use the procedure described in Note 6.

29

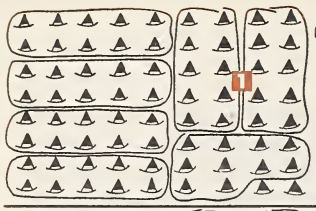
1 Show the children the first picture and ask: "How many pumpkins are there in the first row? How many rows or groups are there like this? How many pumpkins are in each ring? How many are outside the rings? How many pumpkins are there altogether? Count by tens."

2 Have several children take turns reading one line of the text at a time. As they read, make sure they say the number that has been left out of each line of text. In discussion, have them refer back to the picture.

3 Proceed in the same way for the 8 vertical rows of 7 pumpkins and the accompanying text.

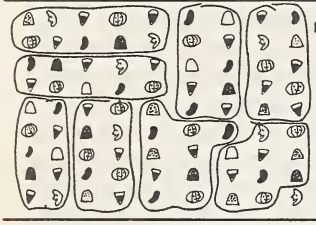
4 Adapt Notes 1, 2, and 3 for Picture H.

5 Adapt the method described in Notes 1 and 2.



J There are ■ rows of 9 hats.
8 nines = ■
8 × 9 = n

There are ■ rows of 8 hats.
9 eights = ■
9 × 8 = n



K There are ■ rows of 9 candies.
9 nines = ■
9 × 9 = n

A 8 × 8 = n	K 7 × 6 = n	A 7 × 7 = n	K 9 × 2 = n
B 8 × 9 = n	L 6 × 9 = n	B 2 × 7 = n	L 9 × 9 = n
C 6 × 8 = n	M 7 × 8 = n	C 3 × 4 = n	M 2 × 3 = n
D 8 × 5 = n	N 6 × 7 = n	D 6 × 2 = n	N 9 × 8 = n
E 9 × 7 = n	O 9 × 5 = n	E 5 × 6 = n	O 7 × 4 = n
F 7 × 7 = n	P 7 × 9 = n	F 4 × 6 = n	P 6 × 9 = n
G 9 × 6 = n	Q 9 × 8 = n	G 2 × 5 = n	Q 8 × 7 = n
H 5 × 9 = n	R 8 × 6 = n	H 7 × 3 = n	R 6 × 6 = n
I 8 × 7 = n	S 9 × 9 = n	I 9 × 7 = n	S 8 × 8 = n
J 5 × 8 = n		J 3 × 9 = n	T 7 × 9 = n

Now you should be able to multiply to get the numbers from 40 to 81.

30

30

1 Ask the pupils how many hats there are in the top row. Ask if there are as many hats in the second row. Ask how many rows of 9 there are. Have them note the rings around groups of 10, and then have someone tell how many hats there are in all.

2 Let the children read the first three lines and use the picture to find the missing numbers.

3 Ask how many hats there are in each up-and-down row. Find out how many rows of 8 there are. Ask pupils how many hats there are in all when the groups of 8 are thought of as one large group. The rings around groups of 10 hats will help show the answer. Have them complete the statements at the right.

4 Adapt Notes 1 and 2.

5 Exercises A to S include the basic facts reviewed in this lesson. Have the pupils give the answers orally first. Then direct them to write the answers.

6 Use the procedure described in Note 5.

31-36 Looking back

Expanded Notes for this lesson are on pages 266-268.

Objectives

The child reviews the division basic facts for groups 40 to 81 (the basic facts with dividends from 40 to 81).

Vocabulary

New words page 32 charms; page 36 important*

Comments

This lesson reviews the division basic facts and the underlying concepts involved in division that were taught in *Seeing Through Arithmetic 4*, pages 105-107 and 114-117.

To help you decide how much work is needed on the basic facts retaught in this lesson, use the first block of exercises at the top of page 36 as an inventory test. If the whole class shows a substantial weakness, use all the time you need to reteach the division basic facts. Since imperfect learning carries over and handicaps the pupils in more difficult work and in the higher grades, make sure now that the children master the basic division facts.

In this lesson the division basic facts are presented in situations in which the size of the equal groups is known and in which the number of equal groups is to be found—that is, the "measurement" type of divisive situation. This leads directly to the usual method of doing division computation as a process of repeated subtracting of *equal groups*. Note that the dividends of all the basic facts represented by pictures in this lesson include 40 or more objects. To enable the child to see a total group of 40 or more objects easily without counting, the objects in the pictures are arranged in horizontal rows of ten.

Be sure the children see that numbers such as 40, 42, 45, etc., have several divisors.

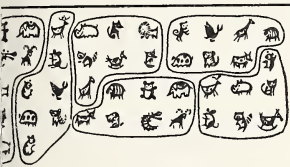
Because this lesson deals only with basic facts, the computational form used for division is not shown here.

Looking back

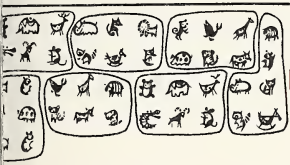
This lesson will help you remember how to divide the numbers from 40 to 81.

A There are 40 birds in all.
The 40 birds are divided into groups of ■.
There are ■ groups of 8.
40 ÷ 8 = n

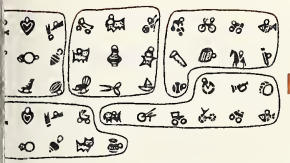
B Now the 40 birds are divided into groups of ■.
There are ■ groups of 5.
40 ÷ 5 = n



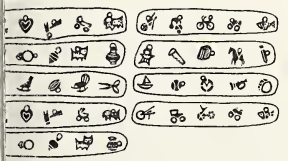
C There are 42 animals in all.
The 42 animals are divided into groups of ■.
There are ■ groups of 7.
42 ÷ 7 = n



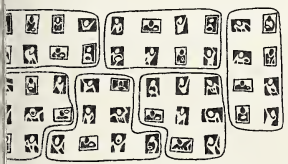
D Now the 42 animals are divided into groups of ■.
There are ■ groups of 6.
42 ÷ 6 = n



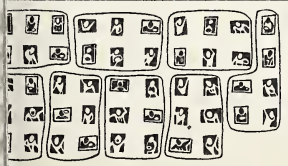
E There are 45 charms in all.
The 45 charms are divided into groups of ■.
There are ■ groups of 9.
45 ÷ 9 = n



F Now the 45 charms are divided into groups of ■.
There are ■ groups of 5.
45 ÷ 5 = n



G There are 48 football cards in all.
The 48 football cards are divided into groups of ■.
There are ■ groups of 8.
48 ÷ 8 = n



H Now the 48 football cards are divided into groups of ■.
There are ■ groups of 6.
48 ÷ 6 = n

31

1 Tell the children to look at Picture A and read the text. Have them note that there are 10 birds in each horizontal row, that there are 4 rows of 10, or 40 birds in all. Ask how many birds are in each ringed group, and how many equal groups there are. Remind them that the rings in the picture help to see the group of 40 separated into groups of 8. Pupils who have difficulty in understanding this should arrange 40 objects in groups of 8 and count the groups. Call on someone to read and complete the statements in the text.

2 Work in the same way with this picture and its text. Have the children observe that this picture shows the same number of birds as the picture above, but that there is a different number in each group and a different number of groups.

3 Proceed in the same way with this picture and its text. First, note that there are 4 groups of 10 and 2 extra objects.

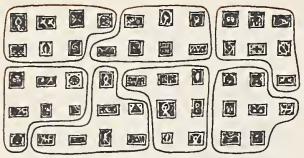
32

1 Tell the pupils to read the text and refer to Picture E. Make sure they note the total number of charms, how many are in each group, and the number of equal groups. Remind them that the rings help them see the 45 charms divided into equal groups of 9. Call on a pupil to read the text aloud and say the missing numbers as he reads. Slow learners may need to work with objects at their desks, rearranging a group of 45 into groups of 9 and counting the groups.

2 Work in the same way with this picture and its text. Compare the groupings with those in the picture above.

3 Proceed in the same way as you did for Picture E.

4 Proceed in the same way as you did for Picture F.



I There are 49 stamps in all.
The 49 stamps are divided into groups of ■.
There are ■ groups of 7.
$49 \div 7 = n$

1

J There are 54 shells in all.
The 54 shells are divided into groups of ■.
There are ■ groups of 9.
$54 \div 9 = n$

2

K Now the 54 shells are divided into groups of ■.
There are ■ groups of 6.
$54 \div 6 = n$

3

4

A $42 \div 6 = n$
B $45 \div 5 = n$
C $48 \div 8 = n$
D $49 \div 7 = n$
E $40 \div 5 = n$
F $54 \div 6 = n$

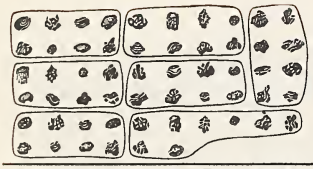
G $42 \div 7 = n$
H $48 \div 6 = n$
I $45 \div 9 = n$
J $40 \div 8 = n$
K $54 \div 9 = n$

5

A $27 \div 3 = n$
B $32 \div 8 = n$
C $36 \div 6 = n$
D $12 \div 4 = n$
E $15 \div 3 = n$
F $35 \div 7 = n$

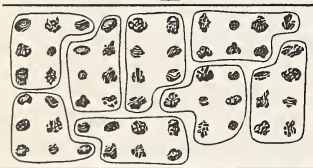
G $16 \div 4 = n$
H $27 \div 9 = n$
I $21 \div 3 = n$
J $24 \div 6 = n$
K $25 \div 5 = n$
L $28 \div 4 = n$

33



L There are 56 stones in all.
The 56 stones are divided into groups of ■.
There are ■ groups of 8.
$56 \div 8 = n$

1



M Now the 56 stones are divided into groups of ■.
There are ■ groups of 7.
$56 \div 7 = n$

2

N There are 63 shells in all.
The 63 shells are divided into groups of ■.
There are ■ groups of 9.
$63 \div 9 = n$

3

O Now the 63 shells are divided into groups of ■.
There are ■ groups of 7.
$63 \div 7 = n$

4

33

1 Tell the pupils to look at Picture I and read the text. Point out that the rings will help them see equal groups of 7. Call on someone to read the statements aloud and say the missing numbers as he reads. Help slow learners to carry out the divisive action with real objects at their desks.

2 Proceed in the same way with this picture and its text.

3 Proceed in the same way with this picture and its text. Compare the groupings with those in Picture J.

4 Call on children to answer Exercises A to K orally. Then have them write the equations with answers or the identifying letters and answers. Note that this block contains only the basic division facts reviewed so far.

5 Call on various children to complete these exercises orally. Then ask them to write the answer for each exercise on their papers.

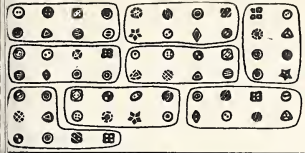
34

1 Direct the children to look at Picture L and read the text. Have them note how many stones there are in all, how many are in each group, and how many equal groups there are. Remind them that the rings in the picture help them to imagine the group of 56 as separated into groups of 8. Pupils who have difficulty understanding what happens should place 56 objects on their desks, rearrange them into groups of 8, and count the number of groups. Call on someone to read and complete the statements in the text.

2 Work in the same way with this picture and its text. Compare the groupings with those in the picture above.

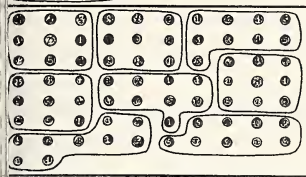
3 Proceed as you did for Picture L.

4 Proceed as you did for Picture M.



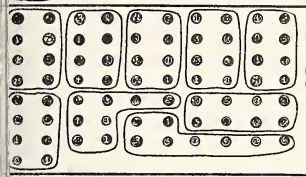
P There are 64 buttons in all.
The 64 buttons are divided into groups of ■.
There are ■ groups of 8.
64 ÷ 8 = n

1



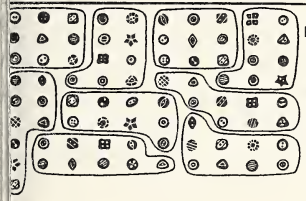
Q There are 72 coins in all.
The 72 coins are divided into groups of ■.
There are ■ groups of 9.
72 ÷ 9 = n

2



R Now the 72 coins are divided into groups of ■.
There are ■ groups of 8.
72 ÷ 8 = n

3



S There are 81 buttons in all.
The 81 buttons are divided into groups of ■.
There are ■ groups of 9.
81 ÷ 9 = n

4

A 48 ÷ 6 = n
B 42 ÷ 7 = n
C 63 ÷ 9 = n
D 54 ÷ 6 = n
E 40 ÷ 5 = n
F 64 ÷ 8 = n
G 45 ÷ 5 = n
H 49 ÷ 7 = n
I 72 ÷ 8 = n
J 42 ÷ 6 = n

1

K 63 ÷ 7 = n
L 81 ÷ 9 = n
M 56 ÷ 7 = n
N 48 ÷ 8 = n
O 45 ÷ 9 = n
P 40 ÷ 8 = n
Q 54 ÷ 9 = n
R 56 ÷ 8 = n
S 72 ÷ 9 = n

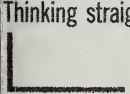
A 16 ÷ 2 = n
B 21 ÷ 3 = n
C 24 ÷ 6 = n
D 14 ÷ 7 = n
E 45 ÷ 9 = n
F 36 ÷ 4 = n
G 9 ÷ 3 = n
H 35 ÷ 7 = n
I 54 ÷ 6 = n
J 30 ÷ 5 = n

2

K 32 ÷ 8 = n
L 12 ÷ 3 = n
M 25 ÷ 5 = n
N 81 ÷ 9 = n
O 18 ÷ 3 = n
P 28 ÷ 7 = n
Q 64 ÷ 8 = n
R 20 ÷ 4 = n
S 27 ÷ 3 = n
T 49 ÷ 7 = n

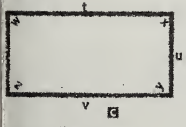
Now you should know the important division equations for the numbers 40 to 81.

Thinking straight



square corner A
right angle

parallel lines B



C

D

Use Pictures A and B to help you answer these questions.
A In Picture C, the sides marked u and ■ are parallel.
B In Picture C, the sides marked t and v are ——.
C In Picture C, the corners marked w, ■, ■, and ■ are right angles.
D In Picture D, which sides are parallel?
E In Picture D, which sides are not parallel?
F In Picture D, which corners are right angles?
G Which corners are not right angles?

35

1 Tell the pupils to refer to Picture P and read the text. Make sure they note the total number of buttons, how many are in each group, and the number of equal groups. Remind them that the rings help them see the 64 buttons divided into groups of 8. Call on a pupil to read the text aloud, saying the missing numbers. Slow learners may need to work with objects at their desks, rearranging a group of 64 into groups of 8 and counting the number of groups.
2 Use the procedures outlined in Note 1 with this picture and text. Compare the groupings with those in the picture above.
3 Proceed as explained in Note 1.
4 Proceed as suggested in Note 2.

36

1 If you used this block of equations for diagnostic purposes at the beginning of the lesson, now have the children write out their answers again as a check on how well they have learned these basic facts.
2 Let the children take turns answering these exercises orally. Then you may ask them to write the equations with answers, or the identifying letters and answers only.

36 Thinking straight

Expanded Notes are not considered necessary for this lesson.

Objectives

The child has experiences that maintain his knowledge of lines and angles and ways to name lines and angles.

Vocabulary

There are no new words.

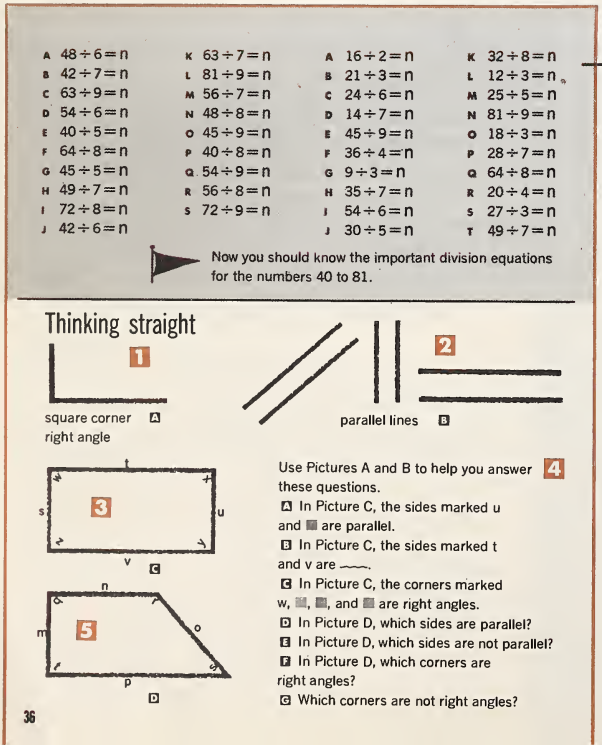
Comments

In the opening lesson of this book the children were introduced to various polygons and were taught how to measure their dimensions. In discussing the polygons they became familiar with such terms as "right angle," "square corner," and "parallel lines." They also were introduced to methods of naming lines for identification.

On page 36 the ideas taught in the first lesson are met again to help the children remember them. The children answer questions in which they must distinguish right angles from other angles and parallel lines from lines that are not parallel. The pupils also encounter the terms "square corner," "right angle," and "parallel line" in the text. They also learn how to label angles by using letters.

Answers

A [s]
B parallel
C [x, y, and z]
D [n and p]
E [m and o]; if some children point out that adjacent sides, like m and n, are not parallel, accept their answers as correct.
F [q and t]
G [r and s]



A 48÷6=n
B 42÷7=n
C 63÷9=n
D 54÷6=n
E 40÷5=n
F 64÷8=n
G 45÷5=n
H 49÷7=n
I 72÷8=n
J 42÷6=n

K 63÷7=n
L 81÷9=n
M 56÷7=n
N 48÷8=n
O 45÷9=n
P 40÷8=n
Q 54÷9=n
R 56÷8=n
S 72÷9=n

A 16÷2=n
B 21÷3=n
C 24÷6=n
D 14÷7=n
E 45÷9=n
F 36÷4=n
G 9÷3=n
H 35÷7=n
I 54÷6=n
J 30÷5=n

K 32÷8=n
L 12÷3=n
M 25÷5=n
N 81÷9=n
O 18÷3=n
P 28÷7=n
Q 64÷8=n
R 20÷4=n
S 27÷3=n
T 49÷7=n

Now you should know the important division equations for the numbers 40 to 81.

Thinking straight

1

square corner A
right angle

2

parallel lines B

3

C

5

D

Use Pictures A and B to help you answer these questions. 4

A In Picture C, the sides marked u and ■ are parallel.
B In Picture C, the sides marked t and v are ——.
C In Picture C, the corners marked w, ■, ■, and ■ are right angles.
D In Picture D, which sides are parallel?
E In Picture D, which sides are not parallel?
F In Picture D, which corners are right angles?
G Which corners are not right angles?

36

36

1 With the aid of Picture A, help pupils recall that a right angle is formed when two straight lines come together to make a square corner. Remind them also that they can tell whether an angle is a right angle or not with a piece of paper or a card with a square corner.

2 Discuss Picture B and bring out that any two straight lines that are the same distance apart are parallel.

3 Call pupils' attention to the fact that each of the four sides of this rectangle and each corner is identified by a letter. Let them compare the sides and angles with the lines and angles in Pictures A and B.

4 Have the children give reasons for their answers to Questions A to G.

5 Make sure the pupils know that Sides m and o are not parallel because the lines would meet if extended upward. Also be sure they know why the corners at r and s are not right angles.

37-38 Looking back

Expanded Notes for this lesson are on pages 268-269.

Objectives

The child learns more about division with remainders and develops skill in mental division.

Vocabulary

New words page 37 arrowheads*

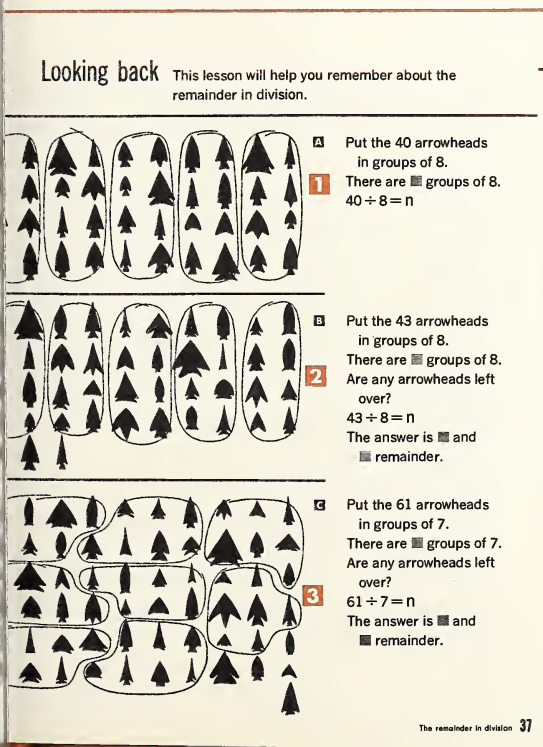
Comments

The child has already had some experience with remainders in division. In this lesson, his experience with remainders is extended to include all of the division basic facts. The work here enables the child to divide to such limits as 49÷5, 59÷6, 69÷7, 79÷8, and 89÷9. For example, 49 is the largest number divisible by 5 within the range of the basic facts involving 5 (49÷5=9 and 4 remainder).

All the division in this lesson involves situations in which the child knows the size of the equal groups and is to find the *number* of equal groups and the remainder, if any. This type of division has been used in all the teaching of the division basic facts in this book and in the preceding books of this program.

In finding the answers to the problems and exercises on these two pages, the child should not be required to use the computational form. Neither should any attempt be made to have the child memorize these examples and answers. He is expected to think of the basic fact he already knows and compute mentally to find the remainder. He may respond by saying the answers aloud or by writing them.

In the pictures on page 37, rings are used to show the equal groups into which the total group has been divided. The number in the remainder can thus be easily seen. You may want to supplement the work with these pictures by providing objects for the children to manipulate. Such a method has the advantage of letting each child make the actual physical division.


Looking back
This lesson will help you remember about the remainder in division.
A
Put the 40 arrowheads in groups of 8.
There are ■ groups of 8.
40 ÷ 8 = n
1
B
Put the 43 arrowheads in groups of 8.
There are ■ groups of 8.
Are any arrowheads left over?
43 ÷ 8 = n
The answer is ■ and ■ remainder.
2
C
Put the 61 arrowheads in groups of 7.
There are ■ groups of 7.
Are any arrowheads left over?
61 ÷ 7 = n
The answer is ■ and ■ remainder.
3
The remainder in division 37

37

1 Ask the children how many arrowheads there are in all. Have them note that there are 10 in each row. Then draw their attention to the fact that the arrowheads have been put into groups of 8. Let them discover that all the arrowheads are included in the 5 groups of 8. Ask, "Are there any left over?" Have someone read and complete the sentences at the right.

2 Ask the children how many arrowheads there are in all and how many are in each ringed group. Let them discover that there are 5 groups of 8 arrowheads and that 3 arrowheads are left over. Either let one child read aloud all the material at the right and say the missing numerals or have the children take turns reading a sentence.

3 Work with this picture and the accompanying text in the same way.

1

A Which numbers can be divided by 5 without a remainder?
35 36 37 38 39 40 41 42 43

B Which numbers can be divided by 8 without a remainder?
64 65 66 67 68 69 70 71 72

C Which numbers can be divided by 4 without a remainder?
26 19 32 11 16 28 39 20 36

D Which numbers can be divided by 7 without a remainder?
21 44 35 50 26 49 63 37 56

E Which numbers can be divided by 3 without a remainder?
14 22 27 18 10 9 15 13 20

F Which numbers can be divided by 8 without a remainder?
58 48 42 32 56 19 24 66 40

G Which numbers can be divided by 2 without a remainder?
16 3 15 18 7 12 19 10 14

H Which numbers can be divided by 6 without a remainder?
42 26 38 48 20 30 54 16 24

I Which numbers can be divided by 9 without a remainder?
29 45 63 73 81 36 84 27 54

2

A Is there a remainder when you divide 55 by 6?

B Is there a remainder when you divide 34 by 4?

C Is there a remainder when you divide 18 by 2?

D Is there a remainder when you divide 53 by 7?

E Is there a remainder when you divide 30 by 5?

F Is there a remainder when you divide 70 by 9?

G Is there a remainder when you divide 29 by 3?

H Is there a remainder when you divide 72 by 8?

3

A	56 ÷ 9 = n	24 ÷ 5 = n
B	17 ÷ 2 = n	31 ÷ 4 = n
C	50 ÷ 6 = n	66 ÷ 7 = n
D	36 ÷ 8 = n	75 ÷ 9 = n
E	33 ÷ 5 = n	14 ÷ 3 = n
F	55 ÷ 7 = n	32 ÷ 6 = n
G	22 ÷ 3 = n	19 ÷ 2 = n
H	38 ÷ 4 = n	65 ÷ 8 = n
I	53 ÷ 8 = n	40 ÷ 5 = n
J	29 ÷ 5 = n	81 ÷ 9 = n
K	84 ÷ 9 = n	58 ÷ 7 = n
L	30 ÷ 7 = n	27 ÷ 3 = n
M	47 ÷ 8 = n	19 ÷ 6 = n
N	46 ÷ 6 = n	26 ÷ 4 = n

Now you should understand that often there is a remainder when you divide.

38

38

1 Have the children take turns naming the numbers that can be divided without remainders. You might also have them write these numerals opposite Letters A to I on their papers. If you think it advisable, this exercise may be extended by having the children take turns giving the quotient and the remainder for the numerals in each exercise. For example, for Exercise A the child would say, "7 and no remainder, 7 and 1 remainder, 7 and 2 remainder, 7 and 3 remainder, 7 and 4 remainder, 8 and no remainder, 8 and 1 remainder, 8 and 2 remainder, 8 and 3 remainder." Or you might say, "Let's look at A, 39." The child should respond, "7 and 4 remainder."

2 Let the children take turns answering these questions orally, or have them write yes or no opposite Letters A to H on their papers.

3 Use these two blocks of exercises for oral work first; then have the children write the answers with remainders.

39 Checking up; Keeping skillful

Expanded Notes are not considered necessary for these lessons.

Objectives

The child tests his knowledge of the multiplication and division basic facts with products and dividends from 40 to 81. He also tests his knowledge of division with remainders. In the "Keeping skillful" exercises, he practices addition and subtraction of large numbers.

Vocabulary

There are no new words.

Comments

The first two "Checking up" tests are on the multiplication and division basic facts just studied. The third and fourth tests include division basic facts and division with remainders. Since these are not speed tests, you should give the children ample time to complete their work.

The children should be expected to know all the basic facts. One of the greatest sources of failure in arithmetic is inability to recall the basic facts. Therefore, if any weaknesses are revealed by these tests, time should be devoted to reteaching before proceeding further.

For such reteaching the pupils may be organized into groups. Those who know the basic facts should be excused from the review. For the children who need reteaching of the multiplication basic facts in the 40 to 81 groups, refer to the lesson on pages 27-30 of the textbook. Pages 31-36 may be reviewed by those who need work with the division basic facts for the same groups. For review on division with remainders, help the children restudy pages 37-38.

The pupils who have a few "trouble spots" may make practice cards for the basic facts they are finding difficult. See Activity 17, pages 363-364 for instructions for making these cards.

In the "Keeping skillful" exercises at the bottom of the page the child reviews other previously learned concepts and skills.

Answers

Checking up:

Remainder is indicated by the abbreviation "rm."

Test 3

A 7 and 3 rm.	I 6 and 3 rm.	O 3 and 1 rm.
B 8 and 1 rm.	J 5 and 3 rm.	P 2 and 4 rm.
C 4 and 2 rm.	K 4 and 5 rm.	Q 3 and 1 rm.
D 5 and 4 rm.	L 7	R 9 and 2 rm.
E 6 and 1 rm.	M 7	S 3 and 3 rm.
F 5	N 6 and 2 rm.	
G 9 and 2 rm.		
H 4 and 3 rm.		

Test 4

A 9 and 1 rm.	I 8 and 1 rm.	O 9
B 9	J 7 and 7 rm.	P 8 and 7 rm.
C 5 and 3 rm.	K 7 and 3 rm.	Q 9 and 3 rm.
D 6 and 6 rm.	L 7 and 1 rm.	R 5 and 6 rm.
E 8 and 4 rm.	M 7 and 2 rm.	S 8 and 5 rm.
F 6 and 5 rm.	N 8	
G 5 and 4 rm.		
H 8 and 1 rm.		

Keeping skillful:

(block 1)	(block 2)	(block 3)
A 839	A 23	A 435
B 160	B 174	B 377
C 127	C 52	C 126
D 900	D 623	D 4604
E 1627	E 225	E 3156
F 549	F 129	F 507
G 2313	G 227	G 1238
H 1712	H 2104	H 2042
I 1200	I 10,675	I 629

Checking up

Test 1	Test 2 **2**	Test 3 **3**	Test 4 **4**
A $8\times8=n$	A $42\div7=n$	A $38\div5=n$	A $82\div9=n$
B $7\times9=n$	B $48\div6=n$	B $25\div3=n$	B $63\div7=n$
C $7\times7=n$	C $40\div8=n$	C $18\div4=n$	C $48\div9=n$
D $8\times5=n$	D $45\div9=n$	D $29\div5=n$	D $60\div9=n$
E $6\times7=n$	E $49\div7=n$	E $13\div2=n$	E $52\div6=n$
F $8\times6=n$	F $54\div9=n$	F $40\div8=n$	F $47\div7=n$
G $5\times9=n$	G $63\div7=n$	G $29\div3=n$	G $44\div8=n$
H $9\times9=n$	H $42\div6=n$	H $27\div6=n$	H $41\div5=n$
I $8\times7=n$	I $40\div5=n$	I $39\div6=n$	I $57\div7=n$
J $6\times8=n$	J $64\div8=n$	J $23\div4=n$	J $70\div9=n$
K $9\times8=n$	K $54\div6=n$	K $37\div8=n$	K $59\div8=n$
L $7\times6=n$	L $45\div5=n$	L $28\div4=n$	L $50\div7=n$
M $5\times8=n$	M $48\div8=n$	M $21\div3=n$	M $44\div6=n$
N $9\times5=n$	N $72\div9=n$	N $32\div5=n$	N $72\div9=n$
O $8\times9=n$	O $56\div8=n$	O $10\div3=n$	O $45\div5=n$
P $6\times9=n$	P $81\div9=n$	P $20\div8=n$	P $71\div8=n$
Q $7\times8=n$	Q $72\div8=n$	Q $16\div5=n$	Q $57\div6=n$
R $9\times7=n$	R $63\div9=n$	R $38\div4=n$	R $41\div7=n$
S $9\times6=n$	S $56\div7=n$	S $30\div9=n$	S $77\div9=n$

Keeping skillful

	5	**5**
J $532+307=n$	A $n+49=72$	A $856-421=n$
I $49+37+74=n$	B $83+91=n$	B $605-228=n$
E $60+26+41=n$	C $28+n=80$	C $300-174=n$
D $189+436+275=n$	D $346+277=n$	D $9730-5126=n$
E $330+794+503=n$	E $584+n=809$	E $7054-3898=n$
F $58+162+329=n$	F $n+608=737$	F $5226-4719=n$
G $897+645+771=n$	G $173+n=400$	G $2003-765=n$
H $254+920+538=n$	H $3896+n=6000$	H $4805-2763=n$
I $341+650+209=n$	I $7994+2681=n$	I $6001-5372=n$

End-of-block tests on multiplication and division basic facts with products and dividends from 40 to 81; division with remainders 39

39

1 Have the pupil write the identifying letter in one column on his paper and at the right in another column write the equation with the answer.

2 Use Note 1.

3 Have the pupil copy the identifying letter in one column on his paper and at the right in another column write the equations with answers and remainders (A $38\div5=7$ and 3 remainder). Or direct him to write only the answer and remainder at the right of the identifying letter (A 7 and 3 remainder). If you wish, let him use "rm." as an abbreviation for remainder.

4 Use Note 3.

5 Have the pupils write the numerals in computational form and add or subtract as necessary.

40-42 Exploring problems

Expanded Notes for this lesson are on pages 270-271.

Objectives

The child learns to use addition to solve problems in which he must find how many items there were at first when the number gone and the number left are known.

Vocabulary

New words page 42 parakeet, materials*, died*, carefully*, fresh*, vegetables*

Comments

In this lesson the children review problems involving situations where they do not know how many objects there were to begin with, but they do know how many were removed and how many are left. This type of problem was originally taught in *Seeing Through Arithmetic 4*, pages 134-137.

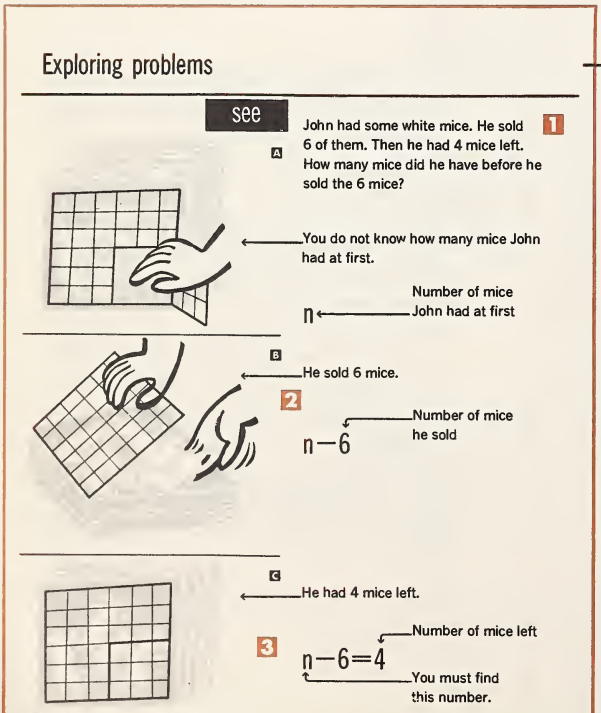
The children learn through pictures and text how to make the equation that symbolizes what happened in the problem situation. They see how to build the equation. The important thing for them to understand in this type of problem is why the *n* is placed first in the equation.

To help the children understand why addition is used to find the numeral that replaces *n* in the equation, they are led to imagine the group that was taken from the original group as put back with the group remaining and that the sum of these two groups is the number they are to find. In a situation where groups of objects do not actually move together into one group, but in which the sum is required, the imagined joining will be indicated by a dotted background (see Picture F on page 41).

Answers

Page 42:

A [$3.23]−$.87=$2.36
B [18]−5=13
C [33]−18=15
D 31+29=[60] or 29+31=[60]
E [$1.93]−$.35=$1.58
F $.16+$.37=[$.53] or $.37+$.16=[$.53]
G $2.78+[$2.22]=$5.00
H 19−7=[12]
I [24]−9=15


Exploring problems
see
John had some white mice. He sold 6 of them. Then he had 4 mice left. How many mice did he have before he sold the 6 mice?
1
A
You do not know how many mice John had at first.
n
Number of mice John had at first
B
He sold 6 mice.
2
n−6
Number of mice he sold
C
He had 4 mice left.
3
n−6=4
Number of mice left
You must find this number.
40 Reteaching of finding how many items there were at first when the number gone and the number left are known

40

1 Have the pupils read this problem and relate it to Picture A. They should realize that they cannot tell from the picture how many mice there were in the box before John sold the 6 mice. They should understand that this is why *n* comes first in the equation. Make sure the pupils observe the subtractive action suggested by the picture.

2 Explain that Picture B shows that 6 mice have been taken out of the large box. Have them relate to the equation the completed action of taking away 6 mice.

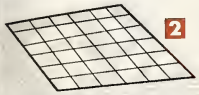
3 This picture shows how many mice John kept for himself, that is, the number left after he had sold 6 of them. Call on someone to explain how the equation shows this.

compute

D

You know that John had 4 mice left.

1 Number of mice left: 4



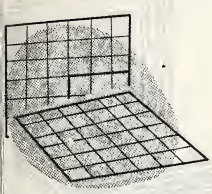
E

You also know that he sold 6 mice.

Number of mice John sold: 6

F

Imagine that the group of 6 mice is put back with the group of 4 mice.



3

4
6
Add 6 to 4.

4
6
10 He had 10 mice.

The sum of 4 and 6 tells how many mice John had to begin with.

4 $n-6=4$

This number is 10. John had 10 mice to begin with.

41

41

1 Explain that Picture D shows the 4 mice John had left. It is the same situation pictured at the bottom of page 40. Point out that the numeral 4 at the right represents this situation.

2 Point out that Picture E shows the other group we know about—the 6 mice John sold.

3 Through discussion, help the pupils see that Picture F shows the two groups of mice—the 4 mice John kept and the 6 he sold. Explain that if we imagine the 6 mice that he sold as put back with the 4, we will know how many there were to begin with. Tell the children what the dotted background suggests [forming one group, or addition]. Direct attention to the computational form shown at the right. Ask the children to add to find the missing number.

4 Let pupils read and discuss this computation. Then let them test the answer in the equation.

Dick made a playground for his parakeet. After he had spent $.87 for boards and sticks, he had $2.36 left in his bank. How much money was in his bank before he bought materials for the playground? 1

Total amount in Dick's bank.

Amount spent for materials.

$-\$.87=\$2.36$

Amount left.

You must find this number.

Add $.87 to $2.36.

There was ■ in Dick's bank before he bought the materials.

Peggy bought some goldfish. Five of them died. Now she has 13 goldfish left. Peggy had how many goldfish before the five fish died? 2

$-5=13$ Add 5 to 13.

Ann says that food for rabbits should be carefully weighed. One day she gave her rabbits 18 oz. of carrots. Then she had 15 oz. of carrots left. How many ounces of carrots did she have to begin with?

$-18=15$ Add 18 to 15. 2

D Ann feeds her rabbits only fresh vegetables. Each day the rabbits need an amount of vegetables equal in weight to their own weight. One rabbit weighs 31 oz. The other weighs 29 oz. How many ounces of fresh vegetables in all should Ann give her rabbits each day? 3

E After Carol paid 35¢ for two mice, she had $1.58. How much money did she have before she bought the mice?

F Nancy bought a bell and some seed for her parakeet. The bell cost $.16. The seed cost $.37. How much did Nancy spend for these two things? 4

G Nancy is trying to save $5 to buy a new cage for her parakeet. So far she has saved $2.78. How much more money does she need to save? 4

H Dick says that his parakeet can say 7 words. Nancy says that her parakeet can say 19 words. Nancy's parakeet can say how many more words than Dick's parakeet? 4

I One Saturday David bought some cans of food for his cats. The cats ate 9 cans during the week. Then there were 15 cans left. How many cans of cat food had David bought? 5

In many of these problems you knew how many things were taken away and how many were left. Then you found how many things there were to begin with.

42

1 Use this problem orally. Develop the equation on the board, and have the children explain what each numeral in it stands for. Stress the fact that after they have made the equation, they should always think what they should do next with the numerals in it, as "Add $.87 to $2.36." Then they should write the numerals in computational form and compute. Let someone test the equation after the answer has been inserted.

2 Use the same procedures with this problem.

3 Let the pupils work the rest of the problems. For each they should write the equation with *n* for the unknown number, compute, and rewrite the equation with the answer.

4 This problem has an equation different from the one introduced in this lesson. Be sure the children write the proper one.

5 Provide answers so that the children may verify their work. Then discuss the problems that caused the most trouble.

43-45 Exploring problems

Expanded Notes for this lesson are on pages 271-273.

Objectives

The child learns to make equations for problems in which he must find how many are gone when he knows the number there were to begin with and the number remaining.

Vocabulary

New words page 43 potholders*; page 44 printed*, spoiled*, George*, press*; page 45 fudge, foreign, pipe*, cleaners*

Comments

In this lesson the children review a type of problem situation which was taught in *Seeing Through Arithmetic 4,* pages 130-133. In this type of situation the number of objects there were to begin with and the number remaining are known. The problem is to find how many were removed. This is a subtractive situation. The pictures and explanations in this lesson will make it clear to the children why they can use the process of subtraction to find how many were removed.

In the "See" step of the first problem on page 43, the pupils learn through pictures and text how to make the equation that represents what happened in the problem situation. The important thing for them to understand in this type of problem is that *n,* which represents the number they are to find (the unknown group that has been removed), is placed immediately to the right of the minus sign.

The "Compute" step shows the children why they can subtract to get the answer. If the children have difficulty with this type of problem, use objects to show just what happens in the problem and what happens in the solution.

Problems A to M on pages 44 and 45 provide practice in handling the subtractive situation discussed above. They also provide practice on the other subtractive situation that is solved by the subtraction process (finding the number left when the original number and the number gone are known), on comparative subtraction (finding how many fewer or more one quantity is than another), and on additive subtraction (finding how many were added or how many more are needed when the original number and the resulting total or desired total are known).

Answers

Exploring problems:

In a problem like J the equation $6.50−$3.69=n should be accepted if the child can justify it.

A 30−[14]=16
B 50−[9]=41
C 63−[25]=38
D 64−[46]=18
E $2.00−[$1.63]=$.37
F 81−[26]=55
G 49−36=[13]
H 37−19=[18]
I $1.50−[$.82]=$.68
J $3.69+[$2.81]=$6.50
K 17−[11]=6
L 96−[88]=8
M 96−24=[72]

Keeping skillful:

(block 1)	(block 2)	(block 3)
A 47	A 58	A 172
B 39	B 23	B 531
C 256	C 195	C 407
D 244	D 232	D 265
E 970	E 1509	E 3548
F 1245	F 13,923	F 2415
G 1277	G 11,099	G 794

see

A

Patsy made 23 potholders. She sold some of them one Saturday. Then she had 11 potholders left. How many potholders had she sold? 1

Patsy made 23 potholders.

23 ← 2 Number of potholders made

B

You do not know how many potholders she sold.

23 − n ← Number of potholders sold

3

There were 11 potholders left.

C

23 − n = 11

Number of potholders left

You must find this number.

4

compute

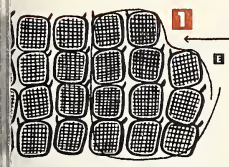
You know that Patsy had 23 potholders to begin with.

D

23 ← Number of potholders to begin with

5

Now turn the page.

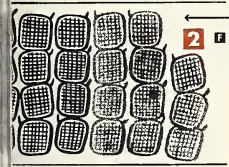


1

Think of a group of potholders equal in number to the potholders left.

E

11 ← Number of potholders left



2

Imagine that this group of 11 potholders is gone.

F

23
11 ← Subtract 11 from 23.
3

3
1
2 ← The number left tells how many potholders Patsy sold. She sold 12.

4

23 − n = 11

This number is 12. Patsy had sold 12 potholders.

Jane made 30 bookmarks. She gave me away. Then she had 16 of them left. How many did she give away? 5

Number of bookmarks she made.

Number she gave away.

Number of bookmarks left.

0 − n = 16

You must find this number.

btract 16 from 30.

ne gave away ■ bookmarks.

B George printed 50 calling cards for his father on his new printing press. He spoiled some and had to throw them away. He had 41 cards left. How many cards had he spoiled? 6

50 − n = 41 Subtract 41 from 50.

C George also printed 63 tickets for a pet show. After he and his friends had sold some tickets, they had 38 left. How many tickets had they sold? 6

63 − n = 38 Subtract 38 from 63.

43

1 Draw attention to Problem A. Then call on someone to relate Picture A to the problem. Make sure the class understands that only the total group is shown in the picture.

2 Explain that this numeral stands for all the potholders Patsy made.

3 Help the children see that this picture indicates that some of the potholders are being removed but that we do not know how many. Have pupils relate the picture to the equation at the right. Make sure they realize that the minus sign indicates that some potholders are being removed and that *n* indicates that we do not know how many.

4 This picture shows the 11 potholders that were left. Have the children discuss how this is shown in the equation at the right.

5 Tell the pupils that here the potholders have been rearranged so that we can easily see how many there were to begin with.

44

1 Explain that the circled group shows the 11 potholders that were left. Tell pupils that any group of 11 potholders could be circled.

2 Have the pupils note that the group dimmed off is the same group of 11 that is circled above. Help them understand that, when we know how many there were in the beginning and how many were left, we can imagine the ones that were left as being removed. Those remaining are the potholders that were sold. Relate this to the computation at the right.

3 Tell the pupils to write the numerals in computational form.

4 Pupils should see that 12 is the answer because it makes the equation correct.

5 Have them read this problem and discuss each number in it. Note the statement below the equation—"Subtract 16 from 30." Tell them to *think* this statement, then write the numerals in computational form to get the answer.

6 Proceed in the same way with this problem.

D Carol and her sister made 64 pieces of fudge one day. The next day only 18 pieces were left. How many pieces of fudge were gone? 1

E Patsy had $2. She bought materials to make potholders. Then she had $.37. How much money had she spent for materials? 2

F David sold some of his 81 foreign stamps to Tom. Then David had 55 foreign stamps. How many foreign stamps did he sell to Tom?

G Betty made 36 pieces of fudge and 49 pieces of peanut candy. She made how many fewer pieces of fudge than pieces of peanut candy? 2

H Jim had 37 baby fish. He sold 19 of them. How many baby fish did he have then? 2

I Jim went to the pet store with $1.50. He bought fish food and plants and came home with $.68. How much money had he spent at the pet store?

J A bird that Jim wants to buy costs $6.50. He has saved $3.69 for it. How much more money does he need? 2

K Ann made 17 dolls with pipe cleaners. She took some dolls to school and kept 6 dolls at home. How many dolls did she take to school? 2

L When Ann started to make dolls, she had 96 pipe cleaners. After she had finished making the dolls, there were 8 pipe cleaners left. How many pipe cleaners had Ann used?

M Twenty-four of the 96 pipe cleaners were white. The rest were red. How many pipe cleaners were red? 2

3 In many of these problems you were finding how many things were given away or used up or sold.

Keeping skillful 4

A $84-n=37$	A $48+n=106$	A $400-228=n$
B $90-n=51$	B $n+29=52$	B $876-345=n$
C $n-176=80$	C $56+139=n$	C $504-n=97$
D $403-n=159$	D $n+704=936$	D $821-556=n$
E $n-628=342$	E $581+n=2090$	E $7837-n=4289$
F $3710-n=2465$	F $4615+9308=n$	F $5080-2665=n$
G $2005-n=728$	G $8673+2426=n$	G $6001-5207=n$

45

45

1 Let the children work independently on Problems D to M without prior discussion. Instruct the pupils to write the equation with *n*, do the computation, and rewrite the equation with the answer in it for each problem.

2 Be sure the pupils see that these problem situations are different from the rest in this problem set. They are to show this difference by the equations they make.

3 When the pupils have finished solving these problems as written work, discuss the problems that have caused the most trouble. Let the pupils help one another with the problems that caused only minor difficulty or with which only a few children had trouble.

4 Assign these exercises as written work. The pupils are to write the numerals in computational form and find the answers.

46-47 Using arithmetic

Expanded Notes are not considered necessary for this lesson.

Objectives

The child solves verbal problems of the various types studied so far in this book. He also learns to solve problems by using answers or information from previous problems.

Vocabulary

New words page 46 pageant, Indians*, Pilgrims, wigwams*, feathers*; page 47 dancing*, begun*

Comments

Since problems in textbooks should simulate real life situations, and since in real life situations problems do not always come singly but in "chains" or sequences, authors of arithmetic texts should provide situations in which the work in one problem depends upon the data or work in preceding problems. Failure to make this provision leaves the child inadequately prepared for the more complex arithmetic situations of his everyday life. This set of problems provides a number of such "chains."

Answers

If a child makes an equation other than the one indicated below for a problem, accept the equation if it results in a correct answer and if he can justify it. In the equations for Problems E and L the order in which the numerals are written is not important.

A $151-87=[64]$
B $87-64=[23]$
C $87-64=[23]$
D $[\$10.80]-\$3.55=\$7.25$
E $\$.65+\$.72+\$4.99=[\$6.36]$
F $\$7.00-\$6.36=[\$.64]$
G $89¢-[44¢]=45¢$
H $42\div 6=[7]$
I $9\times 8=[72]$
J $9\times 6=[54]$
K $27\div[3]=9$
L $72+54+27=[153]$
M $72-27=[45]$
N $72-54=[18]$
O $54-27=[27]$
P $56\div[7]=8$
Q $[35]+18=53$
R $35-18=[17]$
S $27+[33]=60$
T $[32]+24=56$
U $24+[16]=40$
V $24-16=[8]$
W $24-16=[8]$

Using arithmetic 1

A At the Lake School, 151 children were in a pageant. 87 were dressed as Indians, and the others were dressed as Pilgrims. How many children were dressed as Pilgrims?

B There were how many more Indians than Pilgrims in the pageant?

THINK What numbers from Problem A do you need?

C There were how many fewer Pilgrims than Indians?

D The fifth graders collected money to buy materials for the pageant. They spent $3.55 of this money. Then they still had $7.25 left to spend. How much money had they collected?

E Betty and Jim went to the store to buy more materials. They spent $.65 for white paper, $.72 for black paper, and $4.99 for paint. How much money did they spend?

F Jim gave the clerk $7.00 to pay for the paper and paint. How much change should the clerk have given him?

G On the way back to school, Jim said, "I know I had 89¢ when we left the store, but now I have only 45¢. I must have lost some money." How much money had Jim lost?

H 42 children made wigwams for the pageant. 6 children worked on each wigwam. How many wigwams did the 42 children make?

I Bob made 9 Indian headdresses. He used 8 red feathers in each one. How many red feathers did he use in making the 9 headdresses?

J He used 6 blue feathers in each headdress. How many blue feathers did he use altogether?

K He put an equal number of yellow feathers in each headdress. For the 9 that he made, he used 27 yellow feathers. How many yellow feathers did he put in each headdress?

L How many feathers altogether did Bob use in making the headdresses? 3

M He used how many fewer yellow feathers than red feathers?

N He used how many more red feathers than blue feathers?

O He used how many more blue feathers than yellow feathers?

P Each of 8 girls made an equal number of paper collars. Altogether, the 8 girls made 56 collars. How many collars did each girl make?

A Mary sold tickets for the pageant in the morning. In the afternoon she sold 18 more tickets. Then she said, "I sold 53 tickets today." How many tickets had she sold in the morning? 1

B She sold how many fewer tickets in the afternoon than in the morning?

C Jack sold 27 tickets in the morning. In the afternoon he sold more tickets. Then he said, "I sold 60 tickets today." How many tickets had he sold in the afternoon?

D The pageant began with a group of Indians dancing. After 24 more Indians had joined the first group and had begun dancing, there were 56 Indians dancing. How many of them were dancing when the pageant began?

U Don counted some Pilgrims as they marched by. First, 24 Pilgrim boys marched by. Next came some Pilgrim girls. After Don had counted them, he said, "That makes 40 Pilgrims in all." How many of the Pilgrims were girls?

V How many more boys than girls did Don count?

W How many fewer girls than boys did Don count? 2

Looking back

6 tens = 60	A 5 × 8 tens = 40 tens	A 3 × 70 is 3 × 7 tens.
12 tens = 120	40 tens = 40~	3 × 7 tens = ■
6 hundreds = 600	B 4 × 2 hundreds =	B 2 × 600 is
12 hundreds = 1200	8 hundreds	2 × 6 hundreds.
23 tens = 23~	8 hundreds = 8~	2 × 6 hundreds = ■
54 tens = ■	C 3 × 6 tens = ■	C 7 × 50 = n
38 tens = ■	D 7 × 7 tens = ■	D 9 × 60 = n
20 tens = ■	E 2 × 6 tens = ■	E 8 × 200 = n
41 hundreds = 41~	F 8 × 4 tens = ■	F 7 × 400 = n
33 hundreds = ■	G 9 × 3 tens = ■	G 5 × 40 = n
27 hundreds = ■	H 6 × 5 tens = ■	H 6 × 300 = n
64 hundreds = ■	I 5 × 4 hundreds = ■	I 9 × 500 = n
19 hundreds = ■	J 2 × 7 hundreds = ■	J 8 × 80 = n
72 hundreds = ■	K 8 × 3 hundreds = ■	K 8 × 700 = n
90 hundreds = ■	L 7 × 9 hundreds = ■	L 3 × 900 = n

46

1 Have the children read each problem and write the equation that describes what is happening, using *n* to hold a place for the unknown number. Then have them do the necessary computation and rewrite the equation, supplying the numeral that replaces *n*.

2 Call attention to the fact that this problem requires information from Problem A. Warn the children that they will have to determine in subsequent problems, without the help of a "Think" paragraph, whether information is to be taken from previous problems.

3 Note that in this problem the answers from Problems I and J and information from Problem K are needed to find the answer.

47

1 Continue as on page 46 with the problems on this page.

2 Provide answers so that the children may verify their work. Then discuss the problems that caused the most trouble.

47 Looking back

Expanded Notes for this lesson are on page 273.

Objectives

The child reviews what he has learned about the number system as it relates to multiplication. He also reviews multiplication of numbers represented by two-figure and three-figure numerals in which all figures but one are zeros.

Vocabulary

There are no new words.

Comments

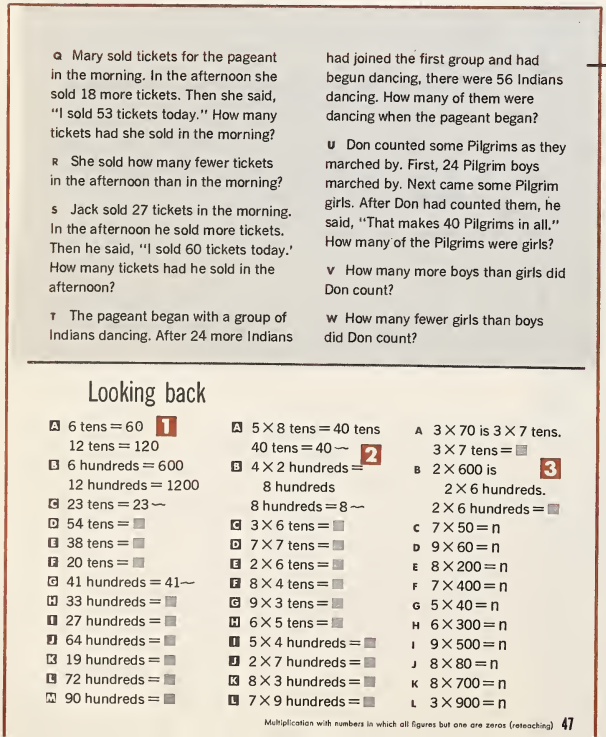
For children who used *Seeing Through Arithmetic 4,* this lesson will be a review of page 138 and pages 140-141 of that book.

On page 47 of this book, the first block of exercises reviews the number system in its relation to multiplication. All fifth-grade pupils should be familiar with the regrouping of tens into hundreds and hundreds into thousands, and know how to write the resulting numerals. In reading such numerals aloud, however, there may be more than one way to say them correctly. For example, Exercise G in the first block may be read either as "forty-one hundred" or as "four thousand, one hundred."

The second block of exercises reviews multiplication of tens and hundreds by multipliers whose numerals have one figure. With this experience the child should have no difficulty in seeing immediately the tens and hundreds he is asked to multiply in the third block of work.

Answers

(block 1)		(block 2)		(block 3)	
C 230	I 2700	A 400	G 270	A 210	G 200
D 540	J 6400	B 800	H 300	B 1200	H 1800
E 380	K 1900	C 180	I 2000	C 350	I 4500
F 200	L 7200	D 490	J 1400	D 540	J 640
G 4100	M 9000	E 120	K 2400	E 1600	K 5600
H 3300		F 320	L 6300	F 2800	L 2700



Q Mary sold tickets for the pageant in the morning. In the afternoon she sold 18 more tickets. Then she said, "I sold 53 tickets today." How many tickets had she sold in the morning?

R She sold how many fewer tickets in the afternoon than in the morning?

S Jack sold 27 tickets in the morning. In the afternoon he sold more tickets. Then he said, "I sold 60 tickets today.' How many tickets had he sold in the afternoon?

T The pageant began with a group of Indians dancing. After 24 more Indians had joined the first group and had begun dancing, there were 56 Indians dancing. How many of them were dancing when the pageant began?

U Don counted some Pilgrims as they marched by. First, 24 Pilgrim boys marched by. Next came some Pilgrim girls. After Don had counted them, he said, "That makes 40 Pilgrims in all." How many of the Pilgrims were girls?

V How many more boys than girls did Don count?

W How many fewer girls than boys did Don count?

Looking back

A 6 tens = 60 **1**
12 tens = 120
B 6 hundreds = 600
12 hundreds = 1200
C 23 tens = 23~
D 54 tens = ■
E 38 tens = ■
F 20 tens = ■
G 41 hundreds = 41~
H 33 hundreds = ■
I 27 hundreds = ■
J 64 hundreds = ■
K 19 hundreds = ■
L 72 hundreds = ■
M 90 hundreds = ■

A 5 × 8 tens = 40 tens
40 tens = 40~ **2**
B 4 × 2 hundreds = ■
8 hundreds
8 hundreds = 8~
C 3 × 6 tens = ■
D 7 × 7 tens = ■
E 2 × 6 tens = ■
F 8 × 4 tens = ■
G 9 × 3 tens = ■
H 6 × 5 tens = ■
I 5 × 4 hundreds = ■
J 2 × 7 hundreds = ■
K 8 × 3 hundreds = ■
L 7 × 9 hundreds = ■

A 3 × 70 is 3 × 7 tens.
3 × 7 tens = ■
B 2 × 600 is **3**
2 × 6 hundreds.
2 × 6 hundreds = ■
C 7 × 50 = n
D 9 × 60 = n
E 8 × 200 = n
F 7 × 400 = n
G 5 × 40 = n
H 6 × 300 = n
I 9 × 500 = n
J 8 × 80 = n
K 8 × 700 = n
L 3 × 900 = n

47

1 Examples A and B show the child how to annex zeros to write hundreds and tens. Have the children respond orally to Exercises C to M by saying the numeral that should come after the equals sign. The wavy line indicates that the numeral should be completed.

2 Use Exercises A and B to develop the idea that tens and hundreds are multiplied just as ones are multiplied. Explain that 400 is just another way of writting 40 tens. Have the children respond orally to Exercises C to L. These exercises may also be assigned as written work.

3 Use Exercises A and B to develop the procedure of multiplying numbers whose numerals end in one or more zeros. Be sure the pupils understand that they should think of 70 as 7 tens and 600 as 6 hundreds when they multiply. Use Exercises C to L orally. Then assign them as written work, if you wish.

48-52 Learning how

Exponded Notes for this lesson ore on pages 274-275.

Objectives

The child reviews multiplication of a number whose numeral has two figures by a number whose numeral has one figure.

Vocabulary

New words page 48 addressing*, invitations*, envelopes; page 50 doughnuts; page 52 salted*, cider

Comments

Since multiplication by a one-figure multiplier was taught in *Seeing Through Arithmetic 4* (pages 142-146), the material in this lesson will be a review for many of the children in your class. You may want to use Exercises E to L in the first block of exercises in the DO step on page 52 to find out how much the children remember about multiplication by one-figure multipliers. Use the results in judging the amount of reteaching you will need to do. However, be sure there is a thorough understanding of multiplication by one-figure multipliers before proceeding to multiplication with multipliers of more than one figure.

Children who complete the inventory test suggested above with evident ease should be allowed to do other work while you are teaching the rest of the class. These children might like to spend their time reading the book described in the footnote below.*

Watch for the child who adds the carried number before he multiplies. This mistake may arise because in addition he was able to add the carried number immediately. Careful teaching of multiplication with carrying will avoid this difficulty.

The process of multiplication is introduced with an example that requires carrying. In this way the child gets an idea of the whole multiplication process. This makes it possible to teach multiplication by one-figure multipliers in one series of lessons rather than in several. This procedure does not give the child an oversimplified idea of the process, and it enables you to use whatever practice material you wish, without determining whether it demands untaught skills.

* *Number Stories of Long Ago*, David Eugene Smith, National Council of Teachers of Mathematics, Washington, D.C., 1955. Exercises on pages 105-108.

The four-step teaching method is used in this lesson as it is throughout this program to develop all computational skills. Steps are provided to help the child SEE (by means of illustrations and explanatory text); THINK (by means of illustrations and questions about a problem); TRY (by means of sample problems and examples worked out with answers for comparison); and DO (by means of practice material without guidance). Special material is provided for each of the four learning steps.

These four steps are based on a teaching method that successful teachers have used consistently in presenting new material. In this book the steps are systematically employed and are implemented by pictures, explanations, questions, examples, and problems. The SEE and THINK steps are intended to be taught under the guidance of the teacher with the aid of the book. The TRY and DO steps are intended to be used independently by pupils. They also furnish a means of providing for individual differences in pupils. The teacher should work with those who need assistance in the TRY step, at the time when they most need help. The better students may work as many problems or examples in the TRY step as they need, then go immediately to the DO step.

Answers

Page 52:

(block 1)

A 72	F 123	K $.92	P 128
B $5.70	G 310	L 105	Q 152
C 96	H $1.12	M 330	R 320
D 96	I 78	N $1.06	S $3.51
E 245	J 188	O 693	T 65

(block 2)

A 456	E 126	I 180	M 152
B 552	F 174	J 594	N 534
C $2.40	G 600	K 370	O $2.00
D 396	H $1.68	L 232	P $6.65

Learning how

In this lesson you will learn how to multiply numbers that have two figures in them.

see

Four girls were addressing invitations to a party at school. Sue said, "If each one of us takes 17 envelopes, we will finish the work in a few minutes." How many envelopes did the girls have to address?

$4 \times 17 = n$ — You must find this number.

Put the 4 groups of 7 envelopes together.

Multiply 7 ones by 4. There are 28 ones.

$$\begin{array}{r} 17 \\ \times\ 4 \\ \hline \end{array}$$

Make 2 new piles of 10 envelopes. You have 8 envelopes left over.

$$\begin{array}{r} 17 \\ \times\ 4 \\ \hline 8 \end{array}$$

28 is 2 tens 8 ones. Write 8 here to show that there are 8 ones in the answer.

Put the 2 new piles of 10 envelopes aside.

$$\begin{array}{r} 17 \\ \times\ 4 \\ \hline 8 \end{array}$$

Remember that you have 2 more tens.

Now put the 4 old piles of 10 envelopes together.

Multiply 10, or 1 ten, by 4. There are 4 tens.

$$\begin{array}{r} 17 \\ \times\ 4 \\ \hline 8 \end{array}$$

Put the 2 new piles of ten with the 4 piles of ten.

You had 2 tens to remember. Now add them to the 4 tens. There are 6 tens in all.

$$\begin{array}{r} 17 \\ \times\ 4 \\ \hline 68 \end{array}$$

Write 6 here to show there are 6 tens in the answer.

The girls had 68 envelopes to address.

$4 \times 17 = 68$

48

1. Have the children look at Picture A and observe four groups of envelopes. Then have them read the problem and relate it to the picture. They should see that there are 4 groups of envelopes and 17 in each group.
2. Have the children look at the equation and explain it in terms of the picture and problem.
3. Get the children to observe that first the ones are put together into one group. They should see that there are 28 ones.
4. Have the children look at the computation. Be sure they know they must multiply the ones first. Relate the multiplication to the picture by noting that 7 is the size of each group and 4 the number of groups.
5. The pupils should observe that the 28 envelopes have been put into 2 piles of 10 with 8 left over.
6. Relate the 8 in the answer to the 8 envelopes left over.

49

1. Have the children observe that the 2 new piles of 10 envelopes each are being pushed aside. Point out that the 2 new tens must be remembered in the computation.
2. Point out that the 4 old piles of 10 envelopes are being pushed together. This action is accomplished in the computation when the children multiply 1 ten in 17 by 4. Be sure they realize that they now have 4 tens, not just 4, and that 4 tens equal 40.
3. Have the class notice that the 2 new piles of 10 envelopes are being put with the 4 old piles of 10.
4. When the remembered 2 tens are added to the 4 tens, there are 6 tens, or 60. Relate the picture to the computation.
5. Have the children note that Picture G shows 68 envelopes. Have them explain what each numeral in the equation represents. Relate this picture and equation to the original equation and problem on page 48.

think

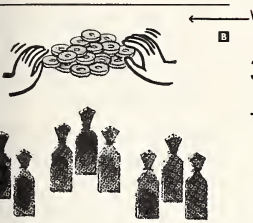
A

Three girls promised to bring doughnuts to the school party. Each girl promised to bring 36 doughnuts. How many doughnuts in all did the girls promise to bring?

1

$3 \times 36 = n$

You must find this number.



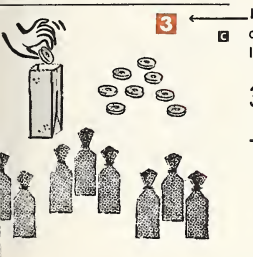
B

What do you put together first?

36
3

What do you multiply first?
How many ones are there?

2



3

C

How many new bags of 10 doughnuts can you make? Will any doughnuts be left over?

36
3
8

18 is 1 ten ■ ones.

4

Why do you write 8 here?

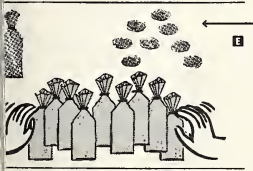
D

What do you do with the new bag of 10 doughnuts?

1

36
3
8

What must you remember?



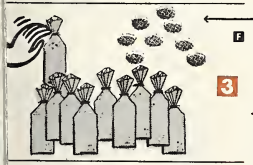
E

Now what do you put together?

36
3
8

Now what do you multiply?
How many tens are there?

2



F

What do you do with the new bag of 10?

3

36
3
108

Why do you write 10 here instead of 9?

G

The girls promised to bring ■ doughnuts to the party.

$3 \times 36 = n$

4

50

1 For this problem, which illustrates the THINK step of the four-step teaching method, the child must answer questions. Have the pupils observe the picture, then read the problem and relate it to the picture. Next they should examine the equation and explain what each part of it stands for in the picture and in the problem.

2 Relate the figures in the ones' column to what is happening in the picture. Let the children answer and discuss the questions.

3 Have the children answer the questions and relate them to the picture.

4 Direct attention to the computation. Be sure the children understand why 8 is put in the answer.

51

1 Ask the children to answer the two questions by relating them to what is happening in the picture. They should understand that they must remember 1 ten, not just 1.

2 Relate the two 3's in the computation to the picture and have the children multiply the tens. Be sure they understand that the 3 in 36 means 3 tens.

3 Relate the picture to the computation. The pupils must realize that the 10 means 10 tens and be able to tell why there are 10 tens and not 9 tens. They should recognize 10 tens as being equal to 1 hundred.

4 Tell the children to read the statement, supplying the name of the number when they come to the screen. Then have them read and complete the equation. Finally, have them relate the statement to Picture G and the equation to the original problem and equation on page 50.

try

A Mrs. Fisher bought 5 lb. of salted peanuts for the party. The salted peanuts cost $.49 a pound. How much did the 5 pounds cost?
$5 \times \$.49 = n$

1

$$\begin{array}{r} \$\ .49 \\ \times\ \ 5 \\ \hline \$2.45 \end{array}$$

Multiply just as you do with any numbers.

← Remember to put a point and dollar sign in the answer.

B $85 \times 8 = n$

2

$$\begin{array}{r} 85 \\ \times\ \ 8 \\ \hline 680 \end{array}$$

← Always multiply by the smaller number.

C $9 \times 48 = n$

$$\begin{array}{r} 48 \\ \times\ \ 9 \\ \hline 432 \end{array}$$

D $3 \times 33 = n$

$$\begin{array}{r} 33 \\ \times\ \ 3 \\ \hline 99 \end{array}$$

do

A Mrs. Black bought 12 packages of paper cups. There were 6 cups in a package. She bought how many cups in all?
$12 \times 6 = n$

B Mrs. Page bought 6 gallons of apple cider at $.95 a gallon. How much did the 6 gallons of cider cost?
$6 \times \$.95 = n$

C A gallon of cider should fill 16 cups. How many cups should 6 gallons of cider fill?
$6 \times 16 = n$

D Mrs. Black also bought 4 packages of paper plates. There were 24 plates in a package. She bought how many plates in all?
$4 \times 24 = n$

3

E $7 \times 35 = n$
F $3 \times 41 = n$
G $62 \times 5 = n$
H $8 \times \$.14 = n$
I $26 \times 3 = n$
J $2 \times 94 = n$
K $4 \times \$.23 = n$
L $7 \times 15 = n$
M $55 \times 6 = n$
N $2 \times \$.53 = n$
O $9 \times 77 = n$
P $4 \times 32 = n$
Q $19 \times 8 = n$
R $5 \times 64 = n$
S $9 \times \$.39 = n$
T $13 \times 5 = n$

4

A $8 \times 57 = n$
B $6 \times 92 = n$
C $3 \times \$.80 = n$
D $44 \times 9 = n$
E $18 \times 7 = n$
F $2 \times 87 = n$
G $75 \times 8 = n$
H $6 \times \$.28 = n$
I $45 \times 4 = n$
J $9 \times 66 = n$
K $5 \times 74 = n$
L $4 \times 58 = n$
M $76 \times 2 = n$
N $89 \times 6 = n$
O $5 \times \$.40 = n$
P $7 \times \$.95 = n$

Now you should be able to multiply any two-figure number by any one-figure number.

52

52

1 Tell the children to read the problem, note the equation, and do the computation. They should then compare their work with the computation shown at the right of the problem. Help them recall that numbers representing money are multiplied in the same way that other numbers are.

2 Ask the children to work Examples B, C, and D and to compare their work with the computation shown in the book. Discuss the idea that they can multiply by the more convenient number, no matter which is expressed first in the equation. If they understand what they are doing, they should go ahead with the problems in the DO step.

3 Tell the children to work Exercises A to T in the usual way, writing the numerals in computational form and finding the answers.

4 Exercises A to P may be worked by children who need additional practice.

53-59 Learning how

Expanded Notes for this lesson are on pages 275-276.

Objectives

The child reviews multiplication of a number whose numeral has three figures by a number whose numeral has only one figure.

Vocabulary

There are no new words.

Comments

There is nothing fundamentally new in this lesson. All the child does is apply to three-figure numbers what he has learned to do in multiplying two-figure numbers (pages 48-52). He should be familiar by now with the four-step teaching method used here.

The pupils who successfully completed the inventory test you gave at the beginning of the previous lesson may be similarly tested before you start work on this lesson. You can use Exercises C to L in the first block of the DO step on page 59 for this test. If the test is satisfactorily completed, you might give these pupils special examples to solve, such as 8×444 and 4×888, 3×666 and 6×333. These children should be encouraged to make up similar pairs of examples to work. They might also work examples, such as 9×999, 7×111, etc.

Answers

Page 59:

(block 1)		(block 2)	
A 288	L 1045	A 966	L 4030
B 864	M $13.00	B 1794	M 1098
C 544	N $13.84	C 1396	N 2250
D 1335	O 4100	D 4249	O $21.90
E 1473	P 1832	E 4344	P $17.36
F 4832	Q $14.35	F 2331	Q 3620
G 2216	R 4023	G 2448	R $25.20
H 2702	S 4218	H 885	S 8325
I 1476	T 1794	I 2124	T $6.95
J 2745	U $26.80	J $31.56	
K 1056	V 5848	K $30.60	

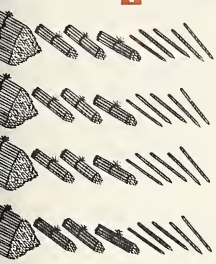
Learning how In this lesson you will learn how to multiply numbers with three figures in them.

see

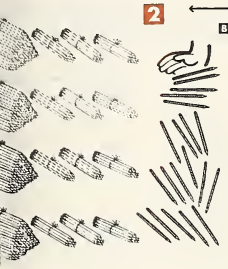
1 A

The boys and girls at the Burns School use about 135 pencils each week. About how many pencils do they use in 4 weeks?

$4 \times 135 = n$ ← You must find this number.



2 B ← First put the 4 groups of 5 pencils together.



$$\begin{array}{r} 135 \\ \times\ 4 \\ \hline \end{array}$$

← Multiply 5 ones by 4. There are 20 ones.

3

Now turn the page.

← Make 2 new bundles of 10 pencils. There are no single pencils left over. C

$$\begin{array}{r} 135 \\ \times\ 4 \\ \hline 0 \end{array}$$

20 is 2 tens. Write 0 here to show that there are no ones in the answer. ←

1 **2**

← Put the 2 new bundles of 10 pencils aside. D

$$\begin{array}{r} 135 \\ \times\ 4 \\ \hline 0 \end{array}$$

Remember the 2 new tens.

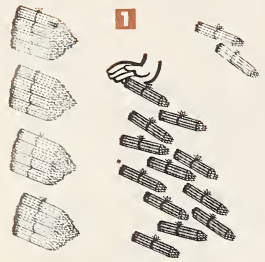
3 **4**

53

1. Have the children observe the 4 groups of 135 pencils, then read the problem and relate it to the picture. The problem should next be related to the equation.
2. Direct the children to observe that the ones are being put together first.
3. Make sure the children observe that 5 (which represents the pencils in each group that are not in bundles of ten) is multiplied by 4 so that they will know how many ones there are in all. Have them note that there are 20 ones.

54

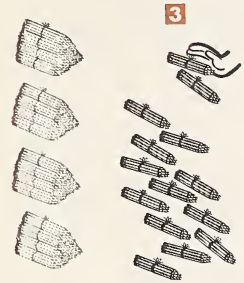
1. Have the children see that the 20 pencils are being made into 2 new bundles of 10 pencils each and that there are no loose pencils left over.
2. Draw attention to the zero in the answer and let someone explain it.
3. The pupils should observe that the 2 new bundles of 10 pencils are being set aside.
4. Discuss the fact that they must remember that they have 2 new tens.



Now put the 4 groups of 30 pencils together. E

135
4
0

Multiply 30, or 3 tens, by 4. There are 12 tens. 2

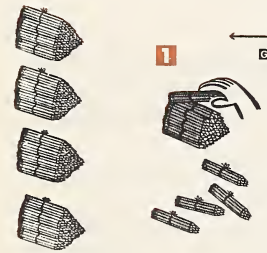


Put the 2 new bundles of 10 pencils with the 12 bundles of 10. F

135
4
0

Remember to add the 2 new tens to the 12 tens. Now there are 14 tens.

Now turn the page.

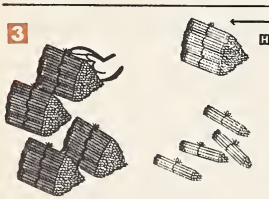


Make a new bundle of 100 pencils. There are 4 bundles of 10 left. Put the new bundle of 100 pencils aside. G

135
4
40

14 tens equal 1 hundred 4 tens. Remember that you have a new hundred. 2

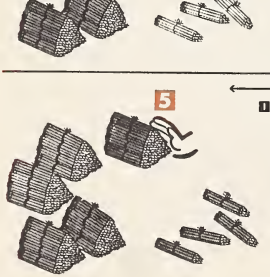
Write 4 here to show there are 4 tens in the answer.



Put the 4 old bundles of 100 pencils together. H

135
4
40

Multiply 100 by 4. There are 4 hundreds. 4



Put the new bundle of 100 with the other hundreds. I

135
4
540

Remember to add the new hundred. Now there are 5 hundreds. 6

Write 5 here to show there are 5 hundreds.

The boys and girls use about 540 pencils in 4 weeks.

$4 \times 135 = 540$

55

1 Have the children observe that the 12 bundles of 10 pencils which are being put together also represent the 4 groups of 30 pencils shown on pages 53 and 54.

2 Draw attention to the numeral the arrow is pointing to and have the children relate to the picture what is being symbolized here. Remind them that the 3 in 135 (which means 3 tens, or 30) is being multiplied by 4, resulting in 12 tens.

3 Have the children observe that now the 2 bundles of 10 that were set aside in Picture D are being put with the 12 bundles of 10, making 14 tens.

56

1 Tell the children to observe that the 14 bundles of 10 pencils are being made into 1 bundle of 10 tens, or 1 hundred, and that this new bundle of 100 is being set aside. Be sure they see that there are 4 bundles of 10 left over.

2 Remind the pupils that they must remember that they have a new hundred. Point out that 4 is written in the answer for the 4 tens that are left over.

3 Have the class observe that the 4 old bundles of 100 pencils are being put together.

4 Remind the pupils that the number in 4 equal groups can be found by multiplying by 4. Have the text read aloud.

5 The children should notice that the new hundred they were to remember is being put with the 4 hundreds.

6 Help the class see that the new hundred is added to the 4 hundreds and that the 5 in the answer means 5 hundreds.

think

The fifth graders are collecting spools to use in making toys. Each of the 3 fifth-grade classes has promised to bring 230 spools to school. How many spools in all have they promised to bring?

A

1

$3 \times 230 = n$ — You must find this number.

B

Are there any single spools to put together? 2

$$\begin{array}{r} 230 \\ \times\ 3 \\ \hline 0 \end{array}$$

Why should you write 0 here? 3

C

What do you put together first? 4

$$\begin{array}{r} 230 \\ \times\ 3 \\ \hline 0 \end{array}$$

What must you do here to find how many tens there are? 5

Now turn the page.

D

1 Can you make a new box of 100 spools?

$$\begin{array}{r} 230 \\ \times\ 3 \\ \hline 90 \end{array}$$

Why should you write 9 here?

E

2

Now what do you put together?

$$\begin{array}{r} 230 \\ \times\ 3 \\ \hline 690 \end{array}$$

What do you multiply now? How do you find how many hundreds there are? 3

Why do you write 6 in the answer?

F

The fifth graders have promised to bring ☰ spools.

$3 \times 230 = n$ 4

57

1. Have the children relate the problem to the picture first and then to the equation. Be sure they see that there are 3 groups of 230 spools.
2. Let the class observe that there are no loose spools to be put together, so there are no ones to multiply.
3. Be sure the children understand that zero is written in the answer because there are no ones.
4. Have the children observe that the 3 groups of 3 bags of spools are being put together and that each bag contains 10 spools. They should think of this as 3 groups of 3 tens, or 3 thirties.
5. Have the class notice that they are going to multiply the 3 in 230 by 3. Be sure they know that the 3 in 230 means 3 tens, and that they will have 9 tens when they multiply.

58

1. Help the children see that 9 tens are not enough to make a new box of 100 spools. Let a child tell why 9 is written in the answer in the tens place.
2. Let the class observe that the 3 groups of 200 spools are being combined.
3. Be sure the children realize that when they multiply the 2 in 230 by 3 that they are multiplying 2 hundreds by 3 and that the result is 6 hundreds. Let the children relate what happens in the computation to what they see in the picture.
4. Have the children read the statement and supply the name of the number when they come to the screen, then read and complete the equation. The statement and equation may then be related to the original problem and equation at the top of page 57.

try

A Mrs. Long says that if each of the 102 fifth graders brings 7 spools to school, there will be enough spools. How many spools does Mrs. Long need?
$102 \times 7 = n$

1

$$\begin{array}{r} 102 \\ \times\ 7 \\ \hline 714 \end{array}$$
← Multiply by the smaller number.

B $9 \times 612 = n$

$$\begin{array}{r} 612 \\ \times\ 9 \\ \hline 5508 \end{array}$$
← Why does the answer have 4 figures?

2

C $5 \times \$3.21 = n$

$$\begin{array}{r} \$\ 3.21 \\ \times\ 5 \\ \hline \$16.05 \end{array}$$
← Put a point and a dollar sign in the answer.

D $927 \times 3 = n$

$$\begin{array}{r} 927 \\ \times\ 3 \\ \hline 2781 \end{array}$$

do

A Mrs. Long sent Tony to the school storeroom to get 2 boxes of pencils. There were 144 pencils in each box. How many pencils in all was Tony to get?
$2 \times 144 = n$

3

B Tony saw 6 new boxes of tablets on a shelf in the storeroom. There were 144 tablets in each box. How many tablets in all were in the 6 boxes?
$6 \times 144 = n$

C	$4 \times 136 = n$	M	$2 \times \$6.50 = n$	A	$322 \times 3 = n$	K	$9 \times \$3.40 = n$
D	$5 \times 267 = n$	N	$8 \times \$1.73 = n$	B	$2 \times 897 = n$	L	$5 \times 806 = n$
E	$3 \times 491 = n$	O	$820 \times 5 = n$	C	$349 \times 4 = n$	M	$183 \times 6 = n$
F	$604 \times 8 = n$	P	$2 \times 916 = n$	D	$7 \times 607 = n$	N	$250 \times 9 = n$
G	$4 \times 554 = n$	Q	$7 \times \$2.05 = n$	E	$543 \times 8 = n$	O	$3 \times \$7.30 = n$
H	$386 \times 7 = n$	R	$447 \times 9 = n$	F	$9 \times 259 = n$	P	$7 \times \$2.48 = n$
I	$2 \times 738 = n$	S	$6 \times 703 = n$	G	$6 \times 408 = n$	Q	$905 \times 4 = n$
J	$9 \times 305 = n$	T	$598 \times 3 = n$	H	$177 \times 5 = n$	R	$8 \times \$3.15 = n$
K	$176 \times 6 = n$	U	$4 \times \$6.70 = n$	I	$708 \times 3 = n$	S	$9 \times 925 = n$
L	$5 \times 209 = n$	V	$731 \times 8 = n$	J	$4 \times \$7.89 = n$	T	$5 \times \$1.39 = n$

4

Now you should be able to multiply any three-figure number by any one-figure number.

59

59

1. Have the children read the problem, note the equation, do the computation, and then compare their work with that shown in the book.
2. Tell the children to work Examples B, C, and D independently, and then compare their work with that in the book.
3. Have the class work Problems A and B first; then direct them to work Exercises C to V by writing the numerals in computational form and finding the answers. The children should be reminded to use the number whose numeral has the fewer figures as the multiplier.
4. Exercises A to T are for the pupils who will profit from more practice.

60 Looking back

Expanded Notes are not considered necessary for this lesson.

Objectives

The child reviews the multiplication by numbers whose numerals end in zero.

Vocabulary

There are no new words.

Comments

This lesson prepares the pupils to understand that when they multiply by the 2 in 26 in an example like 26×34, they are really multiplying by 2 tens, or 20. The partial product is then 68 tens, or 680, not just 68. In this lesson emphasis is focused on the number of zeros that results in the product when numbers are multiplied by 10, 100, 20, 200, and so on.

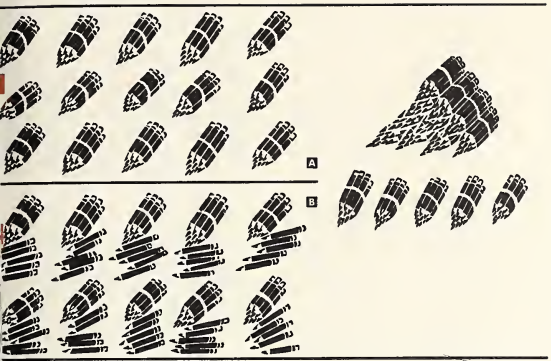
This lesson is based on concepts that the children have already learned in studying the number system. Important here is the understanding of how the number system symbols can be reinterpreted to correspond to a regrouping of objects—for example, that 15 tens can be thought of and written as 10 fifteens. (This idea, introduced in *Seeing Through Arithmetic 3*, was reviewed on pages 11-15 and 159-160 of Book 4.)

Throughout this program the commutative principle is emphasized: when the order of factors is reversed, the product remains the same.

Answers

(block 1)	(block 2)	(block 3)	
C 410	C 7300	A 520	H 4500
D 620	D 4900	B 1200	I 6000
F 5400	F 81,700	C 1650	J 4800
G 2900	H 9300	D 1500	K 14,000
I 620	I 9200	E 1600	L 21,000
J 1000	J 6400	F 1320	M 38,700
K 4200	K 85,800	G 3120	

Looking back



A 15 tens = 10 fifteens	A 15 hundreds = 100 fifteens	A 10 × 52 = n
15 × 10 = 150	15 × 100 = 1500	B 20 × 60 = n 6
10 × 15 = 150 3	100 × 15 = 1500 5	C 50 × 33 = n
B 10 × 19 = 190	B 100 × 26 = 2600	D 20 × 75 = n
C 10 × 41 = 41—	C 100 × 73 = 73—	E 40 × 40 = n
D 10 × 62 = 62— 4	D 100 × 49 = 49—	F 60 × 22 = n
E 10 × 300 = 3000	E 100 × 650 = 65000	G 80 × 39 = n
F 10 × 540 = 540—	F 100 × 817 = 817—	H 100 × 45 = n
G 10 × 290 = 290—	G 300 × 22 = 6600	I 100 × 60 = n
H 20 × 17 = 340	H 300 × 31 = 93—	J 200 × 24 = n
I 20 × 31 = 62—	I 200 × 46 = 92—	K 500 × 28 = n
J 40 × 25 = 100—	J 400 × 16 = 64—	L 700 × 30 = n
K 30 × 140 = 420—	K 200 × 429 = 858—	M 900 × 43 = n

J Multiplying by numbers ending in zero (reteaching)

60

1 Get pupils to observe that this scene shows 15 tens. Let them discover that the picture at the right shows these bundles of ten rearranged so that there is a pile of 10 tens (or 100) and 5 bundles of ten (or 50).

2 Let the children discover that there are 10 groups of 15 and that these 10 groups can be rearranged to make a pile of 10 tens (or 100) and 5 bundles of ten (or 50). This shows them that 15 tens and 10 fifteens are equal.

3 Help the pupils see that these three statements are different ways of saying the same thing. They should observe that the result of multiplying by 10 can be shown by putting a zero at the right of the numeral.

4 Let the pupils complete the answers orally.

5 Bring out that annexing two zeros has the effect of multiplying by 100. Let the children give the answers orally.

6 Have the pupils write the answers. Discuss the use of *n*.

61-64 Learning how

Expanded Notes for this lesson are on pages 276-278.

Objectives

The child reviews multiplication by numbers whose numerals have two figures.

Vocabulary

New words page 61 necklaces*; page 63 gumdrops; page 64 mats, strips*

Comments

Since the multiplication of two-figure numbers by two-figure numbers was taught on pages 161-164 of *Seeing Through Arithmetic 4*, the material in this lesson will be a review for many of the children in your class. As with the earlier lessons on multiplication, you may want to give a short inventory test to find out how much the children remember about this topic. As suggested for the lesson on pages 48-52, you could use part of the first block of exercises in the DO step (Exercises C to K on page 64) for this purpose.

The children who complete Exercises C to K easily should be given other work to do. You might, for example, let them work on problems having multiplicands with four or more figures. Very able children might work examples with three-figure multipliers, since all needed principles for the multiplication of whole numbers have been taught by page 64 of this book. Working with such numbers should not be required of any child, certainly not of average or below average pupils. Able students, however, should be encouraged to generalize and to gain satisfaction from discovering their own powers through working with any whole numbers they want to try. Additional suggestions will be found in the Expanded Notes for this lesson, under the heading "Providing for the able pupil."

The children must apply several ideas about the number system that they have already used. The following three ideas are especially important here. First, numbers represented by two-figure numerals are composed of tens and ones. Second, multiplication may in-

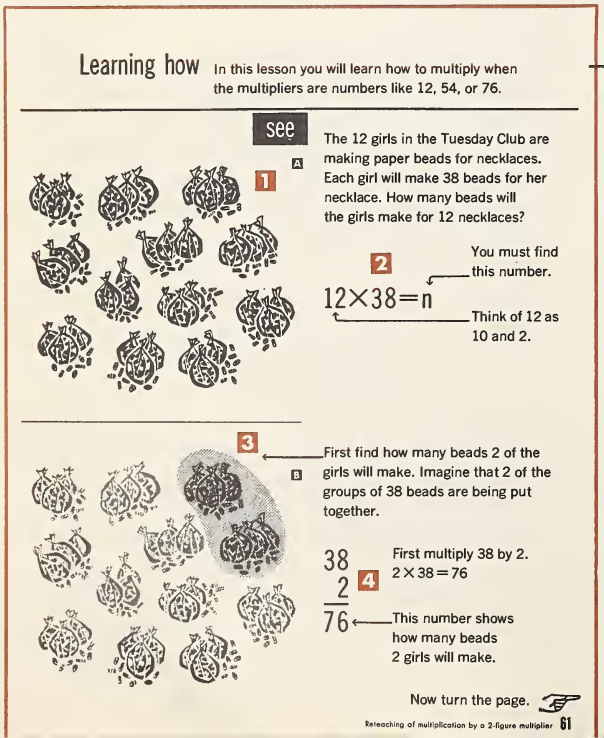
volve the kind of regrouping that is called "carrying." Third, when the tens' figure is taken as the multiplier, the product will represent groups of tens; this product should be written with an annexed zero at the right and will be read as hundreds or thousands. In the multiplication, 12×38, for example, the children should think of the 1 in 12 as 1 ten and of the partial product as 10×38, or 38 tens, or 380. In the multiplication, 63×82 (Exercise E on page 64), they should think of the 6 in 63 as 6 tens, and of the partial product not as 6×82, or 492, but as 60×82, which is 492 tens, or 4920.

In the SEE step, the process has been broken down to show the multiplication involved for each figure in the multiplier, at first by 2 ones and then by 1 ten. Afterwards, the two products are combined. Be sure that as the children study this they understand what each partial product in the computation stands for, as described opposite Picture E on page 62. In the THINK step on page 63, the numerals are shown written in the usual arrangement immediately.

Answers

Page 64:

(block 1)		(block 2)	
A 456	K 1771	A 2842	K $56.54
B $4.20	L 9648	B 2257	L 14,532
C 2584	M 38,272	C $49.50	M 18,288
D 1595	N 21,120	D 1127	N $109.44
E 5166	O $83.44	E $37.80	O 54,270
F 1404	P $121.20	F 1376	P $481.00
G 3895	Q 36,504	G 8836	Q 47,600
H $8.64	R 38,614	H $24.50	R 40,140
I $34.45	S 30,622	I 9420	
J $60.00	T 54,000	J 12,200	


Learning how
In this lesson you will learn how to multiply when the multipliers are numbers like 12, 54, or 76.
see
A
1
The 12 girls in the Tuesday Club are making paper beads for necklaces. Each girl will make 38 beads for her necklace. How many beads will the girls make for 12 necklaces?
2
You must find this number.
12×38=n
Think of 12 as 10 and 2.
3
B
First find how many beads 2 of the girls will make. Imagine that 2 of the groups of 38 beads are being put together.
38
2
76
4
First multiply 38 by 2. 2×38=76
This number shows how many beads 2 girls will make.
Now turn the page.
Reteaching of multiplication by a 2-figure multiplier
61

61

1 Ask the pupils to look at the picture, then read the problem. Get them to see that the picture represents 12 groups of 38 beads and that each group represents the beads one girl will make.

2 Ask a pupil to explain how this equation tells the story described in the problem and the picture.

3 Explain that the 2 in 12 represents 2 girls and that first they will find how many beads 2 of the girls will make. Call on a pupil to tell how the picture shows these beads put together [dotted background].

4 Have the pupils look at this computation. Help them see that 38 is the size of each group and that 2 groups of 38 are being put together first. Ask them to multiply to make sure that 76 is correct. Make certain the children realize that the answer so far shows only how many beads 2 girls will make.

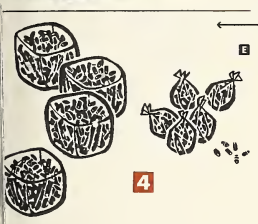
Now find how many beads 10 girls will make. Imagine that the 10 groups of 38 beads are put together.

38
10 ← Multiply the 38 by 10.
380 ← 10 × 38 = 380. This number shows how many beads 10 girls will make.

To find how many beads in all the 12 girls will make, imagine that the beads made by 2 girls and by 10 girls are put together.

76 Add 76 and 380.
380
456 ← The sum of 76 and 380 is 456.

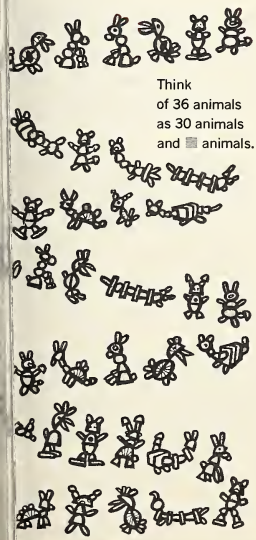
There are 456 beads in all.



38 You should write all
12 of the work in one place, like this.
76 ← Beads 2 girls will make
380 ← Beads 10 girls will make
456 ← Beads all 12 girls will make

The girls will make 456 beads.
12 × 38 = 456

think The girls in the Tuesday Club made 36 animals of gumdrops. They used 14 gumdrops for each animal. How many gumdrops in all did they use?
36 × 14 = n



Think of 36 animals as 30 animals and 6 animals.

14
36 ← First multiply 14 by 6.
First find the number of gumdrops in 6 animals.

14
36
84 ← 6 × 14 = 84. 84 shows the number of gumdrops in 6 animals.

14
36 ← Multiply 36 by 30.
84
Now find the number of gumdrops in 36 animals.

14
36
84
420 ← 30 × 14 = 420. Write 420 under 84 so that you can add. 420 shows how many gumdrops there were in 36 animals.

14
36
84 ┐ Now find the total number
420 ┘ of gumdrops in 36 animals. How do you do this?
504 ← 84 + 420 = 504

The girls used 504 gumdrops in all.

62

1 Point out that the dimmed beads are the 2 groups of beads discussed before, now rearranged to show the total of 76 beads. Help the children decide that next they are to find how many beads 10 girls will make and that the 1 in 12 represents these 10 girls. Call on someone to explain how the picture shows this.

2 Help pupils see that, as before, 38 is the number of beads in each group and that 10 groups are being put together. Pupils should know immediately that 10 × 38 equals 380.

3 Ask the children why the beads 2 girls will make are put with those 10 girls will make. Then have them relate the 76 and the 380 in the computation to the picture.

4 Have a pupil explain how the picture shows 456 beads in all. Make sure the children understand what each number in the computation stands for. Explain that this is the usual arrangement of numerals in multiplication.

63

1 Have the children read this problem and discuss the equation. After they examine the illustration, help them think of 36 as 3 tens (or 30) and 6 ones.

2 Make sure the children understand that first they are to find out how many gumdrops were used in making 6 animals. Have them note that, although they are multiplying by the ones (6), the complete numeral (36) is written because they will use the 3 tens (or 30) later.

3 Have the children multiply 14 by 6. Call on a pupil to read and complete the statements.

4 Make sure the children realize that now they are going to multiply by the 3 tens (30).

5 Have them multiply 14 by the 3 tens and note where the result is written. Ask the pupils why the gumdrops used to make 6 animals are added to those used to make 30 animals.

6 Call on someone to explain each number in the computation.

try

A Each of the 12 girls in the Tuesday Club pays dues of $.15 each month. How much money do they pay in all in dues each month?
12 × $.15 = n

1

$.15
12 ← Think of 12 as 10 and 2.
.30 ← Dues paid by ▩ girls.
1.50 ← Dues paid by ▩ girls.
$1.80 ← Dues paid by ▩ girls. Use a point and dollar sign here.

B 58 × 84 = n

2

84
58
672
4200
4872

C 162 × 43 = n

162
43
486 ← 3 × 162
6480 ← 40 × 162
6966

D 35 × $8.06 = n

$8.06
35
40.30 ← You do not need
241.80 ← to put a point
$282.10 in these numbers.

do

A Mary wanted to make 24 place mats of strips of paper. She needed 19 strips for each place mat. How many strips did she need in all?
24 × 19 = n

3

B Mrs. Fisher bought 12 packages of colored paper for the girls. The paper cost $.35 per package. How much did all the paper cost?
12 × $.35 = n

C 34 × 76 = n
D 29 × 55 = n
E 63 × 82 = n
F 18 × 78 = n
G 41 × 95 = n
H 36 × $.24 = n
I 53 × $.65 = n
J 75 × $.80 = n
K 23 × 77 = n

L 48 × 201 = n
M 416 × 92 = n
N 64 × 330 = n
O 56 × $1.49 = n
P 40 × $3.03 = n
Q 507 × 72 = n
R 86 × 449 = n
S 502 × 61 = n
T 90 × 600 = n

4

A 58 × 49 = n
B 37 × 61 = n
C 66 × $.75 = n
D 23 × 49 = n
E 70 × $.54 = n
F 16 × 86 = n
G 94 × 94 = n
H 49 × $.50 = n
I 15 × 628 = n

J 40 × 305 = n
K 22 × $2.57 = n
L 84 × 173 = n
M 36 × 508 = n
N 57 × $1.92 = n
O 81 × 670 = n
P 65 × $7.40 = n
Q 560 × 85 = n
R 45 × 892 = n

Now you should be able to multiply by numbers with two figures in them.

64

64

1. Discuss this problem and its solution.
2. Assign these examples as written work. Tell the pupils to write the necessary computation, find the answers, and then compare their work with the computation in the book. Direct the children to proceed to the section marked DO just as soon as they are sure they understand the examples.
3. Have Exercises A to T done as written work. Tell the children to write the exercises in computational form and find the answers. Have a set of the answers handy, so that the children may verify theirs.
4. Assign these exercises as written work for those who need extra practice.

65-66 Using arithmetic; Checking up

Expanded Notes are not considered necessary for these lessons.

Objectives

The child makes equations for problems and selects the correct processes to use in finding the answers. He reviews multiplication, including the multiplication of numbers representing amounts of money.

Vocabulary

New words page 65 planned*, favors, taught*, petals, cloth*, crates, built*; page 66 Paul, safes*, drawers

Comments

The problems in "Using arithmetic" will increase the children's ability to write equations that correctly describe the action expressed in problems. In the problems solved by multiplication, the pupils should, of course, be allowed to use the number with the fewer figures as the multiplier to simplify the computation. But it should be pointed out that sometimes, as in Problems F, H, I, J, and K, for example, the number with more figures in its numeral is the number of equal groups, and its numeral would, therefore, appear in the equation at the left of the multiplication sign, as shown in the answer key.

You could let an able student explain why less computation seems to be required if a number made up of repetitions of the same figure is used as the multiplier, even though it may be very much larger than the multiplicand.

The four tests on page 66 provide a means of diagnosing the ability of each child to multiply. Test 1 involves one-figure multipliers, Test 2 includes only two-figure multipliers and multiplicands, Test 3 involves two-figure multipliers and three-figure multiplicands, Test 4 involves numerals that represent amounts of money.

An activity for the able pupils to be used in connection with this lesson is described in the Expanded Notes for page 18 (pages 258-260) and in Activity 9, page 360.

Answers

Using arithmetic:

A $6 \times \$1.80 = [\$10.80]$
B $\$3.60 + [\$1.40] = \$5.00$
C $2 \times \$.75 = [\$1.50]$ or $\$.75 + \$.75 = [\$1.50]$
D $36 \div [9] = 4$
E $36 - 28 = [8]$
F $28 \times 4 = [112]$
G $28 \div [4] = 7$
H $28 \times 3 = [84]$
I $12 \times 6 = [72]$
J $12 \times 8 = [96]$
K $96 \times 6 = [576]$
L $12 \times \$.39 = [\$4.68]$
M $9 \times \$.20 = [\$1.80]$
N $2 \times 14 = [28]$ or $14 + 14 = [28]$
O $13 + 18 = [31]$ or $18 + 13 = [31]$
P $3 \times 8 = [24]$
Q $9 \times 18 = [162]$
R $[51] + 19 = 70$
S $70 + [92] = 162$
T $[\$3.00] - \$2.41 = \$.59$

Checking up:

Test 1	Test 2	Test 3	Test 4
A 388	A 1066	A 4494	A $1.96
B 1480	B 2090	B 7280	B $7.04
C 1955	C 1200	C 4056	C $7.52
D 602	D 1716	D 13,230	D $3.90
E 432	E 1725	E 31,620	E $12.27

Test 1	Test 2	Test 3	Test 4
F 1875	F 2448	F 16,200	F $15.66
G 1674	G 2940	G 12,654	G $6.24
H 1224	H 532	H 22,848	H $21.35
I 2300	I 3534	I 46,250	I $43.68
J 522	J 1900	J 17,936	J $22.33
K 2781	K 7056	K 10,234	K $34.88
L 3584	L 5890	L 36,981	L $252.58
M 2697	M 3264	M 24,840	M $416.00
N 1656	N 6084	N 47,520	N $42.00
O 372	O 3915	O 25,452	O $81.60

Using arithmetic 1

The girls in the Tuesday Club collect .80 a month in dues. How much do ey collect in dues in 6 months?

When the girls had collected $3.60 dues, they planned a party. They eded $5 in all for the party. How uch more money did they need?

Mrs. Young and Mrs. Block each ve the club $.75 to help pay for e party. How much money in all did ey give the club?

The girls made 36 invitations to the rty. The invitations were made 4 girls. Each girl made the same mber of invitations. How many did ch girl make?

Twenty-eight persons said they uld come to the party. The rest re not able to come. How many re not able to come?

The girls planned to make 28 party ors out of pipe cleaners. They eded 4 pipe cleaners for each favor. w many pipe cleaners did they need all the favors?

Seven girls said they would make the favors. Each girl was to make same number of favors. How many s each girl to make?

H Five girls said they would make all the sandwiches. How many sandwiches should they make so that each of the 28 persons at the party could have 3?

I Mrs. Young taught the girls to make paper flowers. Each girl needed 6 petals for one flower. How many petals would all 12 girls need if each girl made just one flower?

J Each girl made 8 flowers. How many flowers in all did the girls make?

K How many petals did the girls need for all the flowers they made?

L Mrs. Block promised to teach the girls to sew. Each of the 12 girls bought one yard of cloth. The cloth cost $.39 a yard. How much did all the cloth cost?

M Nine boys had a club, too. Each boy paid 20¢ a month in dues. How much was collected in dues each month?

N Mrs. Baker let the boys meet in the Bakers' basement. The boys planned to use orange crates to separate their corner of the basement from the rest of the basement. They needed 14 crates for each wall that they built. How many crates did they need for the 2 walls?

65

1 Have the children work independently on Problems A to T (O to T are on page 66). They should write the equation, do the computation, and rewrite the equation with their answer inserted in place of *n*.
2 Warn the children that some problems can be solved only by using information obtained from previous problems in the set.

1

O The boys got 13 orange crates from a grocery store and 18 orange crates from a fruit store. How many crates did they get from the 2 stores?

P Dick's father gave the boys an old record player. Dick, Paul, and Bill promised to bring 8 records each. How many records in all did they promise to bring?

Q Mr. Baker helped the 9 boys build safes for storing their collections. Each boy needed 18 small boxes to use as drawers in his safe. How many boxes in all were needed?

R For a long time the boys saved small boxes. Tony and Don brought 19 boxes to one club meeting. They said, "Now we have 70 boxes." How many boxes did they have before Tony and Don brought the 19 boxes?

S How many more boxes did the boys need so that they could finish their safes?

T Tony's father gave the boys some money to buy paint for the safes. After the boys had spent $2.41 for paint, they had $.59 left. How much money had Tony's father given them? **2**

Checking up **3**

Test 1	Test 2	Test 3	Test 4
A $4\times97=n$	A $26\times41=n$	A $14\times321=n$	A $7\times\$.28=n$
B $2\times740=n$	B $38\times55=n$	B $208\times35=n$	B $4\times\$1.76=n$
C $5\times391=n$	C $15\times80=n$	C $156\times26=n$	C $8\times\$.94=n$
D $86\times7=n$	D $44\times39=n$	D $42\times315=n$	D $6\times\$.65=n$
E $9\times48=n$	E $23\times75=n$	E $60\times527=n$	E $3\times\$4.09=n$
F $3\times625=n$	F $68\times36=n$	F $36\times450=n$	F $2\times\$7.83=n$
G $279\times6=n$	G $70\times42=n$	G $703\times18=n$	G $13\times\$.48=n$
H $8\times153=n$	H $19\times28=n$	H $51\times448=n$	H $35\times\$.61=n$
I $460\times5=n$	I $57\times62=n$	I $74\times625=n$	I $42\times\$1.04=n$
J $6\times87=n$	J $25\times76=n$	J $304\times59=n$	J $29\times\$.77=n$
K $9\times309=n$	K $84\times84=n$	K $86\times119=n$	K $16\times\$2.18=n$
L $512\times7=n$	L $62\times95=n$	L $587\times63=n$	L $73\times\$3.46=n$
M $899\times3=n$	M $34\times96=n$	M $270\times92=n$	M $80\times\$5.20=n$
N $8\times207=n$	N $78\times78=n$	N $55\times864=n$	N $5\times\$8.40=n$
O $4\times93=n$	O $45\times87=n$	O $28\times909=n$	O $96\times\$.85=n$

66 End-of-block tests on multiplication

66

1 The children should continue to work independently on this set of problems begun on page 65. They should write for each of Problems O to T an equation with *n* to hold a place for the answer, then compute and rewrite the equation with the answer inserted in place of *n*.

2 Discuss any problems that caused difficulty.

3 Direct each child to write these exercises in the proper form for computing. Tell them that they should write the numeral with the fewer figures as the multiplier. Give sufficient time for all but the slowest pupils to finish the test. Observe the kind of difficulties each child has and reteach accordingly from pages 47-64.

67 Thinking straight

Expanded Notes are not considered necessary for this lesson.

Objectives

The child learns the distinction between counting a quantity and measuring a quantity and learns reasons for the use of each method.

Vocabulary

New words page 67 Sally*, explain*, very*, grapefruit

Comments

Bring out the fact that counting does not tell anything about the size, shape, or weight of what is being counted—that it gives the number, but not the other characteristics; that measuring by weight or volume, on the other hand, results in information about other characteristics, but does not give necessarily exact information as to number of objects included in the measured quantity. Discuss the meaning of "how many" and "how much." Encourage the children to think of occasions when it might be more important to know how *many* of something one has rather than how much. The following are examples of situations in which it is important to be precise as to number: sharing objects equally, using a definite number of objects (buttons for a coat, shades for the windows in a certain room, books for a class).

Let the children discuss situations in which knowing the total quantity of something by weight is more important than knowing the exact number of items which make up this quantity. For example, each passenger on an airplane is allowed to take a certain weight of baggage, not a certain number of pieces. The airline does not care if this weight is made up of many small pieces or is contained in a single large piece. Then there are the various things commonly sold by weight rather than by the piece, such as coal, or those sold by the contents of the container, such as berries. Mention might be made of the fact that it is more convenient to measure things that are generally used in large quantities than it is to count them. In the case

of small objects, such as berries, the *number* of berries might vary from one standard measure (such as a pint or quart box) to another, depending on the size of the berries. Discuss the fact that we do not really care if there are 105 blueberries or 115 blueberries in a box, but we do care whether there are 105 eggs or 115 eggs in a box.

You might also let the children suggest occasions when it is more convenient to measure a quantity, since counting would be too tedious. (Some examples that might be suggested are such things as grass seed, sugar, sand, salted peanuts, etc.) The preparation of food will suggest many instances of the measurement of things instead of counting. For example, a glass of juice might be made from 1, 2, or 3 oranges, depending on the size and juiciness of the oranges. Discuss why a recipe for a cake may call for a *cup* of egg whites, not a specific *number* of egg whites. Bring out the fact that a standard measuring cup is a uniform measure, while eggs vary in size and weight so greatly that it would be impossible to tell how many egg whites would fill a cup without measuring them.

In connection with Question K, the children might make a collection of containers and instruments for measuring. So that everyone does not bring the same one, have them make a list, and have each child choose one he knows he can bring. In connection with Question M, have the children suggest for each word a kind of thing whose measurement would involve that word.

Answers

A Although Paul picked more *pounds* of oranges, Sally picked a larger number of oranges. If they intend to give them away, Sally will have more oranges to give away than Paul. If they plan to sell them by weight, Paul's oranges will bring more money.

B When sharing them or when you need a certain number

C When buying or selling them, since weight is a more accurate measure than number

D Sally picked *as many* grapefruit as Paul because she picked the same number of them, but she did not pick *as much* grapefruit as he did, since her grapefruit were smaller.

E The weight of the candy each girl bought

F The weight of Dick's apples or the number of apples in the 4 lb. George bought; The question might then be discussed in the same way as Question A.

G Mrs. Fisher bought fewer bottles. You do not know who bought less milk unless you know how much each bottle holds. Perhaps Mrs. Fisher bought a gallon of milk and Mrs. Long only 3 quarts.

H The size of oranges and berries must be considered when selecting suitable methods for selling them. Oranges grow in sizes that make a dozen a convenient quantity to count and use. When oranges are sold by number, they are usually sorted according to size. Since berries are small, a dozen berries would not be a useful amount to buy, and many dozens of berries would require a lot of counting. Therefore, they are sold by the pint or quart, each of which is a useful quantity to buy.

I Eggs, oranges, rolls, cookies, cupcakes, cut flowers, plants, paper plates, etc.

J See answers to Questions A and H.

K Answer will depend on the child's experience. In *Seeing Through Arithmetic 4,* linear measures, dry measures, liquid measures, and measures of weight were studied.

L All children should be familiar with words such as "dozen" and "pair," and, of course, counting words such as "one," "two," etc. Some children may think of other words such as "trio," "duet," "quartette," and "set," "team," "ream," and "gross." If they do mention "set" or "team," discuss the fact that these expressions are not *precise.*

M By the fifth grade, the children should know the following terms: inch, foot, yard, mile, pint, quart, gallon, cup, teaspoon, tablespoon, pound, ton, fluid ounce, peck, bushel.

Thinking straight

1

I picked more oranges than you did, Paul. I have 27, and you have only 24.
A
I don't know about that, Sally! My basket of oranges weighs 12 lb. Your basket weighs only 11 lb. I think I picked more.
12
B

A Did Paul or Sally pick more oranges? Explain your answer.

B When might you want to count oranges?

C When might you want to weigh oranges?

D Paul picked 6 very large grapefruit. Sally picked 6 small grapefruit. Did Sally pick as many grapefruit as Paul? Did she pick as much grapefruit as Paul? Explain your answer.

2

E Carol bought a bag of candy with 25 pieces in it. Kathy bought 5 candy bars. What do you have to know before you can tell who bought less candy?

F Dick bought 10 apples. George bought 4 lb. of apples. What do you have to know before you can tell who bought more apples?

G Mrs. Fisher bought 1 bottle of milk. Mrs. Long bought 3 bottles of milk. Who bought fewer bottles? Who bought less milk? How do you know?

H Oranges are often sold by the dozen, but berries are sold by the pint or quart. Why?

I Name some things that are often sold by the dozen.

J Is it better to count things or to measure things? Explain your answer.

3

K How many different ways of measuring do you know about?

L What words are used for counting things?

M What words are used for measuring things?

67

1 Direct the pupils to look at Pictures A and B and to read what the children are saying. Through discussion help them answer Questions A, B, and C.

2 Continue the discussion with Exercise D and the exercises that follow.

3 In answering Exercise J, be sure the children understand that they must consider the use to which things are to be put, as well as their size. Let them name some things which are better counted and some things which are better measured; then direct them to tell why.

68-74 Learning how

Expanded Notes for this lesson are on pages 280-282.

Objectives

The child reviews division involving one-figure divisors and two-figure quotients.

Vocabulary

New words page 68 grab*

Comments

The process of division has always been one of the greatest stumbling-blocks in arithmetic for children. With the introduction of the two-figure divisor, children begin to have considerable trouble. This lesson on one-figure divisors is important because learning it well will establish the pattern of thought for two-figure divisors.

To eliminate difficulties, certain changes have been introduced in the teaching of division during the past twenty years. Division with one-figure divisors is now customarily introduced by the "long-division" method instead of the "short-division" method. This is done to give the child experience with the complete process before using the short cut. In replacing the short-division method with the long-division method, it was felt, too, that the children would not have to rely so much on remembering numbers.

The importance of helping the child understand what he is doing at every step has led the authors of *Seeing Through Arithmetic* to use a slightly different placement of the numerals in the written work. Partial quotients (numbers that will be combined cumulatively to make the answer) are put at the right in a column. The final answer, which is their total, is put at the bottom of this column instead of above the dividend (the number to be divided). With this arrangement the child can use a partial quotient that is smaller than the maximum partial quotient. This method enables the child to work within the limits of his ability much better than the usual method does. As the study of division continues through the next grades, the child's

thinking will be guided so that he will be able to improve his ability to estimate partial quotients.

To see how the child is taught to write division problems in this program, look at Step E, page 69. Instead of subtracting 3 from 5, as the child would in the old method, he subtracts 30 from 50. He writes the numerals so that they describe the situation as it really is, which enables him to understand each step of the process.

The amount that remains to be divided each time (note the 20 in Steps D, E, and F, page 69) is shown completely. This helps the child to think of the division process as it really is—a process of repeated subtraction.

Pictures are provided for each step of the process for each of the two problems in this lesson. Great care has been taken to relate the numerals in the computation with the pictures. The child thus has a concrete basis to guide his thinking as he is learning.

Note that the explanation of division begins on page 68 with a problem that involves a remainder, so that the children can see all aspects of the process of division from the beginning.

Note also that for the examples on pages 73 and 74, four ways in which children may do the problem are shown. The fourth way shown is the most precise, efficient way to do the computation.

Terms such as *dividend, quotient, partial quotient,* and *partial dividend* are unnecessary in explaining the work and are likely to be confusing to children. They are not used in the text. You will probably want to avoid using them yourself, and you should not require their use by the children.

The extent to which division with a one-figure divisor needs to be retaught depends entirely on the achievement—that is, the accuracy, facility, and understanding of the pupils. It would be advisable to test the children before presenting this lesson. Exercises C to L in the first block of work on page 74 can be used for this test.

Children who have learned the traditional arrangement of division computation should be permitted to use that arrangement if the test shows that they can do the examples correctly in that way. Children who have learned the traditional arrangement but who are unable to divide competently, should be retaught using the method presented here.

The four-step teaching method is used in presenting this lesson. This method enables the teacher to adapt the lesson to the needs of the class. If necessary, the entire lesson may be used to reteach division with a one-figure divisor. If the children show competence and understanding, those steps that seem unnecessary may be omitted.

Answers

Page 74:

(block 1)

A 17
B 14
C 26 and 2 rm.
D 13
E 13 and 4 rm.
F 15 and 3 rm.
G 34 and 2 rm.
H 36
I 38 and 2 rm.
J 66 and 2 rm.
K 22 and 3 rm.
L 27
M 86
N 84 and 7 rm.
O 40 and 1 rm.
P 95
Q 80
R 60 and 4 rm.
S 74 and 5 rm.
T 29
U 68 and 2 rm.
V 89

(block 2)

A 11 and 5 rm.
B 29
C 35 and 3 rm.
D 72 and 2 rm.
E 54 and 4 rm.
F 74
G 48
H 10 and 7 rm.
I 81 and 2 rm.
J 90 and 1 rm.
K 18 and 2 rm.
L 50
M 49
N 56 and 2 rm.
O 84 and 3 rm.
P 29 and 5 rm.
Q 32
R 80 and 3 rm.
S 86 and 4 rm.
T 85 and 5 rm.

Learning how

In this lesson you will learn how to divide when the answer has two figures.

see

Kathy is making surprise packages for the grab bag at a Toy Fair. She has 50 small toys. She wants to put 3 toys in each box. How many boxes of 3 toys will she have? Will any toys be left over?

$50 \div 3 = \square$ ← You must find this number.

Let us start with 10 boxes of 3 toys.

$3\overline{)50}\,|\,10$ ← We will start with 10 groups of 3 toys.

3

To fill 10 boxes, we will use 10×3, or 30 toys.

5

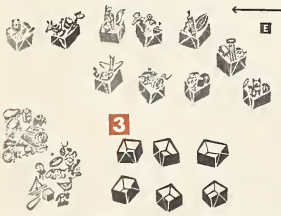
$3\overline{)50}\,|\,10$
$\underline{30}$ ← We write 30 here so that we can subtract it from 50 to find how many toys are left to be put into boxes.

$10 \times 3 = 30$.

How many toys are left to be put into boxes?

$3\overline{)50}\,|\,10$
$\underline{30}$
20 ← There are 20 toys left.

2



There are about 6 threes in 20. Let us try 6 more boxes of 3 toys.

$3\overline{)50}\,|\,10$
$\underline{30}$
$20\,|\,6$ ← We will use 6 more groups of 3 toys.

4

We have used 6×3, or 18 more toys.

$3\overline{)50}\,|\,10$
$\underline{30}$
$20\,|\,6$
$\underline{18}$ ← We write 18 here so that we can subtract to find how many toys are left.

$6 \times 3 = 18$.

6

Now turn the page.

68

1. Ask the pupils to look at Picture A and read the problem and the equation without answering the questions or supplying the missing number. Be sure they realize that Picture A shows 50 toys and that they are going to find out how many groups of 3 toys there will be.
2. Direct attention to Picture B. Explain to the children that they do not know how many boxes of 3 toys they will have, but that they will start with 10 boxes.
3. Draw attention to the computation and relate the 10 to the 10 boxes.
4. Explain that a group of 3 toys has been put aside for each box and that there are many toys left over. Let the children discover that 30 toys are being used.
5. Now direct attention to the computation. Be sure the pupils understand that the 30 is the total of 10 groups of 3 and stands for the 30 toys being used. Explain why 30 is written under the 50.

69

1. Be sure the children observe that 30 toys have been put into boxes and that there are 20 toys left.
2. Relate the subtraction of 30 from 50 to the removal of 30 toys from the original 50 toys. Emphasize that the 20 stands for the number of toys left.
3. Draw attention to the 6 new boxes and make the point that the children are going to use 6 more groups of 3 toys.
4. Relate the 6 to the 6 boxes. Make clear that the 6 means that 6 groups of 3 will be used.
5. Note that 6 groups of 3 toys have been put aside. Let the children discover that 18 toys have been set aside.
6. Be sure the children understand that 18 is the total of 3 sixes. Relate the 18 to the 18 toys in the picture. Explain why 18 is written under 20.

There are only 2 toys left over.

C

$3\overline{)50}\,\big|\,10$

30

$20\;\;6$

18 ← 2

2 ← of 2.

When we subtract, there is a remainder

1

H

There will be 16 boxes of 3 toys.

There will be 2 toys left over.

Add 10 and 6.

$3\overline{)50}\,\big|\,10$

30

$20\;\;6$

18 ← 4

2 ← There will be 16 boxes of 3 toys.

16 ← 2 toys will be left over.

3

$50 \div 3 = 16$ and 2 remainder

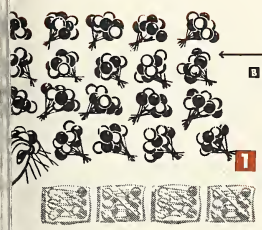
think

A

Sue has 200 lollipops. She is going to divide them into bunches of 8. How many bunches will she have? Will any lollipops be left over?

5

$200 \div 8 = n$ ← You must find this number.

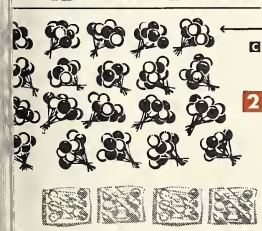


How many bunches of 8 do you try at first?

B

$8\overline{)200}\,\big|\,20$ ← First try ■ eights.

1



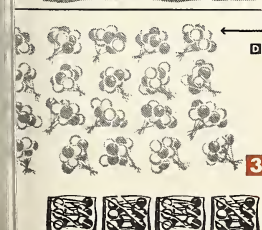
You have used $20 \times$ ■, or ■ lollipops.

C

$8\overline{)200}\,\big|\,20$

160 ← How do you get 160? Why is the 160 written here?

2



How many lollipops are left to be put in bunches of 8?

D

$8\overline{)200}\,\big|\,20$

160 ← 4

40 ← How do you get 40? What does the 40 show?

3

Now turn the page.

70

1 Let the children observe that 18 toys have been put into 6 boxes with 2 toys left over.

2 Relate the subtraction of 18 from 20 to the removal of 18 toys from 20 toys. Be sure that the children understand that the 2 stands for the number of leftover toys.

3 Draw attention to the dotted background. Explain to the children that it helps them think of the group of 10 boxes and the group of 6 boxes as one big group of 16 boxes.

4 Relate the addition of 10 and 6 to the picture. Be sure the children think of the 10 and 6 as 10 groups of 3 and 6 groups of 3 and of the total as 16 groups of 3.

5 Have the pupils read the problem and relate to the picture the 200 lollipops that are to be put in bunches of 8. Be sure they observe that the lollipops in the picture are in bags of 10 lollipops each. Call on someone to explain the equation.

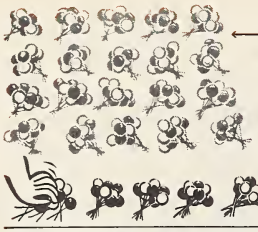
71

1 The pupils should observe the 20 bunches of lollipops in the picture and realize that 8 lollipops have been put in each. Then they should relate the 20 in the computation to the 20 groups of 8 that have been made. Discuss reasons for trying 20 bunches first. Ask if 10 bunches could have been tried. [Yes] Get them to see that a greater number would be left over.

2 The children should note that 160 lollipops in all have been used to make 20 bunches. Call on a pupil to tell which two numbers in the computation are multiplied to make 160.

3 Focus attention on the 40 lollipops still not used.

4 Let a pupil explain the subtraction of 160 from 200 and tell what the resulting 40 means.



How many bunches of 8 can be made from 40 lollipops?

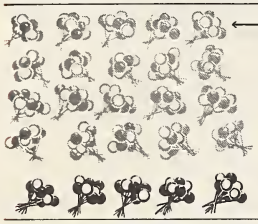
E

$$\begin{array}{r|l} 8\overline{)200} & 20 \\ 160 & \\ \hline 40 & 5 \end{array}$$

1

Why can you use 5 here?

How many lollipops have you used this time?



F

$$\begin{array}{r|l} 8\overline{)200} & 20 \\ 160 & \\ \hline 40 & 5 \\ 40 & \\ \hline & \end{array}$$

2

How do you get this 40?

How many bunches of 8 lollipops will there be? Will any be left over?

G

What do you do now?

$$\begin{array}{r|l} 8\overline{)200} & 20 \\ 160 & \\ \hline 40 & 5 \\ 40 & \\ \hline & 25 \end{array}$$

4

3

What does the 25 mean? Is there a remainder?

$200 \div 8 = n$

try

A $378 \div 5 = n$

1

$$\begin{array}{r|l} 5\overline{)378} & 20 \\ 100 & \\ 278 & 40 \\ 200 & \\ 78 & 10 \\ 50 & \\ 28 & 5 \\ 25 & \\ 3 & 75 \end{array} \qquad \begin{array}{r|l} 5\overline{)378} & 30 \\ 150 & \\ 228 & 20 \\ 100 & \\ 128 & 20 \\ 100 & \\ 28 & 5 \\ 25 & \\ 3 & 75 \end{array} \qquad \begin{array}{r|l} 5\overline{)378} & 50 \\ 250 & \\ 128 & 20 \\ 100 & \\ 28 & 5 \\ 25 & \\ 3 & 75 \end{array} \qquad \begin{array}{r|l} 5\overline{)378} & 70 \\ 350 & \\ 28 & 5 \\ 25 & \\ 3 & 75 \end{array}$$

$378 \div 5 = n$

B $621 \div 9 = n$

$$\begin{array}{r|l} 9\overline{)621} & 10 \\ 90 & \\ 531 & 30 \\ 270 & \\ 261 & 20 \\ 180 & \\ 81 & 9 \\ 81 & \\ & 69 \end{array} \qquad \begin{array}{r|l} 9\overline{)621} & 20 \\ 180 & \\ 441 & 20 \\ 180 & \\ 261 & 20 \\ 180 & \\ 81 & 9 \\ 81 & \\ & 69 \end{array} \qquad \begin{array}{r|l} 9\overline{)621} & 50 \\ 450 & \\ 171 & 10 \\ 90 & \\ 81 & 9 \\ 81 & \\ & 69 \end{array} \qquad \begin{array}{r|l} 9\overline{)621} & 60 \\ 540 & \\ 81 & 9 \\ 81 & \\ & 69 \end{array}$$

$621 \div 9 = n$

C $141 \div 2 = n$

$$\begin{array}{r|l} 2\overline{)141} & 20 \\ 40 & \\ 101 & 50 \\ 100 & \\ 1 & 70 \end{array} \qquad \begin{array}{r|l} 2\overline{)141} & 40 \\ 80 & \\ 61 & 30 \\ 60 & \\ 1 & 70 \end{array} \qquad \begin{array}{r|l} 2\overline{)141} & 10 \\ 20 & \\ 121 & 60 \\ 120 & \\ 1 & 70 \end{array} \qquad \begin{array}{r|l} 2\overline{)141} & 70 \\ 140 & \\ 1 & \end{array}$$

$141 \div 2 = n$

Now turn the page.

72

1 Bring out that all the lollipops were not used in the 20 bunches. Let the pupils determine from this picture how many more bunches of 8 lollipops can be made. They should note where the 5 is placed in the computation.

2 Focus attention on the 5 bunches of 8 lollipops. Ask how many lollipops in all have been used. Be sure they observe that there are no unused lollipops. Let a pupil explain the 40 in the computation.

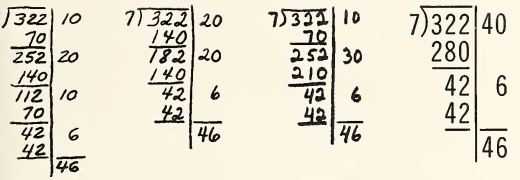
3 The dotted background should indicate to pupils that the two groups of bunches of lollipops are to be combined. They should see that there are in all 25 bunches of 8 lollipops.

4 Make sure the children relate the 20 and the 5 in the partial answers to the two groups of bunches of 8 lollipops in the picture. They should see that the numbers are to be added, just as the groups in the picture were combined. Have someone complete the equation and relate it to the original problem and equation.

73

1 Explain to the pupils that the work for Examples A, B, C, and D (on page 74) shows how different children divided. [The work printed in red shows the greatest skill.] Also explain to them that their own way may be quite different from any of these. Assign Examples A to D as written work. Suggest that each child cover up the work in the book with a piece of paper until he is finished. Tell the children to put the numerals in the proper computational form and find the answers. Tell them to compare their answer with that in the book as they finish each example. Point out that they must get the same answer, but that their solution may be different.

D 322 ÷ 7 = n



322 ÷ 7 = n

1

do

A Tom said that he sold 85¢ worth of balloons when he helped at the Toy Fair. The balloons were sold for 5¢ each. How many balloons did Tom sell?
85 ÷ 5 = n

2

B The sixth-grade boys and girls made 28 cardboard signs for the Toy Fair. They made 2 signs from each piece of cardboard. How many pieces of cardboard did they use for the signs?
28 ÷ 2 = n

C	106 ÷ 4 = n	M	430 ÷ 5 = n	A	93 ÷ 8 = n	K	92 ÷ 5 = n
D	78 ÷ 6 = n	N	679 ÷ 8 = n	B	58 ÷ 2 = n	L	150 ÷ 3 = n
E	95 ÷ 7 = n	O	121 ÷ 3 = n	C	213 ÷ 6 = n	M	196 ÷ 4 = n
F	138 ÷ 9 = n	P	380 ÷ 4 = n	D	290 ÷ 4 = n	N	450 ÷ 8 = n
G	240 ÷ 7 = n	Q	720 ÷ 9 = n	E	382 ÷ 7 = n	O	591 ÷ 7 = n
H	72 ÷ 2 = n	R	364 ÷ 6 = n	F	148 ÷ 2 = n	P	266 ÷ 9 = n
I	306 ÷ 8 = n	S	523 ÷ 7 = n	G	240 ÷ 5 = n	Q	192 ÷ 6 = n
J	200 ÷ 3 = n	T	87 ÷ 3 = n	H	97 ÷ 9 = n	R	323 ÷ 4 = n
K	91 ÷ 4 = n	U	546 ÷ 8 = n	I	488 ÷ 6 = n	S	434 ÷ 5 = n
L	162 ÷ 6 = n	V	801 ÷ 9 = n	J	271 ÷ 3 = n	T	600 ÷ 7 = n

3

Now you should be able to divide when the answer has two figures.

74

1 When the pupils have finished Example D and compared its solution with those given in the book, have them note how the solutions differ.

2 Assign Exercises A to V as written work. Provide answers with which pupils can verify their work. When they have completed the examples, have them compare their solutions with those of two or three other children. Then discuss with the children the solutions of several exercises and decide which is the best way to solve each of them.

3 This block of exercises may be used in the same way for the children who need more practice.

75-76 Using arithmetic

Expanded Notes are not considered necessary for this lesson.

Objectives

The child practices solving various types of verbal problems. He practices his skills in addition, subtraction, and multiplication.

Vocabulary

New words page 75 decorated, crepe, hope*, mended*, o'clock*; page 76 Susan*

Comments

For each of these problems the children are to write an equation that describes the problem situation, using *n* to represent the missing number. Then they are to find the answer by computation and rewrite the equation with the numeral that replaces *n*. Remind the children that in some addition problems two equations are possible and that either is acceptable.

You might remind the pupils that some of the problems in this set require information or answers from preceding problems. This is the type of situation the pupils will often meet outside the arithmetic class.

The "Keeping skillful" exercises should be handled separately.

Answers

Using arithmetic:

A 27 × 12 = [324]
B 324 ÷ [81] = 4
C 94 − 50 = [44]
D 108 ÷ [12] = 9
E 56 ÷ 4 = [14]
F [18] + 18 = 36
G 70 + 36 = [106] or 36 + 70 = [106]
H 70 − 36 = [34]
I 120 ÷ [40] = 3
J 95 − [48] = 47
K 95 × $.15 = [$14.25]
L 200 ÷ [40] = 5
M 200 × $.06 = [$12.00]
N 200 − 75 = [125]
O [64¢] − 35¢ = 29¢
P $2.50 + $.72 = [$3.22] or $.72 + $2.50 = [$3.22]
Q 72¢ ÷ 6¢ = [12]
R 95 − 58 = [37] or 58 + [37] = 95
S 64¢ ÷ [16¢] = 4
T 35¢ + 12¢ + 15¢ = [62¢]
U $.85 + $.85 = [$1.70] or 2 × $.85 = [$1.70]
V 226 − 194 = [32]
W 87 × $.15 = [$13.05]
X $92.75 − $36.67 = [$56.08]

Keeping skillful:

(block 1)	(block 2)	(block 3)
A 1157	A 23	A 261
B 195	B 264	B 735
C 980	C 353	C 684
D 1463	D 326	D 1130
E 482	E 291	E 1824
F 12,823	F 1004	F 2747
G 121	G 929	G 1325
H 77	H 31,276	H 736
I 2625	I 158	I 4212
J 1101	J 2779	J 7524
K 1400	K 13,176	K 24,180

Using arithmetic 1

A The boys and girls decorated the gym for the Toy Fair. They used 27 packages of crepe paper. They cut the paper in each package into 12 strips. How many strips did they have altogether?

B They wanted to use the same number of strips of crepe paper to decorate each of the 4 walls of the gym. How many strips could they use on each wall?

C The gym was 94 feet long and 50 feet wide. The gym was how much longer than it was wide?

D The boys and girls decorated the tables with balloons. They used 108 balloons for 9 tables. They put the same number on each table. How many did they put on each table?

E The boys were going to make toy wagons to sell. They had 56 little wheels to use. They needed 4 wheels for each wagon. How many wagons could they make?

F The children also mended old toys to sell. They mended some one day. The next day they mended 18 more. Dick said, "Now we have mended 36 toys." How many toys did they mend the first day?

G They put the 70 toys they had made on one table and the 36 toys they had mended on another table. 2 How many were on the two tables?

H The children had mended how many fewer toys than they had made?

I Three stores gave 120 new toys to be sold at the Toy Fair. Each store gave the same number of toys. How many toys did each store give?

J At 9 o'clock in the morning there were 95 packages in the grab bag. At noon 47 packages were left. How many packages had been sold between 9 o'clock and noon?

K The grab-bag packages sold for 15¢ each. How much did the children hope to get by selling all 95 packages?

L Five mothers said they would make 200 popcorn balls to sell at the fair. Each mother was to make the same number. How many popcorn balls would each mother make?

M They sold the popcorn balls at 6¢ each. How much money did they hope to get for the 200 popcorn balls?

N Ann sold 75 popcorn balls in the morning. How many popcorn balls were still to be sold in the afternoon?

75

1 Tell the children to read each problem and write the equation. Remind them to use *n* to hold a place for the answer. When they have found by computation the numeral that replaces *n*, the equation should be rewritten with the numeral supplied. Have the children work independently without preliminary discussion of the problems.

2 Remind the pupils that for addition problems like this the numerals may be written in any order in the equation.

O After Bill Fuller had spent 35¢ for a toy boat at the fair, he still had 29¢ left to spend. How much money had he had altogether to spend at the fair?

P Mrs. Fisher spent $2.50 for toys and $.72 for popcorn balls. How much did she spend in all?

Q How many popcorn balls did Mrs. Fisher buy with the 72¢?

R The girls had wrapped 58 of the 95 packages for the grab bag on one day. They wrapped the rest of the packages the next day. How many grab-bag packages did they wrap on the second day?

S Susan paid 64¢ for 4 toy horses at the fair. How much did each of the toy horses cost?

T Tony spent 35¢ for a toy wagon, 12¢ for popcorn balls, and 15¢ for a grab-bag package. How much did he spend for all three of these things?

U Mary and June Fuller each had 85¢ to spend at the fair. How much did both girls have to spend?

V Of the 226 toys the boys and girls had to sell, they sold only 194. How many toys were not sold?

W Only 87 of the grab-bag packages were sold. How much money was collected for these packages?

X The boys and girls had spent $36.67 for materials for the fair. They collected $92.75 for the things they sold. How much money was left after the materials were paid for? 1

Keeping skillful

2

A $731+426=n$
B $82+43+70=n$
C $292+688=n$
D $333+509+621=n$
E $197+76+209=n$
F $7865+4958=n$
G $26+45+50=n$
H $36+18+23=n$
I $904+775+562+384=n$
J $53+421+540+87=n$
K $382+150+639+229=n$

3

A $n+84=107$
B $336+n=600$
C $528-n=175$
D $n-92=234$
E $514-223=n$
F $n-476=528$
G $3071+n=4000$
H $56263-24987=n$
I $n+858=1016$
J $5108-n=2329$
K $17568-4392=n$

4

A $3\times87=n$
B $7\times105=n$
C $9\times76=n$
D $5\times226=n$
E $304\times6=n$
F $41\times67=n$
G $25\times53=n$
H $92\times8=n$
I $54\times78=n$
J $36\times209=n$
K $372\times65=n$

76

1 When the children have worked Problems A to X and have written the final equation for each, let them verify their work. Supply answers, and give individual help where it is needed. The children may work in groups and help one another with problems that are especially difficult.

2 Tell the children to compute and write the answer for each example opposite its identifying letter. Of course, they must write the numerals in correct computational form.

3 For this block, the children, after they have computed, should write the answers opposite their identifying letters or rewrite the equations with the missing numerals supplied.

4 Use Note 2.

77 Thinking straight; Keeping skillful

Expanded Notes are not considered necessary for these lessons.

Objectives

The child learns some techniques for finding partial quotients in division. He extends his skills in multiplication to three-figure multipliers.

Vocabulary

There are no new words.

Comments

The child should be able to use his knowledge of the division basic facts in any division situation. For example, if he is going to divide 610 by 8, he should see 610 as 61 tens. He knows that he cannot divide 61 by 8 without a remainder, and he knows that 56 is the number smaller than 61 that he can divide by 8 without a remainder. The child can learn to look at the first two figures of a numeral and think of the number as tens, hundreds, etc. For example, he should think of 31 in 312 as 31 tens and 48 in 4825 as 48 hundreds. Then he can use his knowledge of division basic facts in estimating how many times the divisor is contained in the number.

You may want to discuss each exercise with the children before they work it; then when they have finished all the exercises, discuss their solutions and discover which child has the best solution. You should not insist that the children use the techniques shown on this page; each child should divide in the way that he finds easiest.

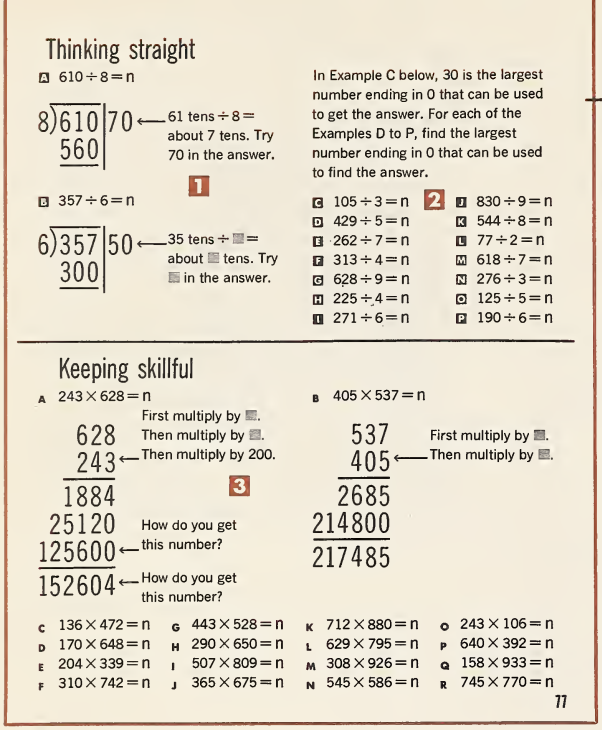
Answers

Thinking straight:

C 30	K 60
D 80	L 30
E 30	M 80
F 70	N 90
G 60	O 20
H 50	P 30
I 40	
J 90	

Keeping skillful:

C 64,192	K 626,560
D 110,160	L 500,055
E 69,156	M 285,208
F 230,020	N 319,370
G 233,904	O 25,758
H 188,500	P 250,880
I 410,163	Q 147,414
J 246,375	R 573,650


Thinking straight
A 610 ÷ 8 = n
8)610 70
560
61 tens ÷ 8 = about 7 tens. Try 70 in the answer.
1
B 357 ÷ 6 = n
6)357 50
300
35 tens ÷ ■ = about ■ tens. Try ■ in the answer.
In Example C below, 30 is the largest number ending in 0 that can be used to get the answer. For each of the Examples D to P, find the largest number ending in 0 that can be used to find the answer.
2
C 105 ÷ 3 = n
D 429 ÷ 5 = n
E 262 ÷ 7 = n
F 313 ÷ 4 = n
G 628 ÷ 9 = n
H 225 ÷ 4 = n
I 271 ÷ 6 = n
J 830 ÷ 9 = n
K 544 ÷ 8 = n
L 77 ÷ 2 = n
M 618 ÷ 7 = n
N 276 ÷ 3 = n
O 125 ÷ 5 = n
P 190 ÷ 6 = n
Keeping skillful
A 243 × 628 = n
First multiply by ■.
Then multiply by ■.
Then multiply by 200.
628
243
1884
25120
125600
152604
How do you get this number?
How do you get this number?
3
B 405 × 537 = n
537
405
2685
214800
217485
First multiply by ■.
Then multiply by ■.
C 136 × 472 = n
D 170 × 648 = n
E 204 × 339 = n
F 310 × 742 = n
G 443 × 528 = n
H 290 × 650 = n
I 507 × 809 = n
J 365 × 675 = n
K 712 × 880 = n
L 629 × 795 = n
M 308 × 926 = n
N 545 × 586 = n
O 243 × 106 = n
P 640 × 392 = n
Q 158 × 933 = n
R 745 × 770 = n
77

77

1. Use the estimating method described in "Comments" to explain Examples A and B.
2. Let the children practice this method by doing Examples C to P. When they have finished, let them discuss their answers. Be sure the children all understand this estimating method, even though some of them may prefer to use some other method of finding the answer.
3. Discuss Examples A and B in "Keeping skillful," and then have the children work independently on Exercises C to R. Supply answers for verifying the work.

78-84 Learning how

Expanded Notes for this lesson are on pages 282-284.

Objectives

The child learns to divide when there are two-figure divisors and two-figure answers.

Vocabulary

New words page 84 loaf*, bread*

Comments

This lesson follows closely the section on one-figure divisors (pages 68-74) because the authors believe there is a real advantage in having children study division with two-figure divisors immediately after working with one-figure divisors. Since the procedure is the same in both cases, the children thus see that there is really nothing different about the method of dividing with two-figure divisors.

The problems in this lesson are confined to the type in which the size of each group to be formed is known and the number of groups is to be found. The lesson is introduced with a problem in which the answer involves a remainder.

In the pictures illustrating the examples, no attempt has been made to show individually each object in the large group of objects that is to be divided. The purpose of the pictures is to show the type of activity rather than the actual number of objects.

The four-step teaching method is again used. Those portions of pictures not under discussion are dimmed off, and the numerals under discussion are in black.

In the TRY step on pages 82 and 83, four ways of doing the work are shown for each example. The fourth solution (in blue) is the most efficient. As mentioned previously, it is desirable to allow the child considerable freedom in choosing the numbers he wants to use as partial quotients.

At this point, the only attempt made to teach the refinements of getting the largest possible partial quotient is by showing the most efficient solution in color. Emphasis here is on the process; how to make

the best estimates for partial quotients will be considered later.

The DO step on page 84 includes one column of exercises with one-figure divisors. Children whose understanding of division is adequate should not, of course, be required to do these exercises.

Answers

Page 84:

(block 1)

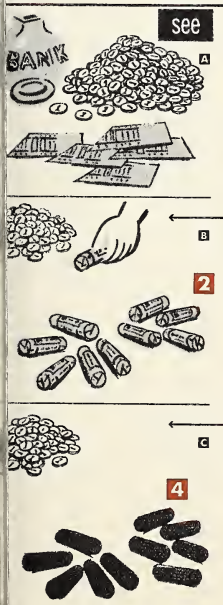
A 7
B 4
C 23 and 20 rm.
D 21 and 2 rm.
E 5
F 12 and 54 rm.
G 7 and 3 rm.
H 52
I 11 and 14 rm.
J 39 and 4 rm.
K 9
L 2 and 40 rm.
M 4 and 12 rm.
N 53
O 32 and 34 rm.
P 17
Q 7 and 2 rm.
R 11 and 14 rm.
S 5
T 7
U 2 and 15 rm.
V 2 and 55 rm.
W 85 and 8 rm.
X 41

(block 2)

A 40 and 4 rm.
B 58
C 81 and 3 rm.
D 75 and 1 rm.
E 74 and 3 rm.
F 93 and 3 rm.
G 53
H 72
I 90
J 92
K 63 and 2 rm.
L 25
M 6 and 9 rm.
N 4 and 3 rm.
O 41 and 46 rm.
P 14 and 5 rm.
Q 82
R 62 and 8 rm.
S 12
T 8 and 5 rm.
U 9 and 7 rm.
V 90

Learning how

This lesson shows how to divide by a number that has two figures.



see

George has saved 864 pennies. He is wrapping them in rolls of 50 to take to the bank. How many rolls of 50 will he have? How many pennies will be left?

1 $864 \div 50 = n$ ← You must find this number.

← Start by making 10 rolls of pennies. For 10 rolls you will use 500 pennies.

2 $50\overline{)864}\ 10$ ← **3** Start with 10 groups of 50 pennies.

500 ← $10 \times 50 = 500$. Write 500 here so that you can subtract to find how many pennies are left.

← Some pennies are left to be wrapped in rolls.

4 $50\overline{)864}\ 10$

500

364 ← 364 pennies are left.

Division by 2-figure divisor; 2-figure quotient

78

1 Have the pupils read the problem and equation without trying to answer the questions or find the missing number. Get them to analyze the equation in terms of the data given in the problem. They should interpret Picture A as illustrating a pile of 864 pennies that are to be wrapped in rolls of 50 in the bank wrappers shown.

2 The children should observe that 10 rolls of 50 pennies each have been made. Ask why 10 is a good number to start with. Ask how many 10 fifties are.

3 Call attention to the partial quotient 10 and the product for 10×50 in the computation. Ask someone if all the 864 pennies were used. Ask him what has to be done to find out how many pennies are left.

4 Point out that making 10 rolls of 50 pennies each did not use up all of the pennies. The pupils should see that after the subtraction has been done 364 pennies are left.

D Try 5 more rolls of pennies. For 5 rolls you will use 250 pennies.

1

$$\begin{array}{r|l} 50\overline{)864} & 10 \\ 500 & \\ \hline 364 & 5 \\ 250 & \\ \hline 114 & \end{array}$$

Try 5 more groups of 50 pennies. 2

5 × 50 = 250. Write 250 here so that you can subtract.

E There are still some pennies left.

3

$$\begin{array}{r|l} 50\overline{)864} & 10 \\ 500 & \\ \hline 364 & 5 \\ 250 & \\ \hline 114 & \end{array}$$

114 pennies are left.

F Try 2 more rolls of pennies. For 2 rolls you will use 100 pennies.

4

$$\begin{array}{r|l} 50\overline{)864} & 10 \\ 500 & \\ \hline 364 & 5 \\ 250 & \\ \hline 114 & 2 \\ 100 & \\ \hline 14 & \end{array}$$

Try 2 more groups of 50.

2 × 50 = 100. Write 100.

Now turn the page.

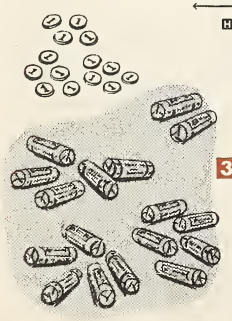
79

G Only 14 pennies are left. That is not enough for another roll of 50 pennies.

1

$$\begin{array}{r|l} 50\overline{)864} & 10 \\ 500 & \\ \hline 364 & 5 \\ 250 & \\ \hline 114 & 2 \\ 100 & \\ \hline 14 & \end{array}$$

2 You cannot try another group of 50. There is a remainder of 14.



H There will be 17 rolls of 50 pennies. 14 pennies will be left over.

Add 10, 5, and 2.

3

$$\begin{array}{r|l} 50\overline{)864} & 10 \\ 500 & \\ \hline 364 & 5 \\ 250 & \\ \hline 114 & 2 \\ 100 & \\ \hline 14 & 17 \end{array}$$

4

There will be 17 rolls of 50 pennies.

14 pennies will be left over.

$864 \div 50 = 17$ and 14 remainder

80

79

1 Help the pupils see that 5 more rolls of pennies have been made. Ask why 5 is a reasonable number to try now. Why not 10?

2 The pupils should note where the 5 and the product of 5 fifties are written.

3 The picture and the computation show that not all the pennies were used. 114 pennies are left. Get the children to see that there are enough pennies left to make more rolls.

4 Ask why two more rolls of pennies have been made. Call attention to the 2 and the product for 2×50 in the computation. Ask if all the pennies were used this time. Ask what has to be done to find out how many pennies are left.

80

1 The children should observe that not all the pennies were used, but that there are only 14 pennies left—not enough to make another roll of 50 pennies.

2 Explain to the pupils that after the 2 groups of 50 pennies, or 100 pennies, were subtracted from the 114 pennies, 14 remained.

3 The pupils should understand that to find how many rolls of pennies in all were made, they must combine the three groups of rolls. [10+5+2] The dotted background should suggest this combining. They can count the total number of rolls and can see the remainder of 14 pennies.

4 Relate the adding of 10, 5, and 2 (which represent the groups of 50) to the combining of the three groups of rolls in Picture H.

think Paul has 650 new stamps for his collection. Each page of his stamp book holds 24 stamps. How many pages can he fill with 650 stamps? Will any stamps be left over? 1

650 ÷ 24 = n ← You must find this number.

A Start by filling 20 pages. For 20 pages you will use 20 × ■, or ■ stamps. 2

$$\begin{array}{r|l} 24\overline{)650} & 20 \\ 480 & \end{array}$$

First try ■ twenty-fours. ← 20 × ■ = 480. Why do you write 480 here?

B How many stamps are left?

$$\begin{array}{r|l} 24\overline{)650} & 20 \\ 480 & \\ \hline 170 & \end{array}$$

3 How do you get 170? What does the 170 show?

C How do you know that you can fill 7 more pages? You will use ■ × 24, or ■ more stamps.

$$\begin{array}{r|l} 24\overline{)650} & 20 \\ 480 & \\ \hline 170 & 7 \\ 168 & \end{array}$$

4 Now try ■ more twenty-fours. ← How do you get 168?

D Are there enough stamps left to fill another page?

$$\begin{array}{r|l} 24\overline{)650} & 20 \\ 480 & \\ \hline 170 & 7 \\ 168 & \\ \hline 2 & \end{array}$$

5 ← There is a remainder of ■.

E How many pages of 24 stamps will there be? How many stamps will be left?

$$\begin{array}{r|l} 24\overline{)650} & 20 \\ 480 & \\ \hline 170 & 7 \\ 168 & \\ \hline 2 & 27 \end{array}$$

What do you do now?

6 There will be ■ full pages of stamps.

■ stamps will be left.

650 ÷ 24 = 27 and ■ remainder

81

1 First ask the pupils to read the problem and the equation without trying to answer the questions or give the missing number. Then call on someone to explain the equation.

2 Let a child tell why 20 is a good choice for a first try at finding how many pages can be filled. [It is easy to multiply by 20.]

3 Have the pupils verify the subtraction of 480 from 650. Ask how many stamps are left.

4 Ask how many stamps were used on 20 pages of 24 stamps each. Ask if this many pages can be filled again, and why not. Have someone explain why 7 pages are tried this time.

5 The children should see that the 2 stamps remaining when 7 more pages have been filled are not enough to fill another page.

6 Have the pupils add the partial quotients. Ask them to explain what each of the black numerals means.

try A 703 ÷ 18 = n 1

$$\begin{array}{r|l} 18\overline{)703} & 10 \\ 180 & \\ \hline 523 & 10 \\ 180 & \\ \hline 343 & 10 \\ 180 & \\ \hline 163 & 5 \\ 90 & \\ \hline 73 & 3 \\ 54 & \\ \hline 19 & 1 \\ 18 & \\ \hline 1 & 39 \end{array} \qquad \begin{array}{r|l} 18\overline{)703} & 20 \\ 360 & \\ \hline 343 & 10 \\ 180 & \\ \hline 163 & 5 \\ 90 & \\ \hline 73 & 2 \\ 36 & \\ \hline 37 & 2 \\ 36 & \\ \hline 1 & 39 \end{array} \qquad \begin{array}{r|l} 18\overline{)703} & 20 \\ 360 & \\ \hline 343 & 10 \\ 180 & \\ \hline 163 & 8 \\ 144 & \\ \hline 19 & 1 \\ 18 & \\ \hline 1 & 39 \end{array} \qquad \begin{array}{r|l} 18\overline{)703} & 30 \\ 540 & \\ \hline 163 & 9 \\ 162 & \\ \hline 1 & 39 \end{array}$$

703 ÷ 18 = n

B 5148 ÷ 63 = n

$$\begin{array}{r|l} 63\overline{)5148} & 20 \\ 1260 & \\ \hline 3888 & 20 \\ 1260 & \\ \hline 2628 & 20 \\ 1260 & \\ \hline 1368 & 20 \\ 1260 & \\ \hline 108 & 1 \\ 63 & \\ \hline 45 & 81 \end{array} \qquad \begin{array}{r|l} 63\overline{)5148} & 30 \\ 1890 & \\ \hline 3258 & 40 \\ 2520 & \\ \hline 738 & 10 \\ 630 & \\ \hline 108 & 1 \\ 63 & \\ \hline 45 & 81 \end{array} \qquad \begin{array}{r|l} 63\overline{)5148} & 50 \\ 3150 & \\ \hline 1998 & 20 \\ 1260 & \\ \hline 738 & 10 \\ 630 & \\ \hline 108 & 1 \\ 63 & \\ \hline 45 & 81 \end{array} \qquad \begin{array}{r|l} 63\overline{)5148} & 80 \\ 5040 & \\ \hline 108 & 1 \\ 63 & \\ \hline 45 & 81 \end{array}$$

5148 ÷ 63 = n

82

1 Have the pupils write Examples A and B in computational form and find the answers. When they have found the answer to an example, they should compare their answer with the one given in the book. Tell the children that if their answers are the same as those shown in the book, their work may be considered correct, even though their work may differ from the four solutions shown for each example.

C 1330 ÷ 35 = n

1

35)1330 10 — 350 — 980 20 — 700 — 280 5 — 175 — 105 3 — 105 — 38

35)1330 10 — 350 — 980 10 — 350 — 630 10 — 350 — 280 8 — 280 — 38

35)1330 20 — 700 — 630 10 — 350 — 280 4 — 140 — 140 4 — 140 — 38

35)1330 30 — 1050 — 280 8 — 280 — 38

1330 ÷ 35 = n

D 609 ÷ 15 = n

15)609 10 — 150 — 459 10 — 150 — 309 20 — 300 — 9 40

15)609 20 — 300 — 309 20 — 300 — 9 40

15)609 10 — 150 — 459 30 — 450 — 9 40

15)609 40 — 600 — 9

609 ÷ 15 = n

E 448 ÷ 56 = n

56)448 2 — 112 — 336 4 — 224 — 112 2 — 112 — 8

56)448 4 — 224 — 224 4 — 224 — 8

56)448 5 — 280 — 168 2 — 112 — 56 1 — 56 — 8

56)448 8 — 448

448 ÷ 56 = n

2

83

1 Tell the pupils to write these examples in computational form and find the answers. Tell them to keep trying to find the shortest and most efficient way for their computation. Let them work independently, and direct them to proceed to the work labeled DO on the next page as soon as they get the correct answers for Examples C, D, and E.

2 After the children have found the answers to Examples A to E (A and B are on page 82), discuss the different solutions shown in the book. Point out that those shown in blue illustrate the quickest ways to get the answers.

do

A Kathy has made 84 cookies. She wonders how many dozen cookies she has made. She has made how many dozen cookies?
84 ÷ 12 = n

1

C 641 ÷ 27 = n
D 968 ÷ 46 = n
E 420 ÷ 84 = n
F 750 ÷ 58 = n
G 220 ÷ 31 = n
H 728 ÷ 14 = n
I 839 ÷ 75 = n
J 862 ÷ 22 = n
K 540 ÷ 60 = n
L 138 ÷ 49 = n
M 304 ÷ 73 = n
N 4240 ÷ 80 = n
O 1218 ÷ 37 = n
P 901 ÷ 53 = n
Q 485 ÷ 69 = n
R 300 ÷ 26 = n
S 390 ÷ 78 = n
T 119 ÷ 17 = n
U 95 ÷ 40 = n
V 231 ÷ 88 = n
W 6043 ÷ 71 = n
X 1599 ÷ 39 = n

B The Fishers are going to make 80 sandwiches. Mrs. Fisher says that they can make 20 sandwiches from 1 loaf of bread. How many loaves of bread will they need?
80 ÷ 20 = n

A 324 ÷ 8 = n
B 116 ÷ 2 = n
C 408 ÷ 5 = n
D 226 ÷ 3 = n
E 521 ÷ 7 = n
F 840 ÷ 9 = n
G 265 ÷ 5 = n
H 432 ÷ 6 = n
I 360 ÷ 4 = n
J 736 ÷ 8 = n
K 443 ÷ 7 = n
L 1125 ÷ 45 = n
M 561 ÷ 92 = n
N 135 ÷ 33 = n
O 2342 ÷ 56 = n
P 299 ÷ 21 = n
Q 5740 ÷ 70 = n
R 1000 ÷ 16 = n
S 1116 ÷ 93 = n
T 637 ÷ 79 = n
U 124 ÷ 13 = n
V 5490 ÷ 61 = n

Now you should be able to divide by any number that has two figures.

Using arithmetic

A One day 15 girls in the 4-H Club canned 6 bushels of tomatoes. Mrs. White told them they would get about 26 quarts of canned tomatoes from each bushel. How many quarts did they expect to get from 6 bushels?

B In all, the girls got 144 quarts of canned tomatoes. They got how many fewer quarts of tomatoes than they expected to get?

C For cooking the tomatoes the girls used a kettle that held enough at one time to fill about 12 quart jars. About how many times had the girls filled the kettle by the time they had finished canning the tomatoes?

D Ann Fuller went to the store to buy 96 more jar lids. The lids were sold in packages of 8. How many packages of jar lids should Ann have bought?

84

1 Assign Exercises A to X as written work. When the pupils have finished, discuss the answers and the different ways of getting them. Let the children put some of their solutions on the board and justify them.

2 Exercises A to V should be assigned to the pupils who need extra practice.

84-85 Using arithmetic

Expanded Notes are not considered necessary for this lesson.

Objectives

The child practices solving various types of problems.

Vocabulary

New words page 84 expect*, kettle*, held*, lids*; page 85 among*, skirt, project, besides, aprons*, rickrack, trim*

Comments

These problems should be assigned without any preliminary discussion. When the children have finished, let them help one another with the problems that only a few pupils missed. Spend most of the discussion time on problems that many of the children missed. This discussion should be guided by consideration of whether the difficulties were in computing or in making equations. If the pupils had difficulty in making the proper equations, it may be helpful to let the children dramatize some of the problems to give them an understanding of what is happening.

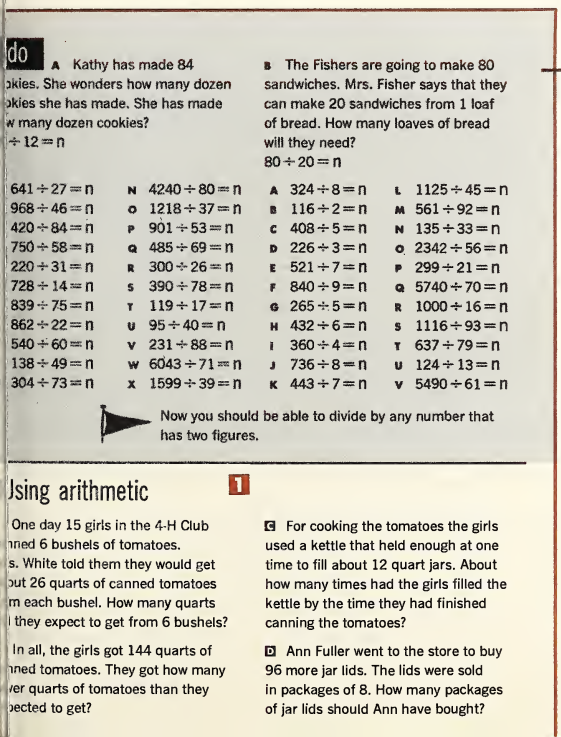
Answers

Accept as correct any order of addends for Problem J.

A $6 \times 26 = [156]$
B $156 - 144 = [12]$
C $144 \div 12 = [12]$
D $96 \div 8 = [12]$
E $[60] + 84 = 144$
F $15 \times 5 = [75]$
G $144 \div [48] = 3$
H $2 \times \$1.17 = [\$2.34]$
I $\$2.82 - \$2.34 = [\$.48]$ or $\$2.34 + [\$.48] = \$2.82$
J $54¢ + 65¢ + 68¢ = [\$1.87]$
K $84¢ \div [28¢] = 3$
L $\$2.82 - \$2.71 = [\$.11]$
M $15 - 8 = [7]$
N $\$3.00 - [\$2.79] = \$.21$

Keeping skillful:

(block 1)	(block 2)	(block 3)
A 8698	A 221	A 3208
B 210	B 1039	B 693
C 917	C 554	C 1472
D 977	D 326	D 675
E 951	E 1943	E 3248
F 1888	F 15,700	F 16,280
G 799	G 370	G 5365
H 2144	H 21,883	H 236,040
I 1033	I 11,467	I 321,685



do

A Kathy has made 84 ookies. She wonders how many dozen ookies she has made. She has made w many dozen cookies? $\div 12 = n$

B The Fishers are going to make 80 sandwiches. Mrs. Fisher says that they can make 20 sandwiches from 1 loaf of bread. How many loaves of bread will they need? $80 \div 20 = n$

$641 \div 27 = n$	N $4240 \div 80 = n$	A $324 \div 8 = n$	L $1125 \div 45 = n$
$968 \div 46 = n$	O $1218 \div 37 = n$	B $116 \div 2 = n$	M $561 \div 92 = n$
$420 \div 84 = n$	P $901 \div 53 = n$	C $408 \div 5 = n$	N $135 \div 33 = n$
$750 \div 58 = n$	Q $485 \div 69 = n$	D $226 \div 3 = n$	O $2342 \div 56 = n$
$220 \div 31 = n$	R $300 \div 26 = n$	E $521 \div 7 = n$	P $299 \div 21 = n$
$728 \div 14 = n$	S $390 \div 78 = n$	F $840 \div 9 = n$	Q $5740 \div 70 = n$
$839 \div 75 = n$	T $119 \div 17 = n$	G $265 \div 5 = n$	R $1000 \div 16 = n$
$862 \div 22 = n$	U $95 \div 40 = n$	H $432 \div 6 = n$	S $1116 \div 93 = n$
$540 \div 60 = n$	V $231 \div 88 = n$	I $360 \div 4 = n$	T $637 \div 79 = n$
$138 \div 49 = n$	W $6043 \div 71 = n$	J $736 \div 8 = n$	U $124 \div 13 = n$
$304 \div 73 = n$	X $1599 \div 39 = n$	K $443 \div 7 = n$	V $5490 \div 61 = n$

Now you should be able to divide by any number that has two figures.

Using arithmetic 1

One day 15 girls in the 4-H Club ined 6 bushels of tomatoes. s. White told them they would get out 26 quarts of canned tomatoes m each bushel. How many quarts they expect to get from 6 bushels?

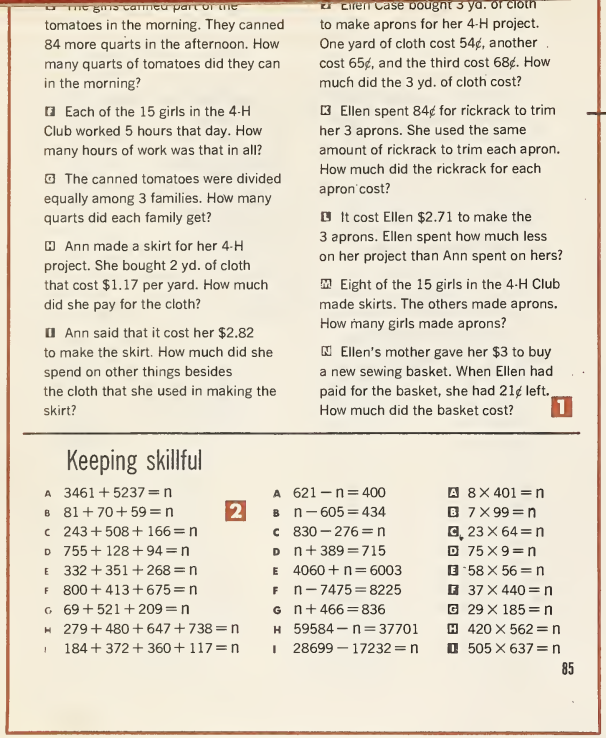
In all, the girls got 144 quarts of ned tomatoes. They got how many ver quarts of tomatoes than they pected to get?

C For cooking the tomatoes the girls used a kettle that held enough at one time to fill about 12 quart jars. About how many times had the girls filled the kettle by the time they had finished canning the tomatoes?

D Ann Fuller went to the store to buy 96 more jar lids. The lids were sold in packages of 8. How many packages of jar lids should Ann have bought?

84

1 Without previous discussion, ask the children to work Problems A to N (E to N are on page 85). Direct them first to write the equation with *n* for the unknown number, then compute, and, finally, rewrite the equation with the answer in place of *n*. Have them label their problems with the identifying letters used in the book.



The girls canned part of the tomatoes in the morning. They canned 84 more quarts in the afternoon. How many quarts of tomatoes did they can in the morning?

F Each of the 15 girls in the 4-H Club worked 5 hours that day. How many hours of work was that in all?

G The canned tomatoes were divided equally among 3 families. How many quarts did each family get?

H Ann made a skirt for her 4-H project. She bought 2 yd. of cloth that cost $1.17 per yard. How much did she pay for the cloth?

I Ann said that it cost her $2.82 to make the skirt. How much did she spend on other things besides the cloth that she used in making the skirt?

J Ellen Case bought 3 yd. of cloth to make aprons for her 4-H project. One yard of cloth cost 54¢, another cost 65¢, and the third cost 68¢. How much did the 3 yd. of cloth cost?

K Ellen spent 84¢ for rickrack to trim her 3 aprons. She used the same amount of rickrack to trim each apron. How much did the rickrack for each apron cost?

L It cost Ellen $2.71 to make the 3 aprons. Ellen spent how much less on her project than Ann spent on hers?

M Eight of the 15 girls in the 4-H Club made skirts. The others made aprons. How many girls made aprons?

N Ellen's mother gave her $3 to buy a new sewing basket. When Ellen had paid for the basket, she had 21¢ left. How much did the basket cost? 1

Keeping skillful

2

A	$3461+5237=n$	A	$621-n=400$	A	$8\times401=n$
B	$81+70+59=n$	B	$n-605=434$	B	$7\times99=n$
C	$243+508+166=n$	C	$830-276=n$	C	$23\times64=n$
D	$755+128+94=n$	D	$n+389=715$	D	$75\times9=n$
E	$332+351+268=n$	E	$4060+n=6003$	E	$58\times56=n$
F	$800+413+675=n$	F	$n-7475=8225$	F	$37\times440=n$
G	$69+521+209=n$	G	$n+466=836$	G	$29\times185=n$
H	$279+480+647+738=n$	H	$59584-n=37701$	H	$420\times562=n$
I	$184+372+360+117=n$	I	$28699-17232=n$	I	$505\times637=n$

85

85

1 Let the children verify their work on Problems A to N (A to D are on page 84) by comparing it with equations and answers you provide for them. Discuss thoroughly any problems that have caused trouble for a majority of the children.

2 Direct the pupils to write the numerals on their papers in the correct form for computing. Be sure to provide answers, so that the children can verify their work when they have finished. Discuss any exercises that cause trouble.

86-88 Exploring problems

Expanded Notes for this lesson are on pages 284-287.

Objectives

The child learns how to solve problems that involve division in which both the divisor and the dividend represent money.

Vocabulary

New words page 88 office*, Cub*, Scouts*, den*, film*

Comments

The child has studied two types of divisive situations up to this time. In one of these types he must find the *number* of equal groups when the size of the equal groups and the total number are known, and in the other type he must find the *size* of the equal groups when the number of groups and the total number are known. Although the equations used to express these two types of problem situations differ, each one is solved by the same computational method.

In working problems involving division of money, the child will again encounter these two types of problem situations. In each case he will work with two numbers to find another number (the answer). When the answer represents the number of groups, the known numbers he works with will always be associated with dollars, dollars and cents, or cents only (see the example on page 86). But when the answer represents the size of the group, *only one* of the numbers with which the child has to work will be associated with dollars, dollars and cents, or cents, since the other known number must be associated with the number of equal groups (see the example on page 87). As noted above, the computational method used to find the answer is the same in both types of problem situations.

Notice that the child is asked to do the computation without the interference of the point. (This is actually the only way he *can* operate with the numbers.) If the child has a thorough understanding of the problem situation he started with, he will interpret his answer

in the only sensible way it can be interpreted when he has finished his computation.

In your teaching it is important to pay special attention to thorough understanding of the problem situation. Before the child begins to compute, he must understand very clearly what it is he is going to find. Sometimes this is how many equal groups there are (for example, how many persons will receive the equal groups of things or how many things can be bought). At other times he is to find how many there are in a group (for example, how many things each person will get, or how much each thing will cost). If he understands the problem situation, knows that division is the process that should be used to find the answer, and knows how to do the division, his interpretation of the answer itself should be correct. In the type of problem situation in which the unknown number is associated with an amount of money, he will know that if there are no more than two figures in the answer, this answer will represent cents, but that if there are more than two figures, the answer will represent dollars and cents.

In some problems there may be a remainder. It will be necessary to stress the meaning of this remainder in problems involving division of money, such as the "How many tickets did she buy?" problem on page 86, so that the children understand that the remainder represents money and not tickets.

Answers

Page 88:

A $4.30÷[$.71]=6 and 4¢ rm.
B $2.25÷[$.75]=3
C $1.50÷$.35=[4] and 10¢ rm.
D $.50÷$.02=[25]
E $.75÷$.15=[5]
F $3.00÷[$.25]=12
G $.98÷[$.49]=2
H $.87÷[$.29]=3
I $2.25÷[$.45]=5
J 72¢÷[6¢]=12
K $1.00÷$.15=[6] and 10¢ rm.
L $2.00÷$.39=[5] and 5¢ rm.

Exploring problems

see

A Movie tickets cost 22¢ each. Sally wonders how many movie tickets she can buy with $2. Will she have any money left?

1

$2.00÷$.22=n
n: Number of tickets Sally can buy; You must find this number.

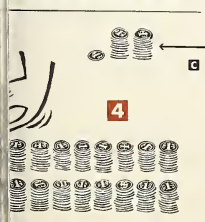
2

compute

3

B Think of the money in pennies.

200 ← Think of $2.00 as 200.



C Find how many groups of 22 pennies there are in 200 pennies.

4

$$22\overline{)200}\ \begin{array}{l}9\\ 198\\ \hline 2\end{array}$$

9 ← Sally can buy 9 tickets.

5

She will have 2¢, or $.02, left over.

Teaching of division involving dollars and cents

86

1 Have the children read the problem and relate it to the picture.
2 The pupils should relate each numeral in this equation to the problem.
3 Tell the children to think of the money in pennies so that they will not have to use the point when they compute.
4 Let the class discover that these pennies, which are now grouped in piles of 10, must be put in groups of 22.
5 Tell the class that if they did not have the actual money to put into piles, they would use numerals to represent the amounts of money and they would divide by 22 because the pennies are to be put into groups of 22. Tell the pupils that they must find out how many of these groups, or piles, there will be. Call special attention to the remainder and to the fact that this remainder represents pennies, not tickets. [9 represents the number of tickets; 2 stands for the pennies that are left over.]

see

A

The 22 boys in the Camera Club want to buy a book. The book costs $2. If the boys share the cost equally, how much must each boy pay? Will anyone have to pay more than his share? If so, how much more will he have to pay?

1

$2.00 ÷ n = 22 ← Number of boys; ← You must find this amount of money.

compute

2

Think of the money they need in pennies.

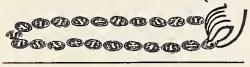
B

200 ← Think of $2.00 as 200.

C

3

Each time the boys pay 1 penny apiece, a group of 22 pennies is used.



The pennies are now in 22 piles of 9. Each boy's share is 9¢. 2 pennies are left over.

D

5

$$\begin{array}{r|l} 22\overline{)200} & 9 \\ 198 & \\ \hline 2 & \end{array}$$

← Each boy's share is 9¢, or $.09.

4

← Someone must pay 2¢ more than his share.

87

A Patsy has $4.30 to spend for 6 presents. She wants to spend the same amount for each present. How much can she spend for each present? Will any money be left over? If so, how much?

1

THINK When you finish dividing, you will have two numbers. One will be the largest amount of money Patsy can spend for each present. The other will be the money left.

$4.30 ÷ n = 6 Divide 430 by 6.

B Mrs. White paid $2.25 for 3 cakes of the same price. How much did each cake cost?

2

$2.25 ÷ n = 3 Divide 225 by 3.

C The girls in the Tuesday Club have $1.50 to spend for colored paper. The paper costs $.35 a package. How many packages can they buy? Will they have any money left? If so, how much?

3

THINK When you finish dividing, one number will be the number of packages of paper. The remainder will be the amount of money left.

$1.50 ÷ $.35 = n Divide 150 by 35.

D George bought 50¢ worth of 2-cent postcards at the post office. How many postcards did he get? Did he have any money left? If so, how much?

4

$.50 ÷ $.02 = n Divide 50 by 2.

E How many 15-cent paper dolls can Carol buy with $.75? Will she have any money left? If so, how much?

F Twelve Cub Scouts bought a plant for their den mother. They shared the cost equally. The plant cost $3. How much was each boy's share? Did anyone have to pay more than his share? If so, how much?

G Mrs. White paid $.98 for 2 lb. of salted peanuts. The peanuts cost how much a pound?

H Tony paid $.87 for 3 goldfish. If the fish were all the same price, how much did each fish cost?

I Five boys earned $2.25 by raking leaves. They shared the money equally. How much was each boy's share? Was any money left over? If so, how much?

J Mary paid 72¢ for 1 dozen doughnuts. The doughnuts cost how many cents apiece?

K How many 15-cent candy apples can Ellen buy with $1? Will she have any money left? If so, how much?

L Don earned $2 to buy film for his camera. The film costs $.39 per roll. How many rolls of film can he buy? Will he have any money left? If so, how much?

5

Now you should be able to divide amounts of money.

87

1 Have the children read the problem and relate it to the equation and picture. Note that this problem involves the same numbers as the previous problem, but now the 22 tells how many equal groups there are, not how many there are in each group.

2 Tell the children that here also the money is to be thought of as pennies.

3 Explain that the pennies are to be put into 22 equal piles [one pile for each of the 22 boys]. Point out that the picture shows that a penny has been put down for each boy and that 22 pennies have been used. Tell the class that each time another penny is put in each of the 22 piles a group of 22 pennies is used.

4 Discuss the division process just as you would for any other example.

5 Help the class see that this picture shows what the 9 and the 2 in the computation stand for.

88

1 Let the children tell what happens each time Patsy puts a penny in each of 6 piles. Have them tell why 430 is divided by 6. Discuss what the answer and the remainder represent.

2 Have the class discuss and explain this problem in the same way as Problem A.

3 Tell the children to find out how many equal groups there are; let them go through a demonstration and discussion to show what happens. [Quantities in these problems have been kept small so that, if you want to, you can use money and work out the answer.]

4 Let pupils work independently on Problems D to L. They should write the proper equation with *n* for the missing number, compute, and finally rewrite the equation with the answer.

5 Discuss any problems that caused trouble. Refer to pages 86 and 87, if necessary.

89-90 Using arithmetic; Checking up

Expanded Notes are not considered necessary for these lessons.

Objectives

The child practices solving types of problems that have been presented so far.

Vocabulary

New words **page 89** trunk*, age*, planet, Mercury, Jupiter, stars*, caterpillars, member*; **page 90** compasses

Comments

Let the children work these problems without any preliminary discussion. When they have completed the set and checked their answers, let them help one another with the problems that only a few children missed. Spend most of the discussion time on problems that many of the children missed.

In connection with the "Checking up" section, refer to "Providing for the able pupil" in the Expanded Notes for page 18 (pages 258-260) and to Activity 9, page 360 for an activity the able pupils might profit from.

Answers

Using arithmetic:

A 41 − 18 = [23]

B 28 + [18] = 46

C 75 ÷ 15 = [5]

D 365 ÷ 88 = [4] and 13 rm., about 4
Do nothing with the remainder in this problem because 13 is not large enough to add another year in rounding off.

E 4380 ÷ 88 = [49] and 68 rm. The answer is "About 50 years."

F 88 ÷ 7 = [12] and 4 rm., 12 weeks and 4 days

G 88 ÷ 30 = [2] and 28 rm., about 3 months

H 170 − 93 = [77] or 93 + [77] = 170

I 81 − 65 = [16]

J 24 − [5] = 19

K $18.00 ÷ [$.75] = 24
We divide 1800 by 24 because we must think of $18 as 1800 cents in order to be able to divide by 24.

L 62 − 24 = [38]

M $3.50 ÷ $.85 = [4] and 10¢ rm.

N 2 × $2.79 = [$5.58] or $2.79 + $2.79 = [$5.58]

O $18.00 − [$14.35] = $3.65

P $2.40 ÷ [$.10] = 24

Q 90 ÷ 6 = [15]

R [171] − 48 = 123

Checking up:

Test 1

A 41 and 1 rm.	E 18 and 4 rm.	I 63
B 19	F 26 and 3 rm.	J 88 and 1 rm.
C 34 and 1 rm.	G 45	K 90 and 3 rm.
D 60 and 6 rm.	H 62 and 4 rm.	

Test 2

A 19 and 7 rm.	E 8	I 36
B 50 and 2 rm.	F 20 and 44 rm.	J 51 and 8 rm.
C 38	G 42 and 3 rm.	K 78 and 23 rm.
D 29 and 7 rm.	H 6 and 15 rm.	

Test 3

A 45 and 1 rm.	E 94 and 4 rm.	I 34 and 2 rm.
B 50 and 3 rm.	F 47	J 6 and 9 rm.
C 33	G 50 and 4 rm.	K 90
D 12 and 49 rm.	H 9	

Test 4

A $.29	E 78 and $.04 rm.	I 24
B 17 and $.05 rm.	F 67	J 32
C 38	G $.03	K $.06 and $.31 rm.
D $.26	H $.07 and $.09 rm.	

Using arithmetic 1

A In science class, the fifth graders learned that a tree trunk has 1 ring for each year of the tree's age. One tree trunk had 41 rings, and another had 18 rings. The first tree was how many years older than the second tree?

B The boys and girls had planted some beans. One day they noticed that 28 bean plants had come up. The next day they saw that, in all, 46 bean plants had come up. How many had come up on the second day?

C The fifth graders wanted to plant 75 seeds of corn. They could plant 15 seeds in a box. How many boxes did they need for the 75 corn seeds?

D The fifth graders learned that the planet Mercury's year is 88 days long. Our year has 365 days. About how many of its years would Mercury have during one of our years?

THINK What do you do with the remainder in the problem? Why?

E They also learned that Jupiter has about 4380 days in its year. About how many of its years would Mercury have during one of Jupiter's years?

THINK Is the remainder large enough to add one more year to your answer?

F Suppose that Mercury had a 7-day week, just as we do. How many weeks would Mercury have in one of its years? How many days would be left over?

G If Mercury had a 30-day month, about how many months would there be in one of its years? How many days would be left over?

H John was reading a book of 170 pages about stars. He had read 93 pages. How many pages did John still have to read to finish the book?

I Patsy and Jane collected different kinds of seeds and pasted them on cardboard to show the class. Patsy had 81 different seeds. Jane had 65 different seeds. Patsy collected how many more seeds than Jane?

J The class had 24 caterpillars in a cage. One day they saw that some of the caterpillars had died. There were only 19 caterpillars left. How many of them had died?

K Twenty-four of the fifth graders belonged to a science club. In September the club collected $18 in dues for the whole year. How much did each club member pay in dues?

THINK Why do you divide 1800 by 24?

L There were 62 fifth graders in the school. How many of them were not members of the science club?

M The members of the science club planned to use $3.50 of the dues to buy magnets. Each magnet cost 85¢. How many magnets could they buy? How much money would they have left over?

N The boys and girls also bought 2 compasses. Each compass cost $2.79. How much did they pay for both of the compasses?

O At the end of the year the science club had $3.65 left of the money that had been collected in dues. How much had been spent during the year?

P The members of the science club went to a museum one day. The tickets for the 24 boys and girls cost $2.40. How much did each ticket cost?

Q Tony was putting pictures of birds in the science club scrapbook. He had 90 pictures. He could put 6 pictures on each page. How many pages did he need?

R The boys and girls in the science club are making a large picture to show what groups of stars they can find at night. They cut out some paper stars to paste on the picture. After they had used 48 stars for the picture, they still had 123 stars left. How many stars had they cut out? 1

Checking up 2

Test 1	Test 2	Test 3	Test 4
A $83 \div 2 = n$	A $596 \div 31 = n$	A $361 \div 8 = n$	A $\$.87 \div n = 3$
B $76 \div 4 = n$	B $902 \div 18 = n$	B $303 \div 6 = n$	B $\$1.24 \div \$.07 = n$
C $103 \div 3 = n$	C $2128 \div 56 = n$	C $2607 \div 79 = n$	C $\$.76 \div \$.02 = n$
D $426 \div 7 = n$	D $732 \div 25 = n$	D $649 \div 50 = n$	D $\$2.34 \div n = 9$
E $94 \div 5 = n$	E $584 \div 73 = n$	E $850 \div 9 = n$	E $\$4.72 \div \$.06 = n$
F $237 \div 9 = n$	F $1004 \div 48 = n$	F $705 \div 15 = n$	F $\$3.35 \div \$.05 = n$
G $270 \div 6 = n$	G $2523 \div 60 = n$	G $1654 \div 33 = n$	G $\$.72 \div n = 24$
H $500 \div 8 = n$	H $495 \div 80 = n$	H $576 \div 64 = n$	H $\$2.19 \div n = 30$
I $567 \div 9 = n$	I $612 \div 17 = n$	I $1600 \div 47 = n$	I $\$6.48 \div \$.27 = n$
J $441 \div 5 = n$	J $4700 \div 92 = n$	J $525 \div 86 = n$	J $\$4.80 \div \$.15 = n$
K $723 \div 8 = n$	K $5015 \div 64 = n$	K $2520 \div 28 = n$	K $\$3.37 \div n = 51$ 3

89

1 Have the children work independently on Problems A to R (Problems L to R are on page 90). They should read each problem and write an equation with *n* in place of the missing number. Then they should compute

90

1 Be sure to spend some time in discussing the problems that cause difficulty. Refer back to the pages in the book where such problems were taught, if necessary.

2 Remind the children to include the remainders in writing their answers.

3 Provide answers so that the children can verify their work. Be sure to discuss any exercises that give trouble.

91 Keeping skillful

Expanded Notes for this lesson are on pages 287-288.

Objectives

The child practices a technique for selecting partial quotients in division.

Vocabulary

There are no new words.

Comments

The technique for estimating the first partial quotient in a division problem introduced on page 77 of this book is developed further in this lesson. The child is taught to look at the first two numerals of the dividend and think of them as representing so many tens or hundreds. Then he can use these numerals to state the dividend of a 'basic fact." He must understand that the answer to this "basic fact" is so many *tens* or *hundreds*, not *ones*.

Discuss Examples A to D with the children. Then have them complete the work that has been started in the book. When they have finished, let them discuss their solutions. Supply answers for these as well as for the exercises in the two blocks at the bottom of the page.

Further work in estimating partial quotients is done in the lesson on pages 205-206.

Answers

(block 1)

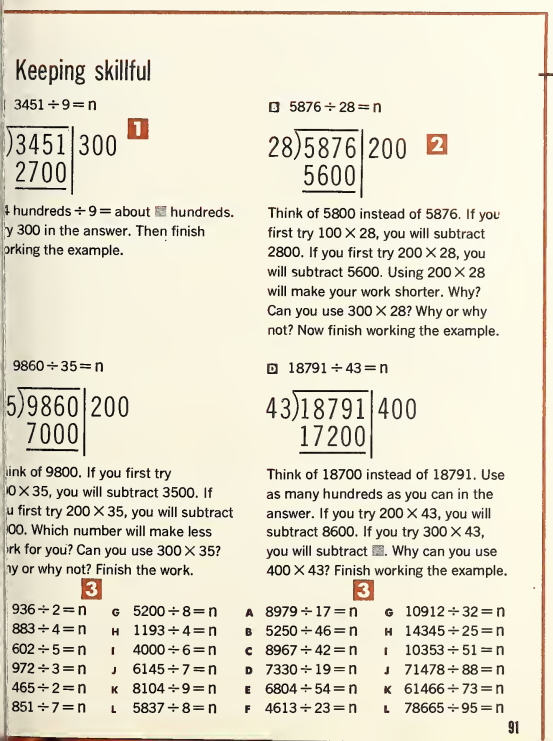
A 383 and 4 rm.	C 281 and 25 rm.
B 209 and 24 rm.	D 437

(block 2)

A 468	E 232 and 1 rm.	I 666 and 4 rm.
B 220 and 3 rm.	F 121 and 4 rm.	J 877 and 6 rm.
C 120 and 2 rm.	G 650	K 900 and 4 rm.
D 324	H 298 and 1 rm.	L 729 and 5 rm.

(block 3)

A 528 and 3 rm.	E 126	I 203
B 114 and 6 rm.	F 200 and 13 rm.	J 812 and 22 rm.
C 213 and 21 rm.	G 341	K 842
D 385 and 15 rm.	H 573 and 20 rm.	L 828 and 5 rm.



Keeping skillful

3451 ÷ 9 = n

)3451 | 300 **1**
2700

4 hundreds ÷ 9 = about ▩ hundreds.
'y 300 in the answer. Then finish
orking the example.

D 5876 ÷ 28 = n

28)5876 | 200 **2**
5600

Think of 5800 instead of 5876. If you first try 100 × 28, you will subtract 2800. If you first try 200 × 28, you will subtract 5600. Using 200 × 28 will make your work shorter. Why? Can you use 300 × 28? Why or why not? Now finish working the example.

9860 ÷ 35 = n

5)9860 | 200
7000

ink of 9800. If you first try
0 × 35, you will subtract 3500. If
u first try 200 × 35, you will subtract
00. Which number will make less
rk for you? Can you use 300 × 35?
y or why not? Finish the work.

D 18791 ÷ 43 = n

43)18791 | 400
17200

Think of 18700 instead of 18791. Use as many hundreds as you can in the answer. If you try 200 × 43, you will subtract 8600. If you try 300 × 43, you will subtract ▩. Why can you use 400 × 43? Finish working the example.

3

936 ÷ 2 = n	G 5200 ÷ 8 = n	A 8979 ÷ 17 = n	G 10912 ÷ 32 = n
883 ÷ 4 = n	H 1193 ÷ 4 = n	B 5250 ÷ 46 = n	H 14345 ÷ 25 = n
602 ÷ 5 = n	I 4000 ÷ 6 = n	C 8967 ÷ 42 = n	I 10353 ÷ 51 = n
972 ÷ 3 = n	J 6145 ÷ 7 = n	D 7330 ÷ 19 = n	J 71478 ÷ 88 = n
465 ÷ 2 = n	K 8104 ÷ 9 = n	E 6804 ÷ 54 = n	K 61466 ÷ 73 = n
851 ÷ 7 = n	L 5837 ÷ 8 = n	F 4613 ÷ 23 = n	L 78665 ÷ 95 = n

91

91

1 Discuss Example A. Point out to the children that it is easy to multiply when all except one of the figures in the partial quotient are zeros. Also get them to see that a large partial quotient saves work. Be sure they understand why thinking of the dividend as so many tens or so many hundreds helps them to estimate a partial quotient. Let the children finish working the example.

2 Discuss Examples B, C, and D in the same way, and let the children finish working them.

3 Assign this block of exercises. When the children have finished working them, let them compare and discuss their solutions. Let them decide which choice of partial quotients is most efficient.

92-95 Looking back; Keeping skillful

Expanded Notes for these lessons are on pages 288-289.

Objectives

The child reviews units of measure and their equivalents. He also reviews changing measures to larger and smaller units.

Vocabulary

New words page 93 objects*, load*, limit

Comments

Units of measure that the children will use in their daily lives are illustrated and discussed in this lesson. Tables listing the equivalents for liquid measures, dry measures, temperature, time, length, and weight are presented, along with accepted abbreviations. Pupils should learn the facts in these tables by using the standard units of measure in the classroom and by their use in exercises, not by sheer memorization.

Have as many as possible of the pictured measuring instruments available in the classroom so that the pupils can handle them and learn about the size of a standard cup, pint, quart, and gallon through use. Let them measure quantities of dried beans, sand, or water. Let them use thermometers to get experience in finding temperatures, and provide a few rulers and yardsticks for measuring lengths, widths, and heights. Small objects in the classroom may be weighed, and the abler pupils might conduct a weighing and measuring project and make a table showing the weights and heights of the children in the class.

Page 92 provides pictures around which discussion may be centered. Some of the illustrations show units of measure. Others show objects that might be measured by using these units.

Besides the tables of measurement equivalents, page 93 presents questions to stimulate thought and discussion about equivalents, use of measures, and use of tables to find information.

Page 94 reviews reduction of measures. Children who have studied the changing of measures by the methods of this arithmetic program should have no difficulty with this work. Others, who may have memorized rules for changing measures, will find explanations in terms of what actually happens (the matching of groups and the combining or separating of equal groups). Much difficulty is caused by failure to relate the procedures of changing measures to familiar problem types. The children must be given help in realizing that the changing of measures involves multiplicative and divisive situations with which they are already familiar and that it requires a type of thinking they have already learned to use in solving non-measurement problems.

In changing units to smaller units, the child should learn to think in terms of the *group* of smaller units that is equal to one of the larger units. Then he must realize that for each larger unit he will have one group of the smaller units. He should easily recognize that finding the number of smaller units in the combined groups is a simple matter of multiplication.

To change units to an equivalent number of larger units, he must ask himself how many of the smaller units there are in a group that equals the larger unit. Knowing the number in the total group and the number in each of the equal groups, he should have no difficulty in realizing that division will provide the answer.

To help the child understand what he is doing and prevent his resorting to trial-and-error procedures, he should be required to write equations for reduction problems. For example, to reduce 18 feet to yards, he should write the equation $18 \div 3 = n$. He should be able to explain that 3 represents the number of feet in each group (yard) and that n represents the number of groups of 3, or the number of yards. To change 6 yards to feet he should write the equation $6 \times 3 = n$. He should be able to explain that 6 represents the number of groups of 3 feet (yards), 3 represents the number of feet in each group (yard), and that n represents the total number of feet.

In working with this lesson, be sure to give the children plenty of opportunities to discuss situations in which they have seen the measuring instruments shown on page 92. Stress the need for scales and standard units in the linear type of measurement, and let the children discuss how the various scales are like and unlike each other. Also discuss good habits and techniques in measuring when using these scales.

Answers

Page 93:

A Liquid measures

B Ice cream, milk, molasses, etc.

C Pictures, desk tops, books, etc.

D Dry measures

E The thermometer at the left is one type of cooking thermometer. It measures from 120° to 300° above zero. The other thermometer is an outdoor temperature thermometer. It measures from 30° below zero to 120° above zero.

F Yes

G Counting table; eggs, ears of corn, oranges, etc.

H Butter, cookies, sugar, etc.

The equations that follow for pages 94 and 95 are those the child would use in changing the measures to other units. The answer is bracketed in each equation. Use your own judgment as to whether or not the child should be required to make an equation for each exercise.

Page 94:

A $2 \times 8 = [16]$

B $8 \div 4 = [2]$

C $48 \times 7 = [336]$

D $72 \div 8 = [9]$

E $5 \times 36 = [180]$

F $96 \div 32 = [3]$

G $2 \times 32 = [64]$

H $27 \times 3 = [81]$

I $42 \div 7 = [6]$

J $128 \div 16 = [8]$

Page 95:

(block 1)

A $72 \div 3 = [24]$

B $64 \div 8 = [8]$

C $19 \times 2 = [38]$

D $180 \times 60 = [10{,}800]$

E $763 \div 7 = [109]$

F $27 \times 12 = [324]$

G $42 \times 36 = [1512]$

H $28 \div 2 = [14]$

I $9 \times 60 = [540]$

J $6 \times 2000 = [12{,}000]$

K $4 \times 5280 = [21{,}120]$

L $192 \div 16 = [12]$

M $22 \times 32 = [704]$

N $17 \times 8 = [136]$

O $60 \div 4 = [15]$

(block 2)

A $84 \div 2 = [42]$

B $96 \div 4 = [24]$

C $39 \times 2 = [78]$

D $600 \div 8 = [75]$

E $34 \times 3 = [102]$

F $36 \div 2 = [18]$

G $156 \div 12 = [13]$

H $72 \div 12 = [6]$

I $4 \times 2000 = [8000]$

J $14 \times 52 = [728]$

K $120 \div 24 = [5]$

L $65 \times 16 = [1040]$

M $8 \times 8 = [64]$

N $13 \times 8 = [104]$

O $52 \times 4 = [208]$

(block 3)

A $135 \div 3 = [45]$

B $47 \times 7 = [329]$

C $58 \div 2 = [29]$

D $1440 \div 60 = [24]$

E $40 \div 8 = [5]$

F $24 \times 12 = [288]$

G $7 \times 24 = [168]$

H $192 \div 32 = [6]$

I $10 \times 5280 = [52{,}800]$

J $144 \div 12 = [12]$

K $32 \div 4 = [8]$

L $420 \div 60 = [7]$

M $15 \times 12 = [180]$

N $144 \div 36 = [4]$

O $12 \times 8 = [96]$

Keeping skillful:

(block 1)	(block 2)	(block 3)
A 875	A 168	A $104.98
B 2574	B 44	B $4.16
C 2696	C 31	C $2.61
D 5208	D 704 and 2 rm.	D $1.33
E 1615	E 899 and 3 rm.	E $13.57
F 1950	F 19 and 11 rm.	F $161.84
G 16,107	G 58 and 17 rm.	G $72.60
H 23,552	H 202 and 8 rm.	H 60
I 5448	I 45 and 86 rm.	I $10.67
J 64,080	J 83 and 7 rm.	J $752.76
K 87,032	K 622	K $4.31
L 90,090	L 914 and 26 rm	L 23
M 97,240	M 205 and 40 rm.	M $2.25

Looking back

This lesson will help you to check up on what you know about measures.

92 Reteaching of measurement equivalents and changing of measures

liquid measures
2 tablespoonfuls = 1 fluid ounce
8 fluid ounces = 1 cup
2 cups = 1 pint
2 pints = 1 quart
4 quarts = 1 gallon
8 pints = 1 gallon

temperature
Water freezes at 32° above 0.
Water boils at 212° above 0.

dry measures
2 dry pints = 1 dry quart
8 dry quarts = 1 peck
4 pecks = 1 bushel
32 dry quarts = 1 bushel

counting
12 things = 1 dozen

time
60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day

calendar
7 days = 1 week
52 weeks = 1 year
12 months = 1 year
365 days = 1 year

length
12 inches = 1 foot
36 inches = 1 yard
3 feet = 1 yard
5280 feet = 1 mile
1760 yards=1 mile

weight
16 ounces = 1 pound
2000 pounds = 1 ton

bushel—bu.
cup—c.
day—da.
dozen—doz.
fluid ounce—fl. oz.
foot—ft.
gallon—gal.
hour—hr.
inch—in.
mile—mi.
minute—min.
month—mo.
ounce—oz.
peck—pk.
pint—pt.
pound—lb.
quart—qt.
second—sec.
tablespoonful—tbs.
ton—T.
week—wk.
yard—yd.
year—yr.

A Would you use the table of liquid measures or the table of dry measures to find how many cups are in a quart?

B What can you think of that is sold by the pint, quart, and gallon?

C What objects do you see in your schoolroom that could be measured with a foot ruler?

D Which table of measures would you use to find how many pecks there are in a bushel?

E What is the lowest temperature that can be measured with each of the thermometers in Picture D?

F Should a truck and a load of rocks that together weigh 6583 lb. go over a bridge that has a load limit of 4 tons?

G Which table above tells the number of things in a dozen? What things have you bought by the dozen?

H Name some foods that are bought by the pound.

92

1 For Picture A, ask the children questions like these: "What do you see in this picture? What do they measure? What measurement ideas do they represent? [7 da.=1 wk., 365 da. = 1 yr., 12 mo.=1 yr., 60 sec.=1 min., 60 min.=1 hr.] How many hours are there in a day?"

2 Adapt Note 1.

3 Ask, "What do you see in this picture? What measuring instrument illustrated above on this page is commonly used to measure what is shown here?"

4 Ask, "What reading on a thermometer does ice water suggest to you? What reading does boiling water suggest?"

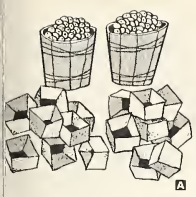
5 Discuss this picture with the children and bring out that it suggests measuring time and distance. Ask them which of the instruments in Pictures A to G can be used in measuring time and distance. Discuss, also, how the sun is used to measure time.

93

1 Do not ask the children to memorize these tables. Instead, let them do some actual measuring in which they use these equivalents. Have as many measuring instruments as possible on hand, so that they can verify the equivalents. (They might pour 4 cups of water into a quart jar.) Conduct experiments showing that water freezes at 32° Fahrenheit (using melting ice cubes in a bowl) and that water boils at 212°. Also let the children make posters illustrating equivalents. Have them show mathematically that if 5280 ft.=1 mi., 1760 yd.=1 mi.

2 After the children study these abbreviations, have them cover the list of abbreviations and copy the tables at the left, using abbreviations.

3 Have the children answer and discuss these questions, using the tables above whenever they need to.



A

B

A A farmer sold 2 pecks of small tomatoes. This was equal to how many dry quarts? **1**

THINK Look at Picture A. Each peck is equal to a group of 8 quarts. 2 pecks are equal to 2 groups of 8 quarts. $2 \times 8 = n$. The 2 pecks of tomatoes were equal to ▩ quarts of tomatoes.

B Another farmer sold 8 pecks of potatoes. This was equal to how many bushels? **2**

THINK Look at Picture B. Each group of 4 pecks is equal to 1 bushel. 8 pecks are equal to ▩ groups of 4 pecks. $8 \div 4 = n$. The 8 pecks of potatoes were equal to ▩ bushels.

C 48 weeks are equal to how many days?

THINK Each week is equal to a group of ▩ days. 48 weeks are equal to 48 groups of 7 days. $48 \times 7 = n$.

D 72 pints are equal to how many gallons?

THINK Each group of 8 pints is equal to ▩ gallon. 72 pints are equal to ▩ groups of 8 pints. $72 \div 8 = n$.

E 5 yards are equal to how many inches?

THINK Each yard is equal to a group of ▩ inches. 5 yards are equal to ▩ groups of 36 inches. $5 \times 36 = n$.

F 96 dry quarts are equal to how many bushels?

THINK Each group of 32 quarts is equal to ▩ bushel. 96 quarts are equal to ▩ groups of 32 quarts. $96 \div 32 = n$.

G 2 bushels are equal to ▩ quarts.

H 27 yards are equal to ▩ feet **4**

I 42 days are equal to ▩ weeks.

J 128 ounces are equal to ▩ pounds.

A	72 ft. = ▩ yd.	**A**	84 pt. = ▩ qt.	**A**	135 ft. = ▩ yd. **1**
B	64 qt. = ▩ pk. **1**	**B**	96 pk. = ▩ bu. **1**	**B**	47 wk. = ▩ da.
C	19 pt. = ▩ c.	**C**	39 qt. = ▩ pt.	**C**	58 tbs. = ▩ fl. oz.
D	180 min. = ▩ sec.	**D**	600 pt. = ▩ gal.	**D**	1440 sec. = ▩ min.
E	763 da. = ▩ wk.	**E**	34 yd. = ▩ ft.	**E**	40 qt. = ▩ pk.
F	27 doz. = ▩ things	**F**	36 c. = ▩ pt.	**F**	24 ft. = ▩ in.
G	42 yd. = ▩ in.	**G**	156 in. = ▩ ft.	**G**	7 da. = ▩ hr.
H	28 tbs. = ▩ fl. oz.	**H**	72 mo. = ▩ yr.	**H**	192 qt. = ▩ bu.
I	9 hr. = ▩ min.	**I**	4 T. = ▩ lb.	**I**	10 mi. = ▩ ft.
J	6 T. = ▩ lb.	**J**	14 yr. = ▩ wk.	**J**	144 things = ▩ doz.
K	4 mi. = ▩ ft.	**K**	120 hr. = ▩ da.	**K**	32 pk. = ▩ bu.
L	192 oz. = ▩ lb.	**L**	65 lb. = ▩ oz.	**L**	420 min. = ▩ hr.
M	22 bu. = ▩ qt.	**M**	8 c. = ▩ fl. oz.	**M**	15 yr. = ▩ mo.
N	17 c. = ▩ fl. oz.	**N**	13 gal. = ▩ pt.	**N**	144 in. = ▩ yd.
O	60 qt. = ▩ gal.	**O**	52 bu. = ▩ pk.	**O**	12 c. = ▩ fl. oz.

Now you should know how to change measures to larger or smaller measures of the same kind.

Keeping skillful

A	$125 \times 7 = n$	**A**	$672 \div 4 = n$ **3**
B	$6 \times 429 = n$	**B**	$396 \div 9 = n$
C	$8 \times 337 = n$	**C**	$186 \div 6 = n$
D	$84 \times 62 = n$	**D**	$2114 \div 3 = n$
E	$17 \times 95 = n$	**E**	$7195 \div 8 = n$
F	$15 \times 130 = n$	**F**	$258 \div 13 = n$
G	$413 \times 39 = n$	**G**	$3729 \div 64 = n$
H	$64 \times 368 = n$	**H**	$5866 \div 29 = n$
I	$24 \times 227 = n$	**I**	$4001 \div 87 = n$
J	$80 \times 801 = n$	**J**	$2995 \div 36 = n$
K	$184 \times 473 = n$	**K**	$34210 \div 55 = n$
L	$234 \times 385 = n$	**L**	$65834 \div 72 = n$
M	$143 \times 680 = n$	**M**	$9060 \div 44 = n$

A	$29 \times \$3.62 = n$ **4**
B	$\$5.05 - \$.89 = n$
C	$73¢ + 46¢ + 62¢ + 80¢ = n$
D	$\$9.31 \div n = 7$
E	$\$20 - \$6.43 = n$
F	$56 \times \$2.89 = n$
G	$132 \times \$.55 = n$
H	$\$300 \div \$5 = n$
I	$\$4.03 + \$2.66 + \$3.98 = n$
J	$164 \times \$4.59 = n$
K	$\$14.08 - \$9.77 = n$
L	$\$3.22 \div \$.14 = n$
M	$\$83.25 \div n = 37$

94

1. Tell the children to refer to Picture A as they read this problem and the explanatory material. Be sure they understand that 8 quart boxes are equal to 1 peck basket. Ask how the picture shows that 2 pecks are equal to 2 groups of 8 quarts. Ask how many 2 groups of 8 are. Help the class determine that this is a multiplicative situation.
2. Have pupils refer to Picture B as they read Problem B and the text labeled "Think." Be sure they understand that 8 is divided by 4 because there are 8 pecks in all and 1 group of 4 pecks is equal to 1 bushel.
3. Discuss Problems C to F, adapting the procedures of Note 1 to Problems C and E, and Note 2 to Problems D and F.
4. Assign Problems G to J as written work.

95

1. Assign these exercises as written work. Ask the children to write an equation for each, using the letter *n* to hold a place for the answer. When they have found the answer, they should rewrite the equation and insert the answer. You might test the ability of the children to read abbreviations by having them read the exercises aloud as they verify their answers. If a child can do these exercises directly without writing equations, permit him to do so.
2. Assign these exercises on multiplication as written work.
3. When the children have finished the multiplication, assign these division exercises as written work.
4. This block of exercises, which involves the four processes and numerals representing money, may also be used as written work. Warn the children that sometimes the answer will not be written as dollars and cents.

96-100 Exploring problems

Expanded Notes for this lesson ore on poges 290-292.

Objectives

The child learns to solve problems that require two or more individual computations (steps) to get the final answer. He learns to make a basic or "master" equation that describes a problem situation in which more than one number is to be found.

Vocabulary

New words page 96 hooks*; page 99 several*; page 100 mail, gathered*, dinner*, glasses*, supplies*, talked*

Comments

Children need special training in learning how to handle problems in which needed facts are not readily apparent. They must learn how to put together and use such "hidden" facts to find the answer. This they can do by building on to what they have already learned about problem solving.

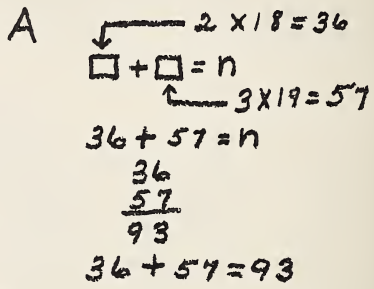
The child should be taught to think somewhat as follows: "What is the main thing I am going to find out in this problem? Are there problems I need to solve before I can get the final answer?"

In *Seeing Through Arithmetic 5,* the child is taught to make a master equation that describes the problem. The making of one master equation causes him to think about the total setting. It helps him to see the problem situation as a whole and is in line with the approach used throughout this program. This approach to problem solving is in direct contrast to traditional methods wherein the child is taught to look at the "parts" and then work toward the whole, frequently without understanding what he is doing.

In problems of two or more steps, as well as in one-step problems, *n* is used in the equation to hold a place for the principal unknown number, or final answer. Screens (suggesting the hiding of numerals) are used to hold a place for the missing numerals that can be found through study of the problem.

The master equation for a problem of two or more steps can indicate any of the four processes and can have many forms: ▦+▦=n, 21−▦=n, 3×▦=n, 8÷▦=n, etc. The first equation above, for example, is for a three-step problem and is used in solving Problem A on page 96. The master equation must be an additive one: ▦+▦=n. The two missing numbers that must be added to get the final answer can be found by multiplication: 2×10, for Tony's hooks and 2×12 for Jim's. When these numbers have been determined, the numerals can be inserted in the master equation, and the answer can then be found by addition.

When you ask the children to solve the problems provided for written work on page 100, you should instruct them to make the master equation for each problem. If they need to compute to get the missing numerals that will replace the screens, have them show this work, as well as the computation for the final numeral that replaces the letter *n*. This will force the children to think before they compute, and seeing the computation will help you locate their difficulties. A suggested way for pupils to show their work is given in the sample below.



Answers

Pages 99-100:

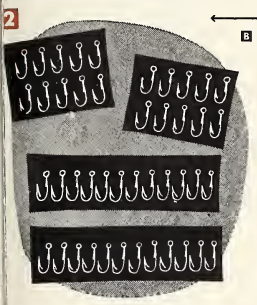
In this answer key the numeral that replaces *n* in the equation is bracketed. The numeral that replaces a screen in the equation is enclosed in parentheses. The child will not use parentheses and brackets. They are merely used to keep things clear in this answer key.

A 3×(80)=[240]
B $4.20÷($.60)=[7]
C (36)+20=[56] or 20+(36)=[56]
D $.88÷($.22)=[4]
E 3×(5)=[15]
F (26)−9=[17]
G 3×(8)=[24]
H (15)×$.02=[$.30]
I (42)−20=[22]
J $10.00−($9.78)=[$.22]
K $.45+($.20)=[$.65]

see

A

1 Tony bought 2 cards of fish hooks with 10 hooks on each card. Jim bought 2 cards of fish hooks with 12 hooks on each card. How many fish hooks did the boys buy in all?



2

B

Think of the hooks Tony bought and the hooks Jim bought as one group.

Total number of hooks Tony bought

Total number of hooks Jim bought

■ + ■ = n

3

Number of hooks the boys bought in all

First you must find the number of hooks Tony bought and also the number of hooks Jim bought.

compute

C

1 Number of cards Tony bought

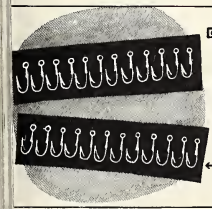
Number of hooks on each card

Number of hooks Tony bought in all

2×10=■

You must find this number.

Think of Tony's hooks as one group.
Multiply 10 by 2. Tony bought 20 fish hooks.



D

2 Number of cards Jim bought

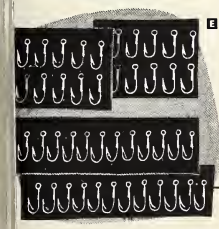
Number of hooks on each card

Number Jim bought in all

2×12=■

You must find this number.

Think of Jim's hooks as one group.
Multiply 12 by 2. Jim bought 24 fish hooks.



E

3 Number of hooks Tony bought

Number of hooks Jim bought

Number they bought in all.

20+24=n

You must find this number.

Think of the fish hooks as one group.
Add 24 to 20.
The boys bought 44 fish hooks in all.

96

1 Have the pupils read the problem and then look at Picture A. Some children will see immediately how many fish hooks each boy bought, but explain that problems are not always so easy and that here we are studying an approach that will work for harder problems.

2 Have the children look at Picture B. Ask how we could find out how many fish hooks the two boys bought if we knew how many each boy bought.

3 Call the pupils' attention to the equation. Explain that the red screen stands for the total number of hooks Tony bought, the blue screen for the total number of hooks Jim bought, and *n* for the total number of hooks both boys bought. Ask how we know that the total is to be found by addition. Then have the children read the text opposite Picture B and discuss why they must first find the number of hooks each boy bought.

97

1 Help the pupils identify the cards of fish hooks in Picture C as those Tony bought. The dotted background should suggest combining. Make sure the pupils see that the two groups of hooks are equal and that multiplication is the correct process to use. Have them study the equation and multiply.

2 Get the children to identify the hooks in Picture D as those Jim bought. Again they must see that 2 cards are to be combined and that, since there are 2 equal groups of 12, multiplication should be used. Have them study the equation and multiply.

3 Have the pupils compare the hooks in Picture E with those in Picture B. Also have them look at the equation opposite Picture B. Ask them if they know what numerals should be where the screens are. Call attention to the equation in E, which shows these numerals inserted for the screens. Then tell pupils to add to find the answer.

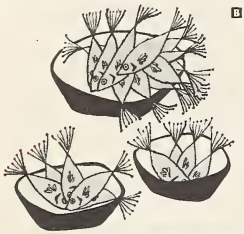
see

A

Together, Tony and Jim caught 21 fish. Tony sold 5 of the fish to his friends, and Jim sold 4 of the fish to his friends. How many fish did the boys have left then? 1

Number of fish the boys caught
Number of fish sold
Number they had left

$21 - \square = n$ 2



B

First you must find the number of fish the boys sold in all.

3

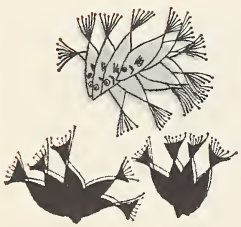
compute 4

C

Number of fish Tony sold
Number of fish Jim sold
Number they sold in all

$5 + 4 = \square$ You must find this number.

Think of the fish Tony and Jim sold as one group.
Add 4 to 5. The boys sold 9 fish.



D

1

Number of fish the boys caught
Number of fish they sold in all
Number of fish they had left

$21 - 9 = n$ You must find this number.

Imagine that the 9 fish are gone.
Subtract 9 from 21.
The boys had ■ fish left.

A Tony fished for 35 minutes in the morning and 45 minutes in the afternoon. He did this every day for 3 days. For how many minutes in all did he fish during the 3 days? 2

Number of days.
Total number of minutes he fished each day.
Number of minutes he fished in all.

3

$3 \times \square = n$

First find the number of minutes Tony fished each day. To do this, add 45 to 35. Put 80 in the equation.
$3 \times 80 = n$
Multiply 80 by 3.
Tony fished ■ minutes in all during the 3 days.

4

B Before the fishing trip Jim earned 40¢ each morning and 20¢ each afternoon for several days. He earned $4.20 in all. For how many days did he work to earn this money? 5

Amount Jim earned in all.
Total amount he earned each day.
Number of days he worked.

6

$\$4.20 \div \square = n$

First find the total amount Jim earned each day. To do this, add $.40 to $.20. Put $.60 in the equation.
$\$4.20 \div \$.60 = n$
Divide $4.20 by $.60.
Jim worked for ■ days to earn the money.

98

1. Let the pupils read Problem A and determine what the central action of the problem is [separating]. Help them see that Picture A shows the 21 fish and the two empty pans into which are to be put the two groups of fish the boys sold.
2. Discuss how this equation shows the main action for this problem.
3. Make sure the children realize that they must know the total number of fish Tony and Jim sold in order to find how many were left. Ask them if combining the number of fish in the two groups the boys sold will give the total number sold.
4. They should see that the dotted background in this picture suggests combining and that the new equation shows the combining of the group of 4 fish with the group of 5 fish. Have pupils read the text.

99

1. Get the pupils to see that the numeral that stands for the total number of fish the boys sold has now been inserted in the first equation. Have them subtract to find how many fish the boys had left.
2. Ask the children to read this problem. Help them see that the master equation should be a multiplication equation.
3. Let the pupils study the equation, tell what part of the problem the screen represents, and tell how to find the number it stands for.
4. Have the pupils take turns reading these sentences. Then have them follow the directions and do the work to find the answer.
5. Have the children read this problem, and help them to determine that they should divide in the master equation.
6. Ask the pupils to explain each part of the equation, read the text below, and do the work to find the missing numbers.

1

c Jim's mother baked 3 doz. sugar cookies and 20 chocolate cookies for the fishing trip. How many cookies did she bake in all?
THINK ▢ + 20 = n
First find how many sugar cookies she baked. 3 × 12 = ▢. What number do you put in the equation?

D One day in town Jim paid 88¢ for some dishes of ice cream with chocolate syrup. The ice cream cost 15¢ a dish. The chocolate syrup cost 7¢ a dish. How many dishes of ice cream with chocolate syrup did Jim pay for?
THINK $.88 ÷ ▢ = n
First find how much each dish of ice cream with chocolate syrup cost. Add 7¢ to 15¢. Where do you put this sum?

E Tony rode his bicycle to town every day for the mail. He rode 2 mi. into town and 3 mi. coming back by a different road. In 3 days, he rode how many miles getting the mail?
3 × ▢ = n

F Jim gathered colored stones for his collection. He gathered 10 stones one day and 16 the next day. Then he gave 9 of these stones to Tony. How many stones did he have left?
▢ − 9 = n

G Every day at dinner each of the 4 persons who went on the fishing trip drank 2 glasses of milk. How many glasses of milk did they drink at dinner during the 3 days?
3 × ▢ = n

H Tony mailed 6 postcards, and Jim mailed 9. Each postcard had a 2¢ stamp on it. How much did the stamps on all these cards cost?
▢ × $.02 = n

I One evening Jim dug 23 worms, and Tony dug 19 worms. They used 20 of these worms the next day. How many worms did they have left?
▢ − 20 = n

J In town one day Jim's father bought groceries for $3.66 and fishing supplies for $6.12. He gave the clerk a ten-dollar bill. How much money should the clerk have given him in change?
$10 − ▢ = n

K He also made a long-distance call. He talked for 5 minutes. The first 3 minutes cost 45¢. The next 2 minutes cost 10¢ a minute. How much did it cost him to talk for 5 minutes?
$.45 + ▢ = n

2

In each of these problems you had to do more than one thing to find the answer to the question in the problem.

100

1 Assign these problems as written work. Tell the children that a master equation is provided for each problem and that they should study each such equation to see how it fits the problem. Ask them to find the numeral to replace the screen, and then to do the necessary work to find the final answer (*n*).

2 When all the children have finished this set of problems, discuss each equation to make sure they see how it applies to the problem. Have the pupils tell what part of the problem each screen stands for and how they found the numeral they inserted. Then assign each problem to a child and have him do the final computation at the chalkboard.

101-104 Exploring problems

Expanded Notes for this lesson are on pages 292-294.

Objectives

The child reviews the meaning of average. He learns to make a "master" equation for each problem involving average.

Vocabulary

New words page 101 loaned; page 102 deliver*; page 104 growth, Thursday*

Comments

In the past, average has been taught as involving two computational procedures, or steps. Children have memorized a term and a rule. In this book, however, children are taught what an average is, and they learn to find it just as they solve any other multiple-step problem.

In Problem A on page 101 the pupils must see that they are to find how much money there would be in each group of coins if each boy had loaned the same amount. This situation can then be linked to the partitive division situation in which the number of equal groups is known but the size of the equal groups is not known. Help the pupils recall the equation for this type of problem, and lead them to make a similar basic or master equation for this problem involving average (▤ ÷ n = 4). Their work in the previous lesson should show them how to proceed from this point.

It is important for the children to realize that no one group or thing need be the same size or amount or length as the average. The children must also realize that sometimes the total amount is already given, and the problem is a simple partitive division problem.

Answers

Page 104:

A (21) ÷ [7] = 3
B 33 ÷ [3] = 11
C (60) ÷ [15] = 4
D (124) ÷ [31] = 4
E $1.62 ÷ [$.27] = 6
F 192 ÷ [16] = 12
G $14.20 ÷ [$3.55] = 4
H (250) ÷ [50] = 5
I (330) ÷ [66] = 5

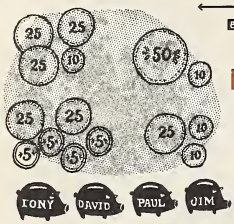
see

A

Four boys started a club. To start the club, each boy loaned money he had saved. Tony loaned $.85, David loaned $.60, Paul loaned $.45, and Jim loaned $.70. Paul said, "Suppose that each of us had loaned the same amount to get that much money. How much would each of us have loaned then?"

1

B



Think of all the money as one amount. Then imagine that the money is to be divided into four equal parts, one for each boy.

2

Total amount of money

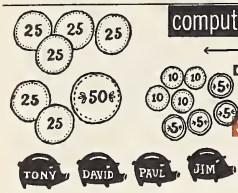
Amount in each part

$\blacksquare \div n = 4$

Number of parts

First you must find the total amount of money the boys loaned.

compute



Find how much money all the boys loaned.

C

$85¢ + 60¢ + 45¢ + 70¢ = \blacksquare$

The sum is 260¢, or $2.60.

3

Put $2.60 in the equation.

$2.60 \div n = 4$

Now turn the page.

D

1

Total amount loaned

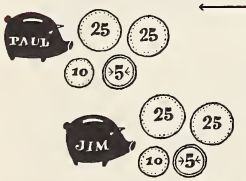
Amount from each boy

Number of boys

$\$2.60 \div n = 4$

Now you must find this number.

Think of the $2.60 as divided into 4 equal parts. How much will there be in each part?



Divide $2.60 by 4.

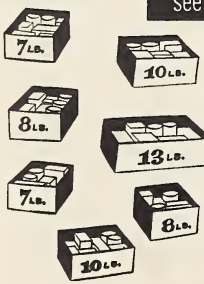
$2.60 divided by 4 equals $.65.

The amount each boy would have loaned is the average amount.

$.65 is the average.

The boys loaned an average of $.65.

see



A

2

To earn money for the club, Paul helped to deliver groceries. The boxes of groceries that he delivered weighed 7 lb., 10 lb., 8 lb., 13 lb., 7 lb., 8 lb., and 10 lb. What was the average weight of the boxes?

101

1 Have the pupils read Problem A and ask them to find in Picture A the group of coins each boy loaned. Point out that if each boy had loaned the same amount of money, each of the groups would have the same value.

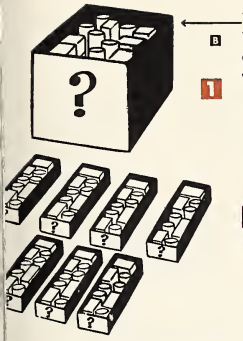
2 Ask the pupils to read the statements beside Picture B. Remind them that the dotted background in the picture should help them think of the coins as one group and that the screen in the equation represents the total amount of money in this group. Point out that next they are to imagine that the money is to be divided into 4 *equal* groups. Then ask the pupils to tell why the letter *n* is used to represent the size of the equal groups and why it is placed after the division sign in the equation.

3 Direct attention to the material labeled C. The pupils should see that the total amount of money is found by combining the groups, and that the total is then put in the master equation in place of the screen.

102

1 Have the pupils study the equation and be ready to explain each part. Let them divide $2.60 by 4 and find the answer. When they have found it, have someone count the coins in each of the rearranged groups in Picture D to see that each group is the same amount as the amount found by division.

2 Call on a pupil to read this problem and relate the picture to it. Get the children to bring out in discussion that finding the average weight means finding what each box of groceries would weigh if each weighed the same. Call on a pupil to tell how many boxes of groceries Paul delivered.



Think of all the groceries in one big box. Then imagine that they have been divided equally among 7 boxes so that each box weighs the same.

B

1

What does this stand for?

What does this stand for?

What does this number stand for?

■ ÷ n = 7

First you must find this number.

compute

C

What do these numbers stand for?

7 + 10 + 8 + 13 + 7 + 8 + 10 = ■

2

What does this stand for?

Why is 63 put here?

63 ÷ n = 7

Divide 63 by 7.

If each box had weighed the same, its weight would have been ■ lb.

The average weight of the boxes was ■ lb.

103

One day the four boys in the club
de for 3 hours on their bicycles.
ey rode 8 mi. the first hour, 7 mi.
e second hour, and 6 mi. the third
ur. What was the average distance
ey rode an hour?

1

Total number of miles they rode.

Average miles an hour.

Number of hours.

■ ÷ n = 3

st find the total number of miles
e boys rode. To do this, add 8, 7,
d 6. Then put the sum in the equation.
÷ n = 3
w divide 21 by ■.
e average distance they rode was
mi. an hour. The average distance
ey rode was ■ mi. per hour.

David said, "I have grown 33 inches
11 years." David has grown an
erage of how many inches a year?

2

Total growth in 11 years.

Average growth per year.

■ ÷ n = 11

Number of years.

netimes the problem tells you
total, and you do not have to find it.

C The four boys in the club went fishing one day. Each boy caught one fish. These fish were 11 in., 18 in., 14 in., and 17 in. long. What was the average length?

3

D These fish weighed 23 oz., 36 oz., 32 oz., and 33 oz. Find the average weight of the fish.

E David earned $1.62 for the club in 6 weeks. He earned an average of how much per week?

F The boys bought a bird book with 192 pages in it. Jim said that he was going to read it in 12 days. To do this, he would have to read an average of how many pages per day?

G In four months the boys earned $14.20 for their club. What were their average earnings per month?

H The boys worked on a clubhouse after school for 5 days. They worked 65 min. on Monday, 45 min. on Tuesday, 45 min. on Wednesday, 35 min. on Thursday, and 60 min. on Friday. They worked an average of how many minutes per day?

I Some old boards the boys used for their clubhouse were 60 in., 72 in., 64 in., 76 in., and 58 in. long. What was the average length?

Now you should understand how to find an average.

103

1 Help the pupils see that in order to find the average weight of the boxes of groceries, they must first think of all the groceries combined. Then the total weight can be redistributed into 7 boxes, or groups, so that each box weighs the same. The pupils should see that this also is a division situation in which the size of equal groups is to be found. Have them answer the questions that refer to the parts of the equation.

2 Have the children find the total weight of the original boxes of groceries. Be sure they can explain why 63 is put in place of the screen in the master equation. Ask them to divide and find the numeral that replaces the letter *n*. Then have the sentences at the bottom of the page read using the known numerals. Ask the children to relate this answer to Picture C.

104

1 Have the pupils read the problem and explain why the equation shown in the book is correct. Then let them add to find the total number of miles the four boys rode. Call on someone to read the last four lines and supply the missing numerals.

2 After Problem B has been read, let a pupil explain why the equation given in the book is correct. Make sure he sees why a screen is not needed. [The total number of inches is already known.] Then ask another pupil what should be done to find the numeral that replaces *n* [divide 33 by 11].

3 Assign Problems C to I as written work. Tell the pupils to write the master equation for each problem first, then do the necessary computation to find the missing numbers. Remind the children that when the total is given in a problem of this kind, a screen is not needed and they can divide immediately.

105-106 Thinking straight

Expanded Notes for this lesson are on page 294.

Objectives

The child develops judgment about the use of remainders in problems involving division.

Vocabulary

New words page 105 eggs*, statements*; page 106 deciding*, market*, though*, done*, sending*

Comments

Dealing with remainders in division has always been difficult for children. The problems in this lesson and the questions about them are designed to help the children come to conclusions about what to do with answers in division when there is a remainder.

Complications arise in considering the use of the remainder. Should the quotient be increased by 1, or should the remainder be ignored? Later, the child will have another decision to make. Should he use the remainder to make a fraction, so that his answer becomes a mixed number?

Ability to handle remainders grows slowly. At this stage in his arithmetic study the child is not ready to use a remainder to make a fraction. He will learn this after he has had more work with fractions.

The important thing to bring out in this lesson is that there is no rule to follow as to what to do about remainders. Some of the problems in this lesson could have several "right" answers. Discussion should bring out this point. What is important is that the child be able to give an acceptable reason for including or not including the remainder in the answer. The use to be made of the answer is, of course, the determining factor. As you discuss these exercises, the children should begin to see what kind of answer is required for a specific type of problem situation.

Answers

A Divide by 12; Statement 5 is the best answer.

B Keep the remainder as part of the answer.

C 14 cartons; The answer is different from the answer to Problem A because we are not interested in the eggs left over.

D The remainder is left unused.

E Divide by 12; 9 dozen eggs and 1 egg left over; keep it; This problem is different from Problem A in that the remainder is much smaller.

F 9 dozen; No, the remainder is ignored. Ignore the remainder because George is selling dozens of eggs, not single eggs. In Problem C there was almost another full dozen.

G It is important that the remainder be included in the answers to Problems A and E because we do not know how the answers are to be used. It is not important to include the remainder in the answers to Problems C and F because the problems ask us only how many *cartons* of eggs, or how many dozens, George will have to sell.

H $1.75÷[$.58]=3 and 1¢ rm.
Two boys should pay $.58 each and one should pay $.59. No; no; If each boy paid only $.58, there would be one more penny to be paid. If each paid $.59, too much would be paid. The remainder is important because it is money that someone must pay. Yes; 1¢ more

I 325÷[40]=8 and 5 rm.
41; 41; 5; 3; There are more rooms with 41 children in them.

J 46¢÷[15¢]=3 and 1¢ rm.
16¢; to offer a bargain so that people will buy more

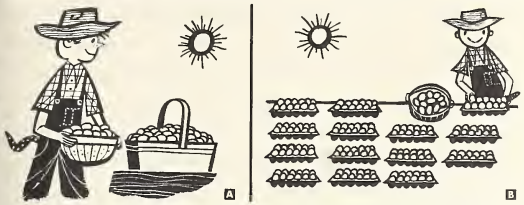
K 280÷25=[11] and 5 rm.
12; 5; Yes, because all of the sheep must be shipped, and 11 trucks are not enough.

L 297÷[42]=7 and 3 rm.
42; yes; yes; The remainder is not important because it is small.

M $5.00÷[$1.66]=3 and 2¢ rm.
No; $1.66; Buy something for 2¢ that they could share or decide that two of the children would get an extra cent apiece.

Thinking straight

This lesson should help you understand how to use remainders in the answers to division problems.



A

B

A In Picture A, George has 179 eggs. What would you do to find how many dozen eggs he has? Which of these statements do you think is the best to use to tell what your answer is?

1 George has 14 dozen eggs. **1**
2 George has 15 dozen eggs.
3 George has about 14 dozen eggs.
4 George has about 15 dozen eggs.
5 George has 14 dozen eggs and 11 eggs left over.

B What do you think is the best thing to do with the remainder in Problem A?

1 Keep the remainder as part of the answer. **2**
2 Do not use the remainder.
3 Use the remainder to make the answer larger.

C In Picture B, George is going to sell his eggs. How many cartons of 1 doz. eggs will he have for sale?

1 The answer is the same as the answer to Problem A. **3**
2 The answer is different from the answer to Problem A.

D What happens to the remainder in Problem C? **4**

1 The remainder is to be left unused.
2 The remainder is used in getting the answer.

E Another day George had 109 eggs. How do you find how many dozen eggs he had? What do you think is the best answer for this problem? What do you do with the remainder? How is this problem different from Problem A? **5**

1

F Suppose George were to sell he eggs he had in Problem E. How nany dozen eggs could he sell? Is your nswer the same as the answer you ave to Problem E? What do you do with he remainder? How is this problem ifferent from Problem C on page 105?

G The size of the remainder is mportant in the answers to Problems A and E, but it is not important in the nswers to Problems C and F. Why?

H George, Tom, and Bill bought baseball that cost $1.75. The boys anted to share the cost equally. How uch should each boy pay? Should ach of them pay 58¢? Should each f them pay 59¢? What is wrong with ach of these answers? How is he remainder important in this roblem? Will one of the boys have o pay more than his share? How much ore? **2**

I There are 325 pupils at Lake School. here are 8 classrooms in the school. /hat is the average number of children er room? Do you think the best nswer is 40 or 41 children per room? ow many rooms could have 41 ildren? How many other rooms could ave 40 children? How do the answers these last two questions help you deciding upon the best answer bout the average number per room? **4**

J A grocer is selling cans of orange juice at 3 cans for 46¢. If you bought 1 can of this orange juice, do you think you would have to pay 15¢ or 16¢ for it? If you think you would have to pay 16¢ for one can, why do you think the grocer sells 3 cans for 46¢ instead of for 48¢?

K Some farmers in Castletown were sending 280 sheep to market in trucks. They could put only 25 sheep in each truck. Did they need 11 or 12 trucks? How large is the remainder in this problem? Even though the remainder is small, do you think you should still use 12 trucks for your answer? Why or why not?

L Jim has a 297-page library book to read in 7 days. If he wants to read about the same number of pages per day, about how many pages should he read in one day? Could Jim read more pages on one day than another? Is an approximate answer good enough for this problem? If so, do you think the remainder is important? **5**

M Mr. Sands gave Kathy, Tony, and Ann $5.00 for helping him in his garden. Could the children share the money equally? How much would each child get? What do you think the children should have done with the 2 cents that would be left over? **6**

105

1 Point out that the problem does not say what use is to be made of the answer, so Statement 1 or 4 or 5 could be considered correct for the answer, but Statement 5 is best.

2 Tell the pupils that the first statement gives the best answer, but that the other two statements might be adequate, depending on the use to be made of the answer.

3 The children should see that the important thing in this problem is to find the number of cartons that can be filled with the eggs George has for sale.

4 Discuss with the children the fact that the remainder is not to be included in the answer because George would not normally sell a partly filled carton of eggs.

5 Help the pupils see that this problem is almost like Problem A. The difference is that the remainder in Problem E is 1, while in Problem A the remainder is 11—almost a dozen.

106

1 Get the children to see that all problems must be read very carefully in order to know what kind of answer is desired.

2 The pupils should see that the cost *cannot* be shared equally, and since the full amount must be paid, one boy will have to pay one cent more than the other two boys.

3 Help the children see that when 325 is divided by 8 the answer is 40 pupils and a remainder of 5 pupils. Explain that 5 of the 8 rooms could have 41 pupils and 3 rooms could have 40 pupils. Since more rooms would have 41 children, then, 41 is the better number for the average.

4 Help the children see that the use of the answer determines which one is chosen.

5 The pupils should observe that the size of the remainder determines whether or not it is used to increase the answer by 1.

6 Let the children discuss what could be done with the 2¢ that were left over.

107-109 Moving forward

Expanded Notes for this lesson are on pages 294-296.

Objectives

The child learns how to find the perimeter of rectangles, triangles, and other polygons.

Vocabulary

New words page 107 polygons, straight*, opposite*, perimeter; page 109 diamond*, county*

Comments

Although the children have previously used the terms polygon, rectangle, square, triangle, right angle, length, and width in their experience with geometric figures (pages 3-11), you should review the meanings of these words with them at this time.

In this text the pupils find "distance around" in a number of instances before the word *perimeter* is introduced (Questions C and D, page 107). The new expression, perimeter, will be even clearer if you use figures cut from construction paper and a long piece of string or a tape measure to explain Pictures D, E, and F on page 107. Seeing how perimeters of polygons drawn to actual size are found will help the children understand how to find perimeters of polygons when the polygons are represented by scale drawings, as on pages 108 and 109.

You will want to emphasize several points in the study of perimeter. First provide experiences so that the child sees that the perimeter is always the sum of the numbers representing the lengths of *all* the sides of a polygon, regardless of how many sides the polygon has. When the sides are of different lengths, all of the numbers representing the lengths must be added to find the perimeter. However, the sides of some polygons are of equal length, and then various short cuts may be taken to find the perimeter. For example, the perimeter of a square may be found either by adding the numbers representing the lengths of the four sides or by multiplying the number representing the length of one side by 4. The perimeter of a pentagon with five equal sides, such as Picture H on page 109, may be found by totaling the numbers which represent the lengths of the five sides or by multiplying the number showing the length of one side by 5. The same method will apply to a hexagon, such as Picture C, which has six equal sides. The perimeter of a rectangle may be found by totaling the numbers showing the lengths of the four sides or, since there are two sides of the same length and two of the same width, the perimeter may be found by multiplying the number showing the length by 2 and the number showing the width by 2 and adding the two products or by adding the numbers representing the length and width and multiplying this sum by 2.

Rather than give the children all these facts at once or give them rules—as has often been done in the past—let them discover the various methods themselves through class discussion and careful examination of the illustrations of the polygons in the lesson.

Answers

Page 109:

A [A] 19 yd. [D] 74 ft. [G] 43 yd.
[B] 110 mi. [E] 44 in. [H] 50 ft.
[C] 36 in. [F] 49 mi.

B By adding the lengths of the sides.

C By adding the lengths of the sides or by multiplying the length of one side by 4.

D By multiplying the length by 2 and the width by 2 and adding the two totals, or by adding the lengths of the four sides, or by adding the length and width and multiplying the result by 2.

E 360 ft. G 1400 yd.

F 108 mi. H 1800 ft.

Moving forward

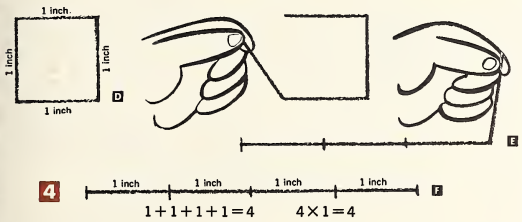
In this lesson you will learn how to find the distance around rectangles, triangles, and other polygons.

Pictures A, B, and C show polygons.
The polygon in Picture A has 4 straight sides. Its opposite sides are equal and also parallel.
It has 4 square corners, or right angles.
This polygon is a rectangle.

2

A. A rectangle with all its sides equal is a square. Which of the three polygons above is a square?

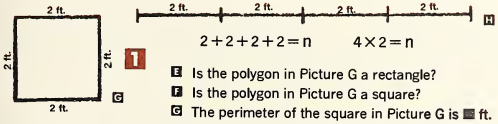
B. The polygon in Picture C has ▪ straight sides. This polygon is a triangle.



E. Pictures E and F show the distance around the square in Picture D. The distance is ▪ in.

5

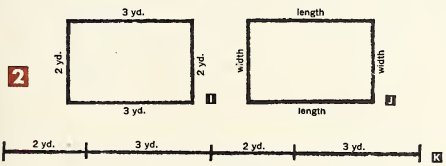
D. The distance around a polygon is the perimeter. The perimeter of the square is ▪ in.



E. Is the polygon in Picture G a rectangle?

F. Is the polygon in Picture G a square?

G. The perimeter of the square in Picture G is ▪ ft.

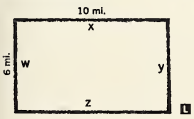


H. Rectangles I and J are the same size. The length of each is ▪ yd.

3

I. The width of each is ▪ yd.

J. The perimeter of each is ▪ yd.



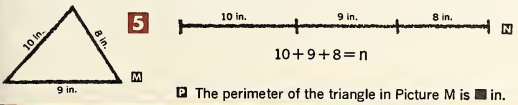
K. Polygon L is a rectangle. Are the opposite sides parallel?

L. Are the opposite sides equal?

M. The side marked z is ▪ mi.

N. The side marked y is ▪ mi.

O. What numbers do you add to find the perimeter?



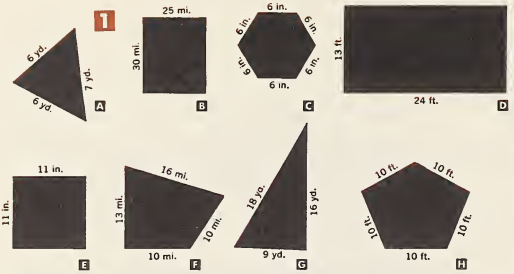
P. The perimeter of the triangle in Picture M is ▪ in.

107

1. Have the children look at Pictures A, B, and C and read the first block of text. Discuss the terms *polygon, parallel,* and *right angle.*
2. Let the children answer Exercises A and B. Discuss the terms *rectangle, square,* and *triangle.*
3. Tell the pupils to note the length of the sides in Picture D. Discuss Picture E. Stress the idea of "straightening out" the square.
4. Point out that Picture F represents the distance around the square in Picture D. Explain that this distance may be found by adding the lengths of the four sides, as shown in the first equation. Have them see that, since the four sides have the same length, the distance around the square can also be found by multiplying the length number of one side by 4, as shown in the second equation.
5. Let someone answer Exercises C and D. Begin using the word *perimeter.*

108

1. Have the children explain what kind of polygon is shown in Picture G and what is shown in Picture H. Then call on someone to answer Exercises E, F, and G.
2. Study Pictures I and J together. Make sure the children see that the length of the rectangle is greater than the width. Have them check to find whether or not the length of the line in Picture K is the same as the distance around Picture I and around Picture J. Let someone read and complete the equation.
3. Ask several children to read Exercises H, I, and J and supply the missing answers.
4. First have the class study Picture L. Then call on pupils to answer Exercises K to O.
5. Ask, "What kind of polygon is shown in Picture M? What does the line in Picture N represent? How many numerals are on the left side of the equation? Why are there three numerals?" Have a child give the perimeter for Picture M.



A Find the perimeter of each polygon shown above.

2 B How can you find the perimeter of any polygon?

3 C What are two ways to find the perimeter of a square?

4 D What are two ways to find the perimeter of a rectangle that is not a square?

THINK One way is to multiply the length by 2 and the width by 2. Then what do you do? What is the other way?

5 E A baseball diamond is a square with each side 90 ft. long. What is the perimeter of a baseball diamond?

F John read that the county in which he lives is in the shape of a rectangle 31 miles long and 23 miles wide. What is the perimeter of the county in which John lives?

G Mr. Banks wants to put a fence around a field that is in the shape of a rectangle. The field is 400 yd. long and 300 yd. wide. How many yards long will the fence be?

H Every night Tony takes his dog for a walk around the block. Tony's father says that the block is a square with each side 450 ft. long. About how far does Tony walk each night when he takes his dog around the block?

Now you should be able to find the perimeter of a polygon if you know the lengths of its sides.

109

109

1 Ask, "To find the perimeter of Picture A, what numbers must you add? What numbers for Picture C? For Picture F? For Picture H?" Have the children write the answers for Exercise A. Then discuss the answers with the whole class.

2 For the answer to Exercise B, have the children state the generalization in their own words.

3 Have the children give two different ways for finding perimeters of squares. Ask the children if any other polygons in Pictures A to H have equal sides. Discuss multiplying the length number of one side of Picture H by 5 and the number representing the length of one side of Picture C by 6.

4 Use Picture D to discuss Exercise D. Be sure the class understands that the two products must be added together.

5 Tell the children to draw free hand a polygon for each of Problems E to H, indicate the lengths of the sides, and find the perimeters. Supply answers for verifying.

110-111 Checking up

Expanded Notes are not considered necessary for this lesson.

Objectives

The child tests his understanding of all problem types taught thus far. He also tests his knowledge of all the computational processes he has learned up to this point. He tests his skill in changing measuring units.

Vocabulary

New words page 110 daisies, pocket*, practicing*, cornet

Comments

On page 110 is an achievement test on problem solving. At least one of each type of problem taught so far is included in this test. (A complete list of problem types is given on pages 388-389 of this *Teaching Guide* and under "Problem solving" in the index of the pupils' book.)

The tests on page 111 are power tests. Give the children sufficient time to enable them to finish the work easily. Use as many of these tests in one day as you think wise, but all of them will probably be too much work for one day.

When the children have completed their work on the tests on pages 110-111, you will be able to see what the individual weaknesses are. Then, by using the Reteaching Chart which follows, you can reteach the problem solving and computational skills that have caused difficulty.

The authors of this book feel that material originally taught in Grade 3 and retaught in Grade 4 should not be further elaborated in a book for Grade 5. The first two inventory tests on page 111, which are on addition that requires carrying and subtraction that requires borrowing, cover materials that fall into this class. There should be very few children who have difficulty with these skills. For these children provide a copy of *Seeing Through Arithmetic 4* for review.

An activity for able pupils that might be used in connection with this lesson is described in the Ex-

panded Notes for page 18, pages 258-260, and in Activity 9, page 360.

Reteaching chart

This chart will help you locate the pages in the pupils' book that may be used for any reteaching that is necessary as revealed by the tests.

Page 110:

Problem	Pages	Problem	Pages
A	25, 68-74	H	21
B	19	I	43-44
C	25, 48-52	J	20
D	101-104	K	24
E	20	L	25, 78-85, 93
F	21	M	20, 61-64, 96-100
G	19	N	40-41, 96-100

Page 111:

Test	Pages
1	Book 4: 20-23, 24-27
2	Book 4: 36-41, 45, 54
3	48-52, 53-59
4	61-64, 77
5	68-74, 91
6	78-84, 91
7	Book 4: 24-27, 36-41 Book 5: 20 (for Ex. A, E, and H) 43-44 (for Ex. B and I) 40-41 (for Ex. C and G) 19 (for Ex. D and F)
8	24 (for Ex. A, D, E, H, and I) 25 (for Ex. B, C, F, and G) 68-74, 78-84, 91
9	92-95

Answers

Page 110:

For purposes of this answer key the numeral that replaces *n* in the equation is bracketed. The numeral that replaces a screen in the equation is enclosed in parentheses.

A $250 \div 3 = [83]$ and 1 rm.
B $\$5.00 - \$3.15 = [\$1.85]$
C $4 \times \$.79 = [\$3.16]$
D $(150) \div [50] = 3$
E $765 + [25] = 790$
F $94¢ - 68¢ = [26¢]$
G $87 + 131 = [218]$ or
$131 + 87 = [218]$
H $3208 - 3050 = [158]$
I $\$10.00 - [\$7.62] = \$2.38$
J $[\$.78] + \$.25 = \$1.03$
K $64 \div [16] = 4$
L $180 \div 60 = [3]$
M $(\$1.80) + [\$1.15] = \$2.95$
N $[67] - (48) = 19$

Page 111:

Test 1	Test 2	Test 3	Test 4
A 559	A 677	A 328	A 1134
B 249	B 644	B 354	B 975
C 1152	C 333	C 162	C 4539
D 831	D 157	D 765	D 6724
E 1779	E 2187	E 2583	E 12,580
F 1303	F 1749	F 240	F 23,374
G 1184	G 3777	G 1440	G 12,738
H 1156	H 4839	H 1480	H 68,766
I 2374	I 39,688	I 3681	I 126,500

Test 5	Test 6
A 27	A 6 and 12 rm.
B 39 and 1 rm.	B 17 and 14 rm.
C 29	C 23
D 50 and 2 rm.	D 130 and 6 rm.
E 32 and 5 rm.	E 44 and 5 rm.
F 60	F 60
G 621 and 5 rm.	G 8 and 5 rm.
H 931	H 520 and 5 rm.
I 751 and 4 rm.	I 70 and 10 rm.

Test 7	Test 8	Test 9
A 180	A 4	A 64 pk.
B 448	B 303 and 12 rm.	B 8000 lb.
C 959	C 88 and 5 rm.	C 4 hr.
D 1322	D 38	D 7 c.
E 152	E 83	E 4 fl. oz., 1 tbs.
F 472	F 27 and 5 rm.	F 920 ft.
G 1403	G 25 and 9 rm.	G 6 ft.
H 1813	H 56	H 128 oz.
I 609	I 981	I 13 yd.

Checking up 1

A Mr. Long has a flower shop. He has 250 bulbs to plant in bowls. He will put 3 bulbs in a bowl. How many bowls will he need? How many bulbs will be left over?

B Mrs. Fisher gave Mr. Long a 5-dollar bill to pay for a plant that cost $3.15. How much money should Mr. Long have given her in change?

C Mrs. Block bought 4 dozen daisies for 79¢ a dozen. How much did she pay for the daisies?

D Patsy Long sometimes helps her father in the flower shop. On Thursday she worked for 45 min. On Friday she worked for 40 min. On Saturday she worked for 65 min. She worked an average of how many minutes per day?

E In September there were 765 pupils at the Burns School. Then some new children came to the school. A month later the school had 790 pupils. How many pupils had been added?

F One month Carol saved 68¢. The same month Sally saved 94¢. Carol saved how much less money than Sally?

G Paul has collected 87 baseball cards and 131 football cards. How many cards in all has he collected?

H The Cub Scouts in Bill's den collected 3050 lb. of paper in a paper drive. Bob's den collected 3208 lb. Bob's den collected how many more pounds of paper than Bill's den?

I Kathy had $10 when she went to the grocery store. She bought groceries and a notebook and came home with $2.38. How much had she spent?

J Tom had some money in his pocket. His father paid him 25¢ for helping in the yard. Then Tom had $1.03 in his pocket. How much money did he have before his father paid him?

K Four girls made 64 pieces of fudge and shared the pieces equally. How many pieces did each girl get?

L Bob spent 180 minutes last week practicing on his cornet. How many hours did he spend practicing on his cornet last week?

M Jim is trying to save $2.95 to buy a radio kit. He has saved $.15 a week for the last 12 weeks. How much more money does he need for the radio kit?

N Mrs. Long baked some cookies. She gave Patsy 4 dozen cookies to take to a school party. Then she had 19 cookies left. How many cookies had she baked? 2

Checking up

Test 1 1
- A $217+342=n$
- B $86+50+69+44=n$
- C $631+149+372=n$
- D $256+75+500=n$
- E $413+829+537=n$
- F $778+59+466=n$
- G $384+272+528=n$
- H $115+634+407=n$
- I $530+993+851=n$

Test 2 1
- A $736-59=n$
- B $889-245=n$
- C $500-167=n$
- D $423-266=n$
- E $6578-4391=n$
- F $2707-958=n$
- G $7000-3223=n$
- H $13645-8806=n$
- I $52463-12775=n$

Test 3 1
- A $4\times82=n$
- B $6\times59=n$
- C $9\times18=n$
- D $3\times255=n$
- E $7\times369=n$
- F $48\times5=n$
- G $2\times720=n$
- H $185\times8=n$
- I $9\times409=n$

Test 4 1
- A $42\times27=n$
- B $25\times39=n$
- C $17\times267=n$
- D $82\times82=n$
- E $34\times370=n$
- F $403\times58=n$
- G $66\times193=n$
- H $157\times438=n$
- I $250\times506=n$

Test 5 1
- A $81\div3=n$
- B $79\div2=n$
- C $116\div4=n$
- D $302\div6=n$
- E $293\div9=n$
- F $300\div5=n$
- G $4352\div7=n$
- H $7448\div8=n$
- I $4510\div6=n$

Test 6 1
- A $90\div13=n$
- B $320\div18=n$
- C $1058\div46=n$
- D $3776\div29=n$
- E $2249\div51=n$
- F $2100\div35=n$
- G $581\div72=n$
- H $34845\div67=n$
- I $1340\div19=n$

Test 7 1
- A $n+420=600$
- B $705-n=257$
- C $n-368=591$
- D $873+449=n$
- E $652+n=804$
- F $700-228=n$
- G $n-502=901$
- H $n+830=2643$
- I $968-n=359$

Test 8 1
- A $124\div n=31$
- B $7284\div24=n$
- C $533\div6=n$
- D $1596\div n=42$
- E $747\div n=9$
- F $221\div8=n$
- G $1659\div66=n$
- H $1008\div n=18$
- I $6867\div n=7$

Test 9 1
- A 16 bu. = ■ pk.
- B 4 T. = ■ lb.
- C 240 min. = ■ hr.
- D 56 fl. oz. = ■ c.
- E 9 tbs. = ■ fl. oz.
- F 6200 ft. = 1 mi. ■ ft.
- G 72 in. = ■ ft.
- H 8 lb. = ■ oz.
- I 39 ft. = ■ yd.

110

1 Give this test without a time limit. Direct the children to make an equation for each problem, write *n* to hold a place for the missing number, then find the missing number and rewrite the equation with the numeral for it in place of *n*. For Problems D, M, and N, the children should also put a screen in the equation and rewrite the equation with the numeral in place of the screen before finding the answer.

2 Supply the children with correct answers, and let them verify their own answers. Determine which problems have given the class the most trouble. Then refer to the Reteaching Chart on page 109, which gives the pages on which each problem type is taught, and do whatever reteaching is necessary.

111

1 Direct the children to write the numerals in the proper form for computing. When most of the children have finished, provide answers for verifying the work. Do not impose any rigid time limit for this test. Provide whatever reteaching is necessary. See the Reteaching Chart.

112-117 Moving forward

Expanded Notes for this lesson are on pages 298-300.

Objectives

The child learns how to express a rate by using two numerals to make a ratio. He learns how to read a ratio when it expresses a rate. He also learns that the same rate may be expressed by several different ratios.

Vocabulary

New words page 112 pair*, express, rate*, ratio; page 114 thought*; page 117 solve*

Comments

When ratios are used in the text of this *Teaching Guide*, they will be written as 12/14, 4/5, etc. When discussing ratios the first numeral (upper in the pupils' book) will be called the *first term*, and the second numeral (lower in the pupils' book) will be called the *second term*. Thus, in the ratio 18/16, 18 is the first term and 16 is the second term. The word *term* is not introduced in the pupils' text at this grade level.

In this lesson the mathematical terms *rate* and *ratio* are introduced in connection with situations that have to do with the common experience of buying things.

Up to this time the children have dealt only with buying items that have their own individual price. Now they will deal with buying things that are sold in groups at a certain rate—that is, so many items for so much money. In such instances, the problem of cost arises when we want to buy one item, or when we want to buy more or fewer items than are sold at a stated given price, or rate.

On page 112 a situation where 12 pieces of candy can be bought for 15¢ is presented. The fact that candy is sold at the same rate when it is bought in larger or smaller quantities than the quantity stated in the price is explained, as well as how to write and read a ratio. The terms *rate* and *ratio* need not be confused if *rate* is used to describe the relationship —in this lesson it is so many items for so much money —and if *ratio* is used as the name of the pair of numerals, written one over the other, to express the rate.

The pictures in this lesson illustrate that several different ratios may be written to express one rate. Items and coins are separated into equal groups to help the children see why many ratios *can* be written for one rate. Thus it is brought out that items may be bought in different amounts at the same rate.

The explanation on page 114 shows how to express the rate as a ratio when the relationship of the two numbers in a rate are thought of in the opposite way —that is, when the amount of money is thought of *before* the number of candies. Again, the different ratios express the same rate, and they are illustrated by pictures of items and coins in equal groups.

This lesson will give the children an understanding of what the two numerals in a ratio mean and how they are related to each other. This knowledge will be used later to solve problems.

Answers

Page 117:

(block 1)

A [A]	F [A]	K [D]	P [E]
B [E]	G [E]	L [F]	Q [A]
C [D]	H [B]	M [B]	R [F]
D [C]	I [D]	N [C]	S [C]
E [C]	J [A]	O [F]	T [B]

(block 2)

A 10/18 or 18/10	G 10/12 or 12/10
B 12/40 or 40/12	H 12/29 or 29/12
C 5/16 or 16/5	I 50/19 or 19/50
D 5/6 or 6/5	J 10/55 or 55/10
E 5/27 or 27/5	K 6/59 or 59/6
F 6/39 or 39/6	

Moving forward

Now you will study rate. You will learn how to use two numerals to express a rate.

1

When Jim answered Don's question, he told him the rate at which the candy was sold. He used two numerals to do this. The rate at which the candy was sold can be expressed in different ways.

2

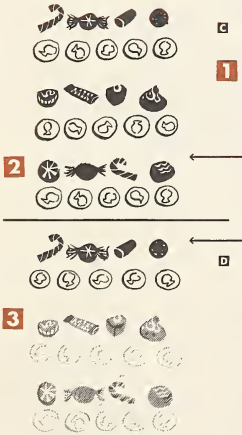
One way to express the rate is to use the numerals 12 and 15. You can say that the candy was sold at the rate of 12 pieces for 15 cents. 3

Here is the way to write 12 and 15 together to express this rate.

12 shows the number of pieces that were sold. → $\frac{12}{15}$ ← 15 shows the number of cents needed.

12 per 15 4

A pair of numerals used in this way is called a ratio. This ratio expresses a rate. The rate is 12 per 15. 5



You can use other numerals to express the rate at which the candy was sold.

1

In Picture C the 12 pieces of candy and the 15 pennies are separated into 3 equal groups. The pennies in each new group will buy the pieces of candy in the group.

2

4 pieces and 5 pennies are in each group.

Think of only 1 of these 3 new groups. 4 pieces of candy will sell for 5 cents.

4

Now you can say that the rate is 4 pieces per 5 cents. Write the ratio this way.

3

4 shows the number of pieces. → $\frac{4}{5}$ ← 5 shows the number of cents needed.

4 per 5

You can now express the rate as 4 per 5.

You can use two other numerals to express the rate. Think of 2 of the 3 new groups. 8 pieces of candy will sell for 10 cents. 6

Now you can say that the rate is 8 pieces per 10 cents. Write the ratio this way.

5

8 shows the number of pieces. → $\frac{8}{10}$ ← 10 shows the number of cents needed.

8 per 10

Now the rate is expressed as 8 per 10.

112

1. Call on a pupil to read the text for Movie A. Let two children act out the parts of Jim and Don. Be sure the children understand that two numerals, 12 and 15, are used to express the rate at which the candy was sold.
2. Have the children first count the pieces of candy in Picture B, then the pennies. Have them relate the text opposite the arrow to the picture.
3. Read the text and note the position of the two numerals which express the rate.
4. Have the pupils note that in reading a rate the word "per" is used in preference to the word "for." Hence, the rate for Picture B is read "12 per 15."
5. Point out again that it takes two numerals to express a rate. Explain that when we pair two numerals like this we call it a ratio.

113

1. Let a pupil read the text. Help the children understand that the candy and pennies have been divided into equal groups to determine the price for a few pieces.
2. Have the children verify the text by counting the pieces of candy and pennies in each group in Picture C.
3. Tell the children to study Picture D and imagine they are going to buy only 4 pieces of candy. Ask them how much the 4 pieces would cost and to note that the other pieces of candy and pennies are dimmed off.
4. Have someone read the text for Picture D. Be sure the children understand what each numeral in the ratio means.
5. Ask the children to look at Picture E and imagine they are going to buy 8 pieces of candy. Ask them to count the pennies to find out how much 8 pieces would cost.
6. Treat the text for Picture E in the same way as the text for Picture D.

F

Picture F shows what happens when more than 12 pieces of candy are sold at the rate Jim paid. It shows that for 20 pieces, 25 cents would be needed. This time you can express the rate as 20 per 25. **2**

What does the 20 show? → 20 / 25 ← What does the 25 show?

20 per 25

A Does 16 per 20 also express the rate shown in Picture F?

B Now you see that 4 per 5, 8 per 10, 12 per 15, 16 per 20, and 20 per 25 all express the same rate.

4 $\frac{4}{5} = \frac{\square}{10} = \frac{\square}{15} = \frac{\square}{20} = \frac{\square}{25}$

When you expressed the rate as 20 pieces for 25 cents, you thought of the number of pieces first. Look at Picture F again. Now think of the amount of money first. When you think this way, you can express the rate as 25 cents for 20 pieces. Write the rate as a ratio this way. **5**

25 shows the number of cents. → 25 / 20 ← 20 shows the number of pieces.

25 per 20

C Now look at 1 group in Picture F. Think of the money first. You can express the rate as 5 cents for ☐ pieces. Write the rate as a ratio this way. **6**

What does the 5 show? → 5 / 4 ← What does the 4 show?

5 per 4

D If you express the rate as 15 per 12, are you thinking first of the money or of the candy?

E 5 per 4, 10 per 8, 15 per 12, 20 per 16, and 25 per 20 all express the same rate.

$\frac{5}{4} = \frac{\square}{8} = \frac{\square}{12} = \frac{\square}{16} = \frac{\square}{20}$ **7**

G

1

Don also expressed a rate when he told Jim how much his candy cost. This rate, too, can be expressed in different ways.

H

You can say that Don's candy was sold at the rate of 12 pieces for 10 cents.

To show the rate, you can write the numerals 12 and 10 as a ratio.

2

12 shows the number of pieces. → 12 / 10 ← 10 shows the number of cents.

☐ per 10

You can express the rate as ☐ per ☐.

I

3

You can use other numerals to express the rate at which Don's candy was sold.

The 12 pieces of candy and the 10 pennies are now separated into 2 equal groups. The pennies in each new group will buy the pieces of candy in the group.

There are ☐ pieces of candy and ☐ pennies in each new group.

Now turn the page.

114

1 Have the children note that this picture shows more candies and pennies than the pictures on page 113 did. Have them give the rate for one of the groups into which the 20 pieces of candy and 25 pennies have been separated. Have them give the rate for 2 groups.

2 Have one pupil read the text and explain what each numeral of the ratio means.

3 Let someone read the question and explain how many groups it takes to make this ratio.

4 Let a pupil show, by referring to the groups in Picture F, that all these ratios express the same rate that 4 per 5 expresses. Let several children supply the missing numerals.

5 Use Note 2. Be sure the pupils understand that the way the numerals are placed in a ratio depends on whether you think of pieces per money or money per pieces.

6 Use Note 2.

7 Adapt the procedures in Note 4.

115

1 Have two children use what Don and Jim are saying as a dialogue. Then let someone read the text for Picture G.

2 Have the children count the pieces of candy and pennies in Picture H. Have them relate the text to the picture and explain the meaning of both numerals in the ratio.

3 Ask a pupil to read the text for Picture I and then fill in the missing numerals after counting the number of candies and pennies in each group in the picture.

Think of only 1 of these 2 new groups. 6 pieces of candy will sell for ■ cents.

You can say that the candy sold at a rate of ■ pieces for ■ cents. You can write the ratio that shows the rate this way.

What does the 6 show? $\frac{6}{5}$ What does the 5 show?

■ per 5

Now the rate is expressed as 6 per ■.

Picture K shows what happens when more than 12 pieces are sold at the rate Don paid. It shows that for 24 pieces, 20 cents would be needed.

You can express the rate now as 24 per 20.

What does the 24 show? $\frac{24}{20}$ What does the 20 show?

■ per ■

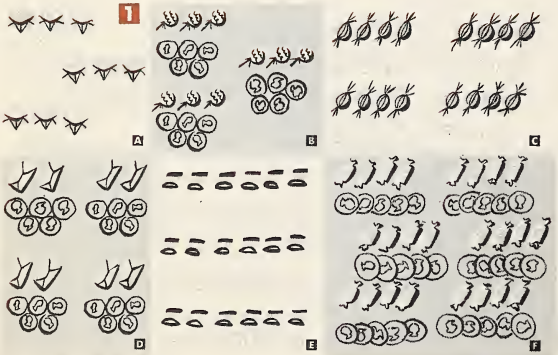
The rate is expressed now as ■ per ■.

Does 18 per 15 also express the rate shown in Picture K?

Do you see that 12 per 10, 6 per 5, and 18 per 15 all express the same rate?

$$\frac{12}{10} = \frac{6}{5} = \frac{18}{15}$$

You can also express the rate at which Don's candy was sold as 5 per 6. What do the 5 and 6 mean?



Each ratio below expresses a rate. For each ratio there is a picture. Find the picture that shows each rate.

A	$\frac{9}{6}$	F	$\frac{3}{2}$	K	$\frac{6}{15}$	P	$\frac{6}{5}$
B	$\frac{12}{10}$	G	$\frac{18}{15}$	L	$\frac{15}{12}$	Q	$\frac{6}{9}$
C	$\frac{4}{10}$	H	$\frac{3}{5}$	M	$\frac{6}{10}$	R	$\frac{4}{5}$
D	$\frac{4}{3}$	I	$\frac{8}{20}$	N	$\frac{12}{9}$	S	$\frac{8}{6}$
E	$\frac{16}{12}$	J	$\frac{2}{3}$	O	$\frac{20}{16}$	T	$\frac{9}{15}$

For each statement below, write the numerals as a ratio to show the rate.

A 10 apples for 18¢
B 12 cookies for 40¢
C 5 candy bars for 16¢
D 5¢ for 6 pieces of candy
E 5 lemons for 27¢
F 6 yards of ribbon for 39¢
G 10¢ for 12 sheets of paper
H 12 lollipops for 29¢
I 50 marbles for 19¢
J 10 flower bulbs for 55¢
K 6 cans of fruit drink for 59¢

Now you know how numerals are used to express rates. Later on you will use rates to solve problems.

116

1. Tell the children to study Picture J and imagine they are going to buy only 6 pieces of candy. Ask them how much the 6 pieces will cost. Have them study the ratio and explain what each numeral means.
2. Let someone count the groups in Picture K and the candies and pennies in each group.
3. Have a pupil read the text for Picture K and explain the meaning of each numeral in the ratio.
4. Help the children relate the groups in Picture K to the ratios in the text.
5. Be sure the children understand that 5 stands for the number of pennies and 6 stands for the number of pieces of candy.

117

1. Let the children have time to study the pictures and decide what ratios can be written for each one.
2. Tell the children to write each ratio given in Exercises A to T and beside each to write the letter of the picture which shows the rate expressed by the ratio. When the children are finished, discuss the exercises with them.
3. Let the children do this work independently also. Tell them that for some of the statements more than one ratio may be written to express the rate. Make sure the children correct any errors they make in their work.

118 Thinking straight

Exponded Notes ore not considered necessory for this lesson.

Objectives

The child gets experience in expressing rates by using ratios, and he learns that the second term of a ratio may be 1.

Vocabulary

New words page 118 speed*, usually*

Comments

In discussing the exercises on this page, make sure the child understands that it is always possible to express a rate situation in two ways. Then explain that in some instances usage determines which of the two is used. In other words, usage determines which term comes first in expressing a rate and, consequently, which term is written above the line in a ratio form. It is essential for the child to realize that he must know what each term in the rate means, so that he can express this meaning properly in writing a ratio. Unless the child understands clearly what each term in the ratio stands for, he will not be able to write ratios in a consistent fashion later on. (On pages 215-220 in the pupils' book, ratios are used in problem solving.)

Answers

Where two answers are given below, as in Exercises A to J, either one may be considered correct. Only one answer has been given in instances where usage strongly favors a particular expression.

Problems	Exercises
A 30	A 45/3
B 45; 45/1	B 87/3, 3/87
C 2 cans per 33¢; yes; not if you know the meaning of each term	C 56/12, 12/56
D 5¢ per 1 lb.; yes; 5¢ per pound is more common.	D 1/2
E 65 mi. per 2 hr.; 65/2	E 8/1
F $6 per 5 da.; 6/5	F 2/19, 19/2
G 8 hr. per 3 da.; 8/3	G 49/1
	H 10/63, 63/10
	I 29/11, 11/29
	J 3/1

Thinking straight

A Sign A shows a speed of ▒ mi. in 1 hr. This rate is usually thought of as 30 miles per hour. The rate is written as a ratio this way.

What does the 30 stand for? → $\frac{30}{1}$ ← What does the 1 stand for?

You do not usually think of the rate as 1 hour per 30 miles.

B Sign B shows a speed of ▒ mi. in 1 hr. Is this rate usually thought of as 45 mi. per hour or as 1 hour per 45 miles? Write this rate as a ratio.

C One way to express the cost shown in Picture C is 2 ▬ per 33 ▬. Can you also express this rate as 33¢ per 2 cans? Does the way you express the rate make any difference?

D One way to express the cost shown in Picture D is 5 ▬ per 1 ▬. Can you also express this rate as 1 lb. per 5¢? Is one way of expressing the rate better than the other? Why?

E Bob's father drove 65 mi. in 2 hr. Would you usually express this rate as 2 hr. per 65 mi. or as 65 mi. per 2 hr.? Which ratio at the right expresses the usual way you think of this rate? $\frac{2}{65}$ $\frac{65}{2}$

F Paul earned $6 in 5 da. Would his rate of earning usually be expressed as $6 per 5 da. or as 5 da. per $6? Write this rate as a ratio.

G Tony worked 8 hr. in 3 da. How would this rate usually be expressed? How would it be written as a ratio?

Write a ratio for each rate below.

A 45 mi. in 3 hr.
B 87¢ for 3 lb.
C 56¢ for a dozen
D 1 tbs. per 2 lb.
E 8¢ each
F 2 for 19¢
G 49¢ a pound
H 10 lb. for 63¢
I 29¢ for 11 oz.
J $3 per box

5

118 Deciding which term of a rate to use first in a ratio: ratios in which the second term is 1

118

1 Relate the problem to Picture A. Let the children discuss the meaning of the numerals in each term of the ratio and the reason for writing 30 above the line.

2 Relate the problem to Picture B and let a child write the ratio on the chalkboard.

3 Bring out that a ratio for this situation may be written in either of two ways. Make sure the pupils see that in this example one way of stating the price is as good as the other.

4 Discuss Problems D to G. Pay special attention to situations in which the rate is commonly expressed in one way rather than another.

5 Tell the children that although ratios for these rates *may* be written in two ways, there are some that are usually written in one of these ways.

119-123 Moving forward

Expanded Notes for this lesson are on pages 300-302.

Objectives

The child learns to use a ratio to compare one quantity with another. He learns the meaning of each term in the ratio and how to read it.

Vocabulary

New words page 119 comparison; page 122 sailboats*, motorboats*, steamboats*

Comments

Work with ratios is extended in this lesson to include the comparison of a quantity with another quantity. Making comparisons of this kind is a common experience in our everyday lives. Most of us spend a lot of time comparing what we have, what we do, how long it takes us to do it, etc., with the similar achievements of our friends. Newspapers and magazines are full of statistical data and illustrations using comparisons of all kinds. Learning to compute with ratios and learning the related terminology is an accomplishment the children will have occasion to use the rest of their lives.

In previous lessons the children learned that more than one ratio can be written for a rate. In this lesson they will learn that more than one ratio can be written to express a comparison. Two groups of objects are compared at the start. By separating each of the two sets of objects into the same number of equal groups, it is shown that several ratios may be written for the same comparison. A set of the smaller groups of objects becomes the basis for writing the new ratio.

Just as the children learned that one set of ratios could be written for expressing the number of items per so much money and another set for expressing so much money per so many items, so, too, is it possible to write two sets of ratios when comparing one group of items with another group. We may think either of planes compared with boats, or boats compared with planes. It is important for the pupils to realize that either group can be compared with the other group — that is, either term can be the first term of the ratio.

On pages 119-121 two groups of objects are compared and used to show how to write, read, and interpret the ratio; and then, by separating the objects into equal groups, the children are shown how to use other pairs of numerals to write different ratios for the same comparison. The picture on page 121 shows what happens when the number of objects is increased by including more groups, and the accompanying text explains some of the equivalent ratios that can be written for the original comparison. It also shows some of the ratios that can be written when the comparison is made the other way around.

On page 122 three pictures are used for making comparisons. Exercises A to E are detailed steps that can be followed by you to explain writing ratios for Picture A. These same steps can be used in studying Pictures B and C. If you let the children in your class use objects for counting and grouping, it will give them better understanding of the work on these pages. Using a different number of objects than shown here and forming other groups will give the children still further practice in visualizing comparisons and relating ratios that can be written to what they have done with objects.

Answers

Page 122:

A 16/24

B 2 sailboats and 3 motorboats; 2/3; 3/2

C 6 sailboats and 9 motorboats; 6/9; 9/6

D 4/6, 8/12, 10/15, 12/18, 14/21

E 6/4, 12/8, 15/10, 18/12, 21/14, 24/16

F 15/20

G 3/4, 6/8, 9/12, 12/16

H 10/8, 15/12, 5/4, 20/16, 25/20

I 20/25, 8/10, 4/5, 12/15, 16/20

Page 123:

(block 1)

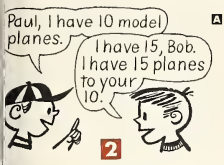
A [D]	D [E]	G [E]	J [C]	M [F]	P [B]
B [C]	E [A]	H [D]	K [F]	N [B]	
C [A]	F [F]	I [B]	L [D]	O [A]	

(block 2)

A 10/4 or 4/10	E 9/12 or 12/9	I 8/12 or 12/8			
B 4/10 or 10/4	F 6/10 or 10/6	J 24/15 or 15/24			
C 27/18 or 18/27	G 18/12 or 12/18	K 10/14 or 14/10			
D 4/1 or 1/4	H 12/8 or 8/12	L 20/16 or 16/20			

Moving forward

Now you will learn how to use a ratio to compare one group with another. 1



A Paul compared his planes with Bob's by using two numerals, 15 and 10.

2

3 ← This picture shows both boys' planes.

B You can express the comparison that Paul made by writing the numerals 15 and 10 as a ratio. Your ratio will look like this. 4

Number of Paul's planes that are compared with Bob's planes → $\frac{15}{10}$ ← Number of Bob's planes that Paul's planes are compared with

15 to 10 5

C 6

You can use other numerals to compare Paul's planes with Bob's.

Now Paul's 15 planes and Bob's 10 planes are separated into 5 equal groups. 3 of Paul's planes and 2 of Bob's are in each group.

Now turn the page.

1 D Think of one of these groups. Now you can compare Paul's planes with Bob's by saying that there are 3 of Paul's planes to 2 of Bob's.

The ratio that shows this comparison is written this way. 2

Number of Paul's planes that you are thinking about now → $\frac{3}{2}$ ← Number of Bob's planes that Paul's planes are compared with

3 to 2

You can now express the comparison of Paul's planes to Bob's as 3 to 2.

3 E When you think of three of the groups, you can use other numerals to express the comparison of Paul's planes to Bob's.

Now you can express the ratio as 9 planes to 6 planes. Write the ratio this way.

Number of Paul's planes you are thinking about → $\frac{9}{6}$ ← Number of Bob's planes that Paul's planes are compared with

9 to 6

The comparison of Paul's planes to Bob's is now expressed as 9 to 6.

119

1 Explain to the class that in the preceding lesson they used ratios to express rate. In this lesson they will use ratios to make a comparison between two quantities.

2 Let two children read the dialogue of Bob and Paul; then point out that the two boys are comparing the number of planes they have. Use objects to represent each boy's planes.

3 Have the children count the planes in each group in Picture B. Let someone tell which group belongs to Paul and which to Bob.

4 Have a child read the text for Picture B and explain what each numeral in the ratio means.

5 Tell the children to note that the word *to* can be used in reading ratios that express comparisons. Ask them how this differs from reading ratios that express rates.

6 Tell the class to look at Picture C and note the number of equal groups, as well as the number of Paul's planes and Bob's planes in each group.

120

1 Have the class study Picture D. Be sure they observe that each group of planes includes 3 of Paul's and 2 of Bob's. Explain that since we are especially interested in one of these groups, the others are dimmed off. Get them to understand that in each group there are 3 of Paul's planes to 2 of Bob's. Ask questions to get the pupils to see that the ratio 3/2 expresses this. Emphasize the idea that the comparison between the planes is the same as it was on page 119, but that different numerals are used to express it. Stress the idea that the ratio 15/10 equals the ratio 3/2.

2 Call on several children to read the entire text for Picture D and explain the numerals in the ratio.

3 Ask: "How many groups are we considering now? How many of the planes are Paul's? How many are Bob's?" Read the text and ask children to explain the numerals in the ratio in terms of the picture.

Picture F shows that the boys bought some more planes. Now there are 18 of Paul's planes to 12 of Bob's. Write the comparison as a ratio this way.

What does the 18 show? → $\frac{18}{12}$ ← What does the 12 show?

18 to 12

A Does 6 to 4 also express the comparison of Paul's planes with Bob's? What part of Picture F shows this?

B Does 12 to 8 also express the comparison of Paul's planes with Bob's?

C 3 to 2, 6 to 4, 9 to 6, 12 to 8, 15 to 10, and 18 to 12 all express the same comparison.

$\frac{3}{2} = \frac{\blacksquare}{4} = \frac{\blacksquare}{6} = \frac{\blacksquare}{8} = \frac{\blacksquare}{10} = \frac{\blacksquare}{12}$

When you expressed the comparison as 18 planes to 12 planes, you compared Paul's 18 planes with Bob's 12 planes. Now look at Picture F again. This time compare Bob's 12 planes with Paul's 18 planes.

Number of Bob's planes you are thinking about → $\frac{12}{18}$ ← Number of Paul's planes that Bob's planes are compared with

12 to 18

Look at 1 group in Picture F and compare Bob's planes with Paul's. You can express the comparison now as 2 planes to 3 planes. Write the comparison as a ratio this way.

What does the 2 show? → $\frac{2}{3}$ ← What does the 3 show?

2 to 3

D If you express the comparison as 8 to 12, are you comparing Bob's planes with Paul's or Paul's planes with Bob's?

E 2 to 3, 4 to 6, 6 to 9, 8 to 12, 10 to 15, and 12 to 18 all express the same comparison.

$\frac{2}{3} = \frac{\blacksquare}{6} = \frac{\blacksquare}{9} = \frac{\blacksquare}{12} = \frac{\blacksquare}{15} = \frac{\blacksquare}{18}$

121

A

B

C

A Look at Picture A. There are ■ sailboats. There are ■ motorboats. Finish the ratio at the right to compare all the sailboats with all the motorboats. $\frac{16}{\blacksquare}$

B Think of one group of sailboats and motorboats. There are ■ sailboats and ■ motorboats. Write the ratio to compare sailboats with motorboats. Write the ratio to compare motorboats with sailboats.

C Think of three groups of sailboats and motorboats. There are ■ sailboats and ■ motorboats. Write the ratio to compare sailboats with motorboats. Write the ratio to compare motorboats with sailboats.

D Use three other pairs of numerals to compare the sailboats with the motorboats.

E Use three other pairs of numerals to compare the motorboats with the sailboats.

F For Picture B write the ratio to compare all the cars with all the engines.

G Use two other pairs of numerals to compare the cars with the engines.

H Which pairs of numerals below show the comparison of planes with steamboats in Picture C?

$\frac{10}{8}$ $\frac{3}{4}$ $\frac{1}{2}$ $\frac{15}{12}$ $\frac{5}{4}$ $\frac{2}{3}$ $\frac{20}{16}$ $\frac{25}{20}$

I Which pairs of numerals below show the comparison of steamboats with planes?

$\frac{3}{4}$ $\frac{20}{25}$ $\frac{5}{6}$ $\frac{8}{10}$ $\frac{8}{5}$ $\frac{4}{5}$ $\frac{12}{15}$ $\frac{16}{20}$

121

1 Ask the children how Picture F differs from Picture E [one more group of 5 planes]. Determine the number of each boy's planes. Develop the idea that as long as Paul has 3 planes to 2 of Bob's, the relationship is the same, but a different ratio is written.

2 Relate the ratio to Picture F.

3 Let several pupils answer Questions A and B.

4 Have the children use the groups in Picture F to complete the ratios in Exercise C.

5 Read the text to the class and explain that Bob's planes can also be compared with Paul's.

6 Tell the class to count all the black planes in Picture F and then all the red ones. Have someone explain the new ratio.

7 Read the text and relate the ratio to one group in Picture F.

8 Let several children answer Question D and complete the ratios in Exercise E.

122

1 Tell the class to count the sailboats and motorboats in Picture A and to explain how each group of 5 is alike. Then have someone read the text and supply the missing answers.

2 Call on one child to read Exercise B and use one group in Picture A for determining the missing numerals. Then ask all the children to write the two ratios. Discuss the ratios.

3 Proceed in the same way as you did for Exercise B.

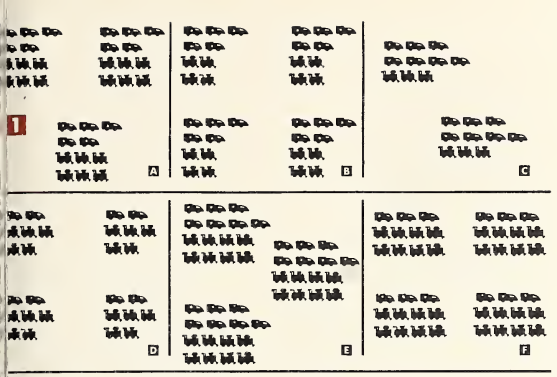
4 Discuss together the other ratios that can be written for Exercises D and E.

5 Have the children note the number of engines and cars in Picture B. Also have them explain how the groups are alike.

6 In connection with Exercises F and G, discuss the ratios that can be written for Picture B.

7 Treat Picture C in the same way as Picture B.

8 Have the children write out their answers for Exercises H and I.



Each ratio below expresses a comparison. Find a picture that belongs with each ratio. For example, Ratio A can be used to compare 6 of the new engines with 15 of the old engines in Picture D.

3

A	$\frac{6}{15}$	E	$\frac{5}{6}$	I	$\frac{16}{20}$	M	$\frac{12}{32}$
B	$\frac{3}{7}$	F	$\frac{9}{24}$	J	$\frac{14}{6}$	N	$\frac{12}{15}$
C	$\frac{12}{10}$	G	$\frac{7}{8}$	K	$\frac{8}{3}$	O	$\frac{15}{18}$
D	$\frac{21}{24}$	H	$\frac{10}{4}$	L	$\frac{2}{5}$	P	$\frac{5}{4}$

Now you should be able to write a ratio to compare one group with another.

For each comparison given below, use the numerals to write a ratio.

4

A 10 model cars with 4 model cars
B 4 model cars with 10 model cars
C 27 stamps with 18 stamps
D 4 dolls with 1 doll
E 9 pictures with 12 pictures
F 6 books with 10 books
G 18 plants with 12 plants
H 12 pencils with 8 pencils
I 8 pencils with 12 pencils
J 24 records with 15 records
K 10 cents with 14 cents
L 20 erasers with 16 erasers

123

123

1 Give the class time to study the six pictures and note the total number of objects of each kind. Also have them note that the groups in each picture are equal. Discuss how many objects of each color they would have if they had 1 group, 2 groups, 3 groups, etc. Then discuss the various ratios that can be used for each picture.

2 Read the text to the class and make sure the children understand the example that is given.

3 Ask the children to write on a sheet of paper each ratio and the letter of the picture that illustrates it. Go over the answers with the children, and have them explain their answers by pointing out how the pictures illustrate the ratios.

4 Tell the pupils to write at least one ratio for each comparison. Some children may write several different ratios for each comparison.

124-125 Using arithmetic

Expanded Notes are not considered necessary for this lesson.

Objectives

The child solves problems of various types taught so far in this book, particularly multiple-step problems and those requiring approximate answers.

Vocabulary

New words page 124 raised*, fed*, grew*, harvested*, applesauce*, raisin, budget, allowed*, clothing*; page 125 spring*, since*, laid*

Comments

In the work on pages 124 and 125, the child solves problems of various types. When an approximate answer is called for in a problem solved by division, he should exercise judgment in considering the remainder. Some problems require him to round off the answer.

Since to start with a detailed discussion may cause children to lose interest in working problems of this kind, it is wise to delay the discussion until each child has tried to solve the problems. Point out, however, that the data needed to solve a problem may not be given in the problem. Solution of some problems may depend on previous answers or on data given in earlier problems.

When the time for discussion comes, you may wish to pay special attention to those problems which require approximate answers. Problem J on page 124 is especially worthy of classroom consideration, since good arguments may be produced in favor of either of the two answers suggested. For example, one might reason like this: "The $8 remainder would make almost $1 more per month; so to the nearer whole dollar, $42 is the better answer." On the other hand, if one considers the stated fact that $500 has been allowed for clothing, then this requirement must be treated with respect. In budgeting, no harm is done if one spends less than has been planned, but to spend over the budget may be disastrous. From this point of view, $41 would be considered the better answer.

Answers

Using arithmetic:

A $13 \times 6 = [78]$

B $780 \div 56 = [13]$ and 52 rm., about 14 bu.; 14 is the better answer because the remainder, 52 lb., is almost another bushel.

C $75 \times 56 = [4200]$

D $2990 \div [230] = 13$

E $(32) + 95 = [127]$ or $95 + (32) = [127]$

F $95 - 32 = [63]$

G $32 \div 2 = [16]$

H $16 - 9 = [7]$

I $12 \times \$95 = [\$1140]$

J $\$500 \div [\$41] = 12$ and \$8 rm.; If it is important not to exceed the budget, \$41 is the better answer. If you want to find an average to the nearer whole dollar, \$42 is the better answer. Two answers are given for Problem K, using both \$41 and \$42.

K $\$325 - (\$136) = [\$189]$ or $\$325 - (\$137) = [\$188]$

L $\$1140 - \$500 = [\$640]$

M $120 - [75] = 45$

N $120 \div 15 = [8]$

O $8 \times 7 = [56]$

P $(6) \times 7 = [42]$ or $56 - [42] = (14)$ or $[42] + (14) = 56$

Q $(95) \div [19] = 5$

R $30 \times 19 = [570]$

S $[45] - 11 = 34$

Keeping skillful:

	(block 1)		(block 2)
A	32 tbs.	M 180 sec.	A 487
B	9 ft.	N 8 wk.	B 64
C	2 pk.	O 36 yd.	C 7871
D	40 pt.	P 60 mo.	D 30,628
E	4 yr.	Q 2 pt.	E 1626
F	240 min.	R 5 pk.	F 31 and 15 rm.
G	6 lb.	S 180 in.	G 319
H	16 pk.	T 1460 da.	H 188
I	32 fl. oz.	U 1 mi.	I 47
J	3 da.	V 40 fl. oz.	J 20,732
K	16,000 lb.	W 128 oz.	K 208,164
L	2 bu.	X 168 hr.	L 15,885

Using arithmetic 1

A Paul and George raised 13 pigs for a 4-H Club project. They gave each pig 6 lb. of feed each day. How many pounds of feed did they use in one day?

B In a month the boys fed the pigs about 780 lb. of shelled corn. One bushel of shelled corn weighs 56 lb. About how many bushels of shelled corn did they feed the pigs that month?

THINK Which is the better answer, about 13 bu. or about 14 bu.? Why?

C The boys grew the corn they used for feeding the pigs. The corn that they harvested gave them about 75 bu. of shelled corn. About how many pounds of shelled corn did they get?

D When the boys took their pigs to be sold, they learned that the 13 pigs weighed 2990 pounds. What was the average weight of the pigs?

E Sally canned 64 pt. of applesauce and 95 qt. of tomatoes for a 4-H project. She canned how many quarts of applesauce and tomatoes in all?

THINK 64 pt. = ▩ qt. What should you do next?

F Sally canned how many more quarts of tomatoes than quarts of applesauce?

G Sally's mother said that she would use about 2 qt. of applesauce per week. The applesauce that Sally had canned would last about how many weeks?

H Sally and Mary baked 16 loaves of bread for the 4-H Club fair. 9 of the loaves were white bread, and the others were raisin bread. They baked how many loaves of raisin bread?

I As a 4-H project, Jim and Kathy planned a family budget for a year. They figured that their family would spend an average of \$95 per month for food. How much did they think their family would spend for food in one year?

J Jim and Kathy allowed \$500 for clothing for the year. They allowed an average of about how much per month for clothing?

THINK Which is the better answer, \$41 or \$42? Why?

K The family's average income per month is \$325. About how much money will be left each month after they have paid for food and clothing?

THINK Which problem tells you about how much they will spend per month for food? How do you know about how much they will spend for clothing?

124

1 Let the pupils work Problems A to K independently. Point out that some problems require use of numbers given in previous problems, and for some of them, missing numbers must be found by computation before the answer can be found. Tell the children first to write an equation with *n* for the unknown number (and a screen if the problem is a multiple-step one), then to compute and rewrite the equation with the answer.

L Jim and Kathy think their family will spend about how much less in one year for clothing than for food?

M In the spring Jim had 120 tomato plants to set out. He set out some of the plants one morning. At noon he still had 45 plants to set out. How many plants did he set out in the morning?

N Jim planted the 120 tomato plants in rows of 15 plants each. How many rows did he set out?

O Jim hoped that his tomatoes would be ready to eat in about 8 weeks. He hoped that they would be ready in about how many days?

P One day he said that his tomatoes should be ready in about 2 wk. About how many days had it been since Jim had planted the tomatoes?

Q Kathy took care of 30 chickens for her 4-H project. Her chickens laid 21 eggs one day, 17 eggs the second day, 23 eggs the third day, 18 eggs the fourth day, and 16 eggs the fifth day. They laid an average of how many eggs per day?

R Use this average to find about how many eggs Kathy's chickens would lay in one month of 30 days.

S One month the family used 11 doz. of the eggs that Kathy's chickens had laid. Kathy still had 34 doz. eggs left to sell. How many dozen eggs had her chickens laid that month?

Keeping skillful

3

A 16 fl. oz. = ▩ tbs.
B 108 in. = ▩ ft.
C 32 dry pt. = ▩ pk.
D 5 gal. = ▩ pt.
E 208 wk. = ▩ yr.
F 4 hr. = ▩ min.
G 96 oz. = ▩ lb.
H ▩ pk. = 4 bu.
I 4 c. = ▩ fl. oz.
J ▩ da. = 72 hr.
K 8 T. = ▩ lb.
L 64 dry qt. = ▩ bu.
M ▩ sec. = 3 min.
N 56 da. = ▩ wk.
O 108 ft. = ▩ yd.
P 5 yr. = ▩ mo.
Q ▩ pt. = 4 c.
R 40 dry qt. = ▩ pk.
S 5 yd. = ▩ in.
T 4 yr. = ▩ da.
U ▩ mi. = 5280 ft.
V 5 c. = ▩ fl. oz.
W 8 lb. = ▩ oz.
X ▩ hr. = 7 da.

4

A $n + 628 = 1115$
B $6208 \div n = 97$
C $8000 - 129 = n$
D $76 \times 403 = n$
E $n - 624 = 1002$
F $604 \div 19 = n$
G $143 + 69 + 107 = n$
H $536 + n = 724$
I $1316 \div n = 28$
J $n + 21030 = 41762$
K $249 \times 836 = n$
L $20100 - n = 4215$

125

125

1. Let the pupils work Problems L to S independently.
2. Discuss each problem with the class, beginning with Problem A on page 124. When discussing Problems B and J, be sure to have pupils explain why one answer is better than another.
3. The pupils may work these exercises on paper. Some of the most competent pupils may prefer to take turns doing them orally. You may want to allow the children to use the table on page 93 for reference.
4. Use this block of exercises for those children who need more practice in computation.

126-131 Moving forward

Expanded Notes for this lesson are on pages 302-304.

Objectives

The child learns that a pair of numerals may be used to express a fraction and how to read and write such a fraction numeral. He also learns that different pairs of numbers may be used for the same fraction.

Vocabulary

New words page 126 denominator, numerator; page 129 green*; page 131 arranged*

Comments

This is the third lesson in which the children learn about the uses of number pairs. On pages 112-117 the children learned that a pair of numerals may be a ratio that expresses a rate. On pages 119-123 they learned that a pair of numerals may also be a ratio that expresses a comparison. Now they are going to learn that a pair of numerals may be a fraction numeral that expresses a fraction.

From the beginning of the study of arithmetic children use expressions for fractions, rates, and comparisons, but they do not fully understand their meanings, nor do they know how to write them with numerals. In this book all of these number pairs are written in the same way—one numeral above the other. These lessons are designed to help the child become familiar with this form and the three different meanings.

When the child studied rates, he learned that the ratio is merely a way to symbolize or express the rate and that several different ratios can be used to express the same rate. Again, when he studied comparisons, he learned that more than one ratio can be used to symbolize the result of a comparison. In this lesson the child learns that a fraction represents part of a whole, that the fraction numeral is a way to symbolize the fraction, and that more than one fraction numeral can be used to symbolize or express the same fraction. For example, $\frac{4}{8}$, $\frac{2}{4}$, $\frac{1}{2}$, and $\frac{8}{16}$ all express the same fraction. At this point, no attempt

should be made to develop the computational procedure for the reduction (or changing) of fraction numerals. That will be done later. In this lesson the main idea to be developed is that the same fraction can be expressed in different ways.

On pages 126-128 the idea of fractions of wholes, or units, or things that can be thought of as one is developed. Picture F on page 128 shows how different fraction numerals may be used to express the same fraction.

On pages 129-131 the fraction idea is used to represent the relation of part of a group of discrete objects to the whole group. Here the group is thought of as a whole, or unit, and consequently, when referring to it, do not be concerned with the number of objects in it. Likewise, when referring to the part of the group, do not be concerned with the number of objects. This concept needs careful attention; it is of great importance.

Use orally as much as possible the expressions *fraction numeral, denominator,* and *numerator,* which are used throughout this lesson.

Answers

Page 131:

A $\frac{2}{3}$, $\frac{4}{6}$, $\frac{8}{12}$

B $\frac{3}{4}$, $\frac{6}{8}$

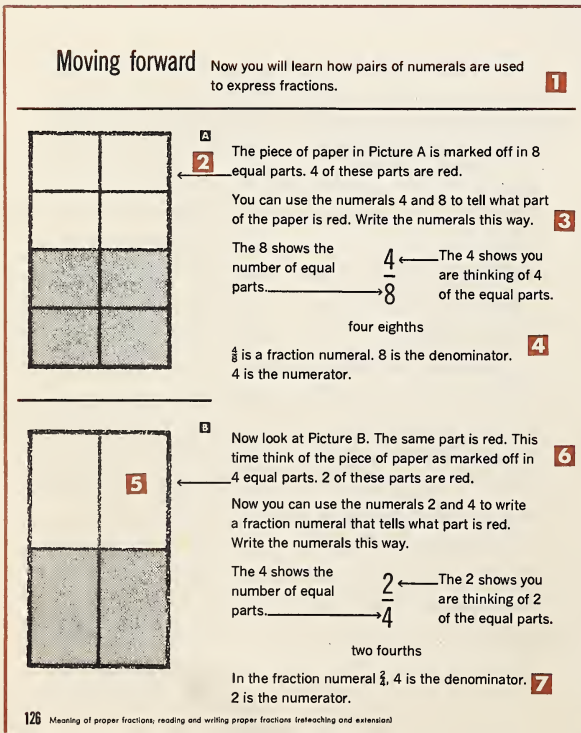
C fifteenths; $\frac{4}{5}$, $\frac{12}{15}$

D tenths, twentieths; $\frac{3}{5}$, $\frac{6}{10}$, $\frac{12}{20}$

E ninths; $\frac{2}{3}$, $\frac{6}{9}$

F fourths, eighths; $\frac{1}{4}$, $\frac{2}{8}$

G [G] $\frac{2}{10}$; [H] $\frac{2}{3}$, $\frac{4}{6}$, $\frac{12}{18}$; [I] $\frac{1}{2}$, $\frac{2}{4}$, $\frac{4}{8}$



Moving forward

Now you will learn how pairs of numerals are used to express fractions. 1

A

2 The piece of paper in Picture A is marked off in 8 equal parts. 4 of these parts are red.

You can use the numerals 4 and 8 to tell what part of the paper is red. Write the numerals this way. 3

The 8 shows the number of equal parts. → $\frac{4}{8}$ ← The 4 shows you are thinking of 4 of the equal parts.

four eighths

$\frac{4}{8}$ is a fraction numeral. 8 is the denominator. 4 is the numerator. 4

B

Now look at Picture B. The same part is red. This time think of the piece of paper as marked off in 4 equal parts. 2 of these parts are red. 6

Now you can use the numerals 2 and 4 to write a fraction numeral that tells what part is red. Write the numerals this way.

The 4 shows the number of equal parts. → $\frac{2}{4}$ ← The 2 shows you are thinking of 2 of the equal parts.

two fourths

In the fraction numeral $\frac{2}{4}$, 4 is the denominator. 2 is the numerator. 7

126

1 Read the "Moving forward" statement and discuss it. Remind the children that they used two numerals to express rate and comparison.

2 Ask into how many equal parts the piece of paper in Picture A has been divided. Ask how many of the equal parts are red. Have someone read the first two sentences.

3 Ask another pupil to read the text that follows. Have the children note the placement of the numerals that represent the number of equal parts (the denominator) and the number of parts under consideration (the numerator).

4 Have someone read the fraction numeral. Be sure the children understand the terms *fraction numeral, denominator,* and *numerator.*

5 Tell them this piece of paper is the same size as that in Picture A, but it is divided into a different number of equal parts. Have them note that the same part of the paper is red.

6 Adapt the procedures in Note 3.

7 Adapt the procedures in Note 4.

C

There are other fraction numerals you can use to tell that the same part of the paper is red. Look at Picture C. This time think of the piece of paper as marked off in 2 equal parts. 1 of these parts is red.

This time you can use the pair of numerals 1 and 2 to tell what part is red.

The 2 shows the number of equal parts. $\frac{1}{2}$ ← The 1 shows you are thinking of 1 of the equal parts.

one half

2 is the denominator of this fraction numeral.
1 is the numerator.

D

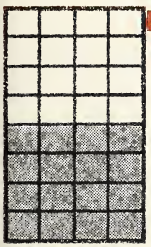
Picture D shows how to get still another fraction numeral that tells what part is red. Think of the piece of paper as marked off in 16 equal parts. 8 of these parts are red.

Now you can use the numerals 8 and 16 to tell what part is red.

The 16 shows the number of equal parts. $\frac{8}{16}$ ← The 8 shows you are thinking of 8 of the equal parts.

eight sixteenths

16 is the denominator of this fraction numeral.
8 is the numerator.



E

Picture E shows the same paper with the same part red. Think of the paper as marked off in 32 equal parts. ▩ of these parts are red.

To tell what part of the paper is red, you can use the numerals 16 and 32.

What does the 32 show? $\frac{16}{32}$ ← What does the 16 show?

sixteen thirty-seconds

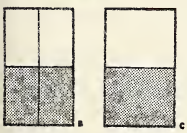
What is the denominator of this fraction numeral? What is the numerator?

F

Here are the five pictures you have been working with. In each picture the same part of the paper is red. The same fraction of the paper is red.

The fraction numeral $\frac{4}{8}$ tells what fraction of Picture A is red. What fraction numeral tells what fraction of Picture B is red?

For each of the other three pictures, write the fraction numeral that tells what fraction is red.



You can use several different fraction numerals to tell what fraction of the paper is red. You can use $\frac{4}{8}$, $\frac{2}{4}$, $\frac{1}{2}$, $\frac{8}{16}$, or $\frac{16}{32}$.

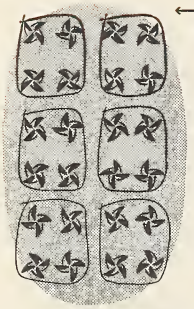
$\frac{4}{8}=\frac{2}{4}=\frac{1}{2}=\frac{8}{16}=\frac{16}{32}$

127

1. Have the children see that again the same part of the same-sized paper is red, but this time the paper has been marked off in two equal parts. Ask one or more children to read the text.
2. Call on someone to read the text and explain the meaning of the numbers used to form the fraction.
3. Read the fraction numeral together several times. Be sure everyone understands what numerator and denominator mean.
4. Adapt the procedures in Note 1.
5. Adapt the procedures in Note 2.
6. Adapt the procedures in Note 3.

128

1. Ask the children to study Picture E and be ready to supply the numeral for the screen.
2. Ask, "What do 16 and 32 stand for in this situation?"
3. Have someone read the fraction numeral. Then let someone answer the questions.
4. Let one of the pupils read the text. Explain that these are the same pieces of paper they have been studying. Be sure they understand that, although each paper is divided into a different number of equal parts, the same part is red.
5. Let a child read the text and give orally a fraction numeral that tells what fraction of Picture B is red.
6. Tell the children to write fraction numerals for Pictures C, D, and E. Discuss the answers.
7. Be sure the pupils know that all five fraction numerals express the same fraction. Relate each fraction numeral to its own picture.



G

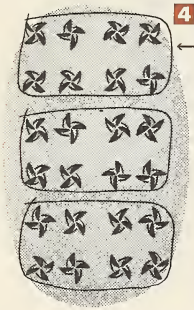
You can also use different fraction numerals to express a fractional part of a group. Look at Picture G. First think of the pinwheels as one big group. The rings show that you are to think of the pinwheels as separated into 6 equal groups. 4 of these equal groups are green. 1

You can tell what fraction of the pinwheels is green by writing a fraction numeral. Use the numerals 4 and 6. 2

The 6 shows the number of equal groups. → $\frac{4}{6}$ ← The 4 shows you are thinking of 4 of the equal groups.

four sixths 3

6 is the denominator of the fraction numeral $\frac{4}{6}$.
4 is the numerator.



4 H

Picture H shows the same pinwheels. Now think of them as separated into 3 equal groups. 2 of the equal groups are green. 5

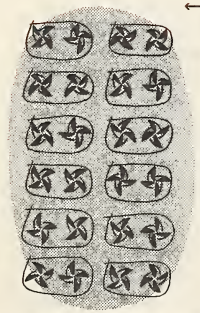
Use the numerals 2 and 3 to tell what fraction of the pinwheels is green.

The 3 shows the number of equal groups. → $\frac{2}{3}$ ← The 2 shows you are thinking of 2 of the equal groups.

two thirds 6

■ is the denominator of this fraction numeral.
■ is the numerator.

Now turn the page.



I

Picture I shows how to get another fraction numeral that tells what fraction of the pinwheels is green. Now think of the pinwheels as separated into 12 equal groups. ■ of the equal groups are green. 1

The 12 shows that the pinwheels are separated into ■ equal groups. 2 → $\frac{8}{12}$ ← The 8 shows you are thinking of ■ of the equal groups.

3 eight twelfths

■ is the denominator of this fraction numeral.
■ is the numerator.

G

H

I

4 J

Here are the three pictures of pinwheels you have been working with. In each picture the same fraction of the pinwheels is green.

The fraction numeral $\frac{4}{6}$ tells what part of the pinwheels in Picture G is green. What fraction numeral tells what part in Picture H is green? 5

What fraction numeral tells what part of the pinwheels in Picture I is green?

You can use several different fraction numerals to tell what fraction of the pinwheels is green. You can use $\frac{■}{■}$, $\frac{■}{■}$, or $\frac{■}{■}$. 6

$\frac{4}{6}=\frac{2}{3}=\frac{8}{12}$

129

1 Have one of the children read the text to the class. Point out that the dotted background means the pinwheels are to be thought of as one big group. Have the pupils note that 4 of the 6 equal groups of pinwheels are green.

2 Be sure the children understand that each number in the pair relates to certain *groups* and not to the objects.

3 Let the children read the fraction numeral and the terms relating to it.

4 Point out that this time the whole quantity of pinwheels has been divided into a different number of equal groups, but that the number in each group is still equal.

5 Have one or more children read the text and explain the fraction numeral representing the groups of green pinwheels.

6 Ask a pupil to read the fraction numeral and supply the correct numerals in place of the screens in the last two sentences.

130

1 Let someone read the text and supply the missing answer. Have all the children count the number of green groups and the number of black groups.

2 Let another child read the text, supply the answers, and explain the fraction numeral.

3 Have a pupil read the fraction numeral and supply the missing numerals in the sentences below.

4 Tell the children to study the three pictures and note that in each the same fraction of pinwheels is green, no matter how many equal groups the pinwheels have been divided into.

5 Call on pupils to read the text and answer the questions.

6 Tell the children to relate these statements to the three pictures at the left and then supply the numerals in place of the screens. Stress that more than one fraction numeral can be written to express the same fraction.

A The paper in Picture A has been divided to show thirds, sixths, and twelfths. Write three fraction numerals that tell what fraction of the paper is green.

B The plants in Picture B are arranged to show fourths and eighths. Write two fraction numerals that tell what fraction of the plants are in green pots.

C Picture C shows fifths and ——. Write two fraction numerals that tell what fraction of the piece of paper is green.

D Picture D shows fifths, ——, and ——. Write three fraction numerals that tell what fraction of the piece of paper is green.

E The plants in Picture E are arranged so that you can see thirds and ——. Write two fraction numerals that tell what fraction of the plants are in green pots.

F Picture F shows —— and ——. Write two fraction numerals that tell what fraction of the paper is green.

G For each picture below, write as many fraction numerals as you can that tell what fraction of the piece of paper is green. For Picture G you can write $\frac{1}{5}$ and $\frac{2}{10}$.

Now you should be able to read and write fraction numerals and should understand what they mean.

131

131

1 Study Picture A together. Point out that the paper is divided into parts of three different sizes. Tell the children that three fraction numerals can be written to express the fraction of the paper that is green. Have someone read the text.

2 Help the children see that the plants in green pots are a fraction of all the plants, and that this fraction is really the same when they think of 4 groups of plants as when they think of them as 8 separate plants. Make sure all the children know how to write two fraction numerals to express the fraction of the plants that is green.

3 Discuss together the answers for Exercise C.

4 Let the children work independently on Exercises D to G. Then discuss their answers with them and help anyone who makes an error see how to correct his mistake.

132-133 Thinking straight

Expanded Notes for this lesson are on page 304.

Objectives

The child learns to use fractions, rates, and comparisons.

Vocabulary

New words page 132 mine*, Andy's*; page 133 supper*, following*, exercises, meat*, hiked, pie*

Comments

The three lessons just completed have introduced children to three uses of pairs of numbers that are written in the same way. For each of these three uses, the numerals representing these pairs of numbers are written in the way that is commonly thought of as a fraction. Yet such a pair of numbers can be interpreted as a rate, the result of a comparison, or a fraction, and for each it has a different meaning.

This lesson brings the three uses of pairs of numbers together and serves to call the children's attention to the fact that they should be careful in reading numerals that they find written in this way until they know what they stand for. You will have to tell children that until they know what the situation actually is, they cannot even be sure of how to read the numerals. At first it is suggested that they should read a pair of numerals like $\frac{5}{6}$ as "five over six" until they are sure of the situation. In a situation where they know that the numerals express a fraction, they will read them as "five sixths." When the pair of numerals expresses a comparison, the pupils should use the language of comparison, and read them as "five to six." If the pair is used to express a rate, pupils should read them as "five per six."

Answers

Page 132:

A The 5 equal parts; the 3 green parts; a fraction numeral; three fifths

B The number of lollipops; the number of pennies, or the cost of 3 lollipops; a ratio; three per five

C Bill's marbles; Andy's marbles; a ratio; three to five

D The money Ann earned; the money Susan earned; a comparison; six to five

Page 133:

E The number of miles driven; number of hours; a rate; 98 per 3

F The number of equal pieces in the cake; number of pieces used; a fraction; seven twelfths

G The number of cans sold; price for 2 cans; a rate; 33 per 2

(block 2)

Children may write the rates and comparisons in either of two ways, 2/9 or 9/2. Only one is shown in the key. Of course, they must read them as they are written.

A $\frac{1}{2}$; fraction; one half

B 9/2; rate; 9 per 2

C 2/5; comparison; 2 to 5

D 2/37; rate; 2 per 37

E $\frac{10}{16}$; fraction; ten sixteenths

F 8/16; comparison; 8 to 16

G 2/3; rate; 2 per 3

H 4/7; comparison; 4 to 7

I $\frac{3}{6}$; fraction; three sixths

J $\frac{4}{5}$; fraction; four fifths

K 5/2; rate; 5 per 2

L 30/50; comparison; 30 to 50

M $\frac{4}{6}$; fraction; four sixths

N $\frac{1}{8}$; fraction; one eighth

O Fraction numeral: A, E, I, J, M, N
Ratio: B, C, D, F, G, H, K, L

Thinking straight

Look at each of the three pictures at the left. Notice that, in each picture, 3 is written over 5 as one numeral. You cannot tell what this numeral stands for in each picture until you understand the picture. 1

If the numeral is a fraction numeral, you read it "three fifths." If it is a ratio that expresses a rate, you read it "three per five." If it is a ratio that expresses a comparison, you read it "three to five." Until you know what the numeral stands for, you can read it "three over five." 2

A Look at Picture A again. What does the 5 stand for? What does the 3 stand for? Did John write a fraction numeral or a ratio? How do you read the numeral John wrote? 3

B Now look at Picture B. What does the 3 stand for? What does the 5 stand for? Did Susan write a fraction numeral or a ratio? How do you read the numeral Susan wrote?

C Look at Picture C. What does the 3 stand for? What does the 5 stand for? Did Bill write a fraction numeral or a ratio? How do you read the numeral Bill wrote?

D Ann earned $6, and Susan earned $5. In the numeral at the right, what does the 6 stand for? What does the 5 stand for? Does the numeral express a rate, a comparison, or a fraction? Read the numeral. 4

$\frac{6}{5}$

132

1 Have the pupils read this paragraph and examine Pictures A, B, and C. Ask them to note in what ways the pictures are alike and in what ways they are different. Then discuss the pictures.

2 Ask a pupil to read this paragraph aloud. Call attention to the fact that before he knows what the pair of numbers stands for it may be read "three over five."

3 Call on the children to answer Questions A, B, and C. If they need help in reading the numeral, let them refer to the paragraph above.

4 If necessary, tell the children that here they are comparing the amount Ann earned with the amount Susan earned. Let them refer to the explanation in the second paragraph again if they need help in reading the numeral.

E Mr. Brown drove 98 mi. in 3 hr. In the numeral at the right, what does the 98 stand for? What does the 3 stand for? Does the numeral express a comparison, a rate, or a fraction? Read the numeral. $\frac{98}{3}$

F Mrs. Brown cut a cake into 12 equal pieces. She used 7 pieces for supper. In the numeral at the right, what does the 12 stand for? What does the 7 stand for? Does the numeral express a rate, a comparison, or a fraction? Read the numeral. $\frac{7}{12}$ 1

G A grocer sold 2 cans of beans for 33¢. In the numeral at the right, what does the 2 stand for? What does the 33 stand for? Does the numeral express a rate, a fraction, or a comparison? Read the numeral. $\frac{33}{2}$

For each of the following exercises, do these things. Write the pair of numerals together as one numeral. Decide whether the numeral expresses a rate, a comparison, or a fraction. Write the words you use to read the numeral. For Exercise A you write: "$\frac{1}{2}$, fraction, one half." 2

A June cut an orange into 2 equal pieces. She ate 1 of the pieces.

B In 2 hours Jim rode 9 mi. on his bicycle.

C Bob had $2, and Paul had $5.

D May's mother bought 2 cans of orange juice for 37¢.

E Susan folded a piece of paper into 16 equal squares. She colored 10 of the squares yellow.

F Nancy made 8 place mats. Mary made 16 place mats.

G Mrs. White used 2 teaspoonfuls of salt for 3 pounds of meat.

H Bill had 4 white mice, and John had 7 white mice.

I Patsy broke a chocolate bar into 6 equal pieces. She gave 3 of the pieces to Carol.

J Don sawed a board into 5 equal pieces. He used 4 of the pieces.

K In 2 hr. the Boy Scout troop hiked 5 mi.

L Andy practiced on his cornet for 30 min. Jim practiced for 50 min.

M June cut a pie into 6 equal pieces. Her family ate 4 of the pieces.

N Mrs. Bridges cut a small cake into 8 equal pieces. Jim ate 1 of the pieces.

O For which of the exercises above did you write a fraction numeral? For which did you write a ratio? 3

133

133

1 For Exercises E, F, and G, have the children read each exercise and answer the questions. If they need help in reading the numerals, let them refer to the second paragraph on page 132.

2 Discuss these directions, and have the class explain why the answer "$\frac{1}{2}$, fraction, one half" is correct for Exercise A. Then assign Exercises A to O as written work.

3 When everyone has finished the exercises, discuss those that caused the most difficulty. For some of the situations, it might be helpful if you draw pictures on the chalkboard or use objects.

134-135 Using arithmetic

Expanded Notes are not considered necessary for this lesson.

Objectives

The child solves problems of the types taught thus far. In the "Keeping skillful" exercises, he uses equations based on additive and subtractive situations. He maintains his skills in the addition of two-figure, three-figure, and four-figure numbers. He also uses equations based on multiplicative and divisive problem situations.

Vocabulary

New words page 134 expensive, least*, album, developed*, allowance, grandfather*

Comments

Each problem in this set should be thought through and solved by the children before any general class discussion takes place. This procedure keeps the children from losing interest. If the work has been discussed, it presents no challenge.

Be sure that the children write an equation for each problem. This compels them to think about what is happening in the problems before they compute. The equations they write also show very clearly what their thinking has been. Individual and common difficulties can be cleared up by discussion after the pupils have finished the problem set.

In the discussion, point out that information is given in Problem D that need not be used in solving the problem. Some children may treat it as a multiple-step problem. Point out that the answer can be found by using the information given about one roll of film.

The "Keeping skillful" exercises may take considerable rime. Assign only as many exercises from each block of work as you see fit.

Answers

Using arithmetic:

In these answers the numeral that replaces *n* in the equation is bracketed. The numeral that replaces a screen is enclosed in parentheses.

A $2.89 + $5.49 + $3.98 + $2.96 = [$15.32]
B $5.49 − $2.89 = [$2.60]
C 4 × $1.89 = [$7.56]
D $.47 ÷ [$.05] = 8 and $.07 rm.; about $.06
E $10.00 − $3.79 = [$6.21]
F $8.00 ÷ ($2.00) = [4], 4 weeks
G $8.00 ÷ $.75 = [10] and $.50 rm., about 11 weeks
H [$1.25] − ($.90) = $.35
I ($1.60) ÷ [$.40] = 4
J [$3.30] + $2.00 = $5.30
K $5.30 + [$2.70] = $8.00
L $3.00 − [$.87] = $2.13
M $.87 ÷ [$.29] = 3
N (24) − 4 = [20]
O 18 + 10 + 15 + 12 = [55]
P 3 × $.40 = [$1.20]
Q $1.20 ÷ [$.08] = 15
R 112 + [23] = 135

Keeping skillful:

(block 1)

A 1224	G 7801	M 13,482	S 36,262
B 3609	H 250	N 901	T 8723
C 39	I 7760	O 368	U 12,777
D 275	J 26,576	P 13,258	
E 2473	K 3420	Q 13,733	
F 5542	L 1345	R 32,109	

(block 2)

A 185	F 1241	K 285	P 2298
B 1331	G 483	L 819	Q 1361
C 1053	H 1111	M 1520	R 2768
D 7387	I 224	N 228	S 6824
E 627	J 1716	O 1819	T 2520

(block 3)

A 65,096	H 89,856	O 39,576
B 66	I 128 and 17 rm.	P 65
C 70 and 59 rm.	J 276	Q 183,396
D 424,296	K 689,640	R 384 and 4 rm.
E 48	L 103 and 12 rm.	S 782,460
F 95	M 58	T 508
G 53,465	N 318 and 65 rm.	U 513,400

Using arithmetic 1

A Jim, Bob, Bill, and Tony each had a camera. Their cameras cost $2.89, $5.49, $3.98, and $2.96. How much did all the cameras cost?

B The most expensive camera cost how much more than the least expensive camera?

C Each of the boys bought an album for pictures. Each album cost $1.89. How much did all four albums cost?

D Tony sent 2 rolls of pictures to be developed and printed. There were 8 pictures on each roll. The cost was $.47 per roll. About how much did it cost to develop and print each picture?

E Bob was given $10 for his birthday. He bought a developing and printing set for $3.79. How much of the $10 did he have left?

F Jim planned to save his earnings and allowance to buy a better camera. His allowance was $.75 a week, and he earned $1.25 a week. How long would it take him to save enough to buy a camera priced at $8?

G Jim found that he could save only $.75 each week. At this rate, about how long would it take him to save enough to buy the camera he wanted?

H Bill earned some money by raking leaves. On his way home, he bought 2 rolls of film for 45¢ a roll. Then he had $.35 left. How much money had he earned by raking leaves?

I Bob paid 75¢ for 2 rolls of film, and Jim paid 85¢ for 2 rolls. What was the average cost per roll of the four rolls?

J After Jim had saved some money, his grandfather gave him $2. Then Jim had $5.30. How much money had Jim saved before his grandfather gave him the $2?

K How much more money did Jim need before he could buy the camera that he wanted?

THINK What was the price of the camera?

L Tony earned $3 one week. He bought 3 rolls of film. Then he had $2.13 left. How much had Tony paid for the film he bought?

M What did Tony pay per roll for the film he bought?

N Tony sent 3 rolls of film to be developed and printed. There were 8 pictures on each roll. When the pictures came back, he said, "Four of my pictures are spoiled." How many of his pictures were not spoiled?

134

1 Ask the children to work independently on Problems A to R (O to R are on page 135). Direct them first to write the equation with the letter *n* holding a place for the unknown number (and a screen or screens if the problem is a multiple-step one), then to compute, and, finally, to rewrite the equation with the answer in place of the letter *n*. Have them label their work for each problem with its identifying letter.

O Each of the boys brought his best pictures to school. Jim brought 18, Bob brought 10, Bill brought 15, and Tony brought 12. How many pictures in all did the boys bring to school?

P Bob took a good picture of his dog. Then he had 3 large pictures made. They cost 40¢ each. How much did the 3 large pictures cost?

Q Jim had 15 prints made of one of his best pictures. He paid $1.20 for the 15 prints. How much did he pay per print?

R Jim had 112 pictures in his album. He put in more pictures, and then he had 135 pictures in his album. How many pictures had he added to his album?

1

Keeping skillful 2

	Find the sum.	
A $n+2876=4100$	A 28, 64, 77, 16	A $206\times316=n$
B $6004-2395=n$	B 586, 29, 716	B $5544\div n=84$
C $101-n=62$	C 338, 214, 501	C $6569\div93=n$
D $529+n=804$	D 6218, 327, 842	D $568\times747=n$
E $n-680=1793$	E 95, 103, 429	E $1776\div n=37$
F $4637+905=n$	F 287, 624, 330	F $7220\div76=n$
G $n-1570=6231$	G 54, 100, 27, 302	G $85\times629=n$
H $775+n=1025$	H 704, 325, 82	H $208\times432=n$
I $6276+1484=n$	I 66, 97, 13, 48	I $7441\div58=n$
J $48253-21677=n$	J 1002, 498, 216	J $25392\div n=92$
K $n+6220=9640$	K 99, 104, 62, 20	K $840\times821=n$
L $2345-n=1000$	L 18, 46, 703, 52	L $4853\div47=n$
M $n-7482=6000$	M 746, 214, 560	M $4698\div n=81$
N $1747-n=846$	N 42, 68, 105, 13	N $27413\div86=n$
O $8632+n=9000$	O 664, 215, 940	O $97\times408=n$
P $75040-61782=n$	P 831, 269, 403, 795	P $5655\div n=87$
Q $n+57601=71334$	Q 128, 374, 299, 560	Q $348\times527=n$
R $27931+n=60040$	R 678, 925, 379, 786	R $7300\div19=n$
S $84010-n=47748$	S 2738, 694, 3392	S $805\times972=n$
T $n-3641=5082$	T 822, 340, 507, 851	T $7112\div n=14$
U $61712-48935=n$		U $680\times755=n$

135

135

1 Provide the children with equations and answers for Problems A to R (A to N are on page 134) and let them verify their work. Discuss thoroughly the problems that caused trouble. Give individual help where it is needed.

2 Assign only as many of the exercises in each of these three blocks of work as you feel the children can do in the time they have. Provide answers so that the children can verify their work, and discuss any exercises that have caused trouble. You might handle Blocks 2 and 3 in this way: assign Exercises A to J of each block; when completed and verified, make notes of general and individual difficulties and clear these up. Then return to the remaining exercises several days or weeks later—whenever it is convenient.

136-137 Thinking straight

Expanded Notes for this lesson are on pages 304-305.

Objectives

The child learns that dividing or multiplying both the divisor and the dividend by the same number does not change the quotient. He learns to recognize odd and even numbers. He also learns several tests of divisibility that will help him choose common divisors.

Vocabulary

New words page 136 odd*

Comments

The children should understand that dividing or multiplying both the divisor and the dividend by the same number does not alter the quotient, and they should understand that both terms of a fraction can be divided or multiplied by the same number without altering the relationship of the terms. This kind of understanding is basic in division involving decimal fractions and in working with fractions.

This lesson gives several tests of divisibility that are useful in choosing a common divisor. Some of the children may already know that a number divisible by 2 is called "even," or that a number whose numeral ends in 0 or 5 is divisible by 5. Most children will find the tests of divisibility interesting, but they should not be required to memorize them.

Answers

(block 1)

It is not considered necessary to provide answers for Exercises A to L, N, R, S, and T.

M 240; 108; 630

O 532; 136; 220; 424

P 615; 480; 335; 900

Q Both 24 and 168 are even numbers.

(block 2)

A 2, because 82 is an even number

B 3; The sum of its figures (2+3+1) can be divided by 3 without a remainder.

C 5, because 700 ends in zero and all numbers ending

in zero are divisible by 5; 2, because 700 is an even number; also 4, because 700 ends in two zeros and numbers ending in two zeros are divisible by 4

D 5, because 590 ends in zero; also 2, because 590 is an even number

E 5, because 735 ends in 5 and all numbers ending in 5 are divisible by 5; also 3, since the sum of its figures can be divided by 3 without a remainder

F None of these tests of divisibility works for 253.

G 2, because 532 is an even number; also 4, because 32 can be divided by 4 without a remainder

H 2, because 946 is an even number

I 3; The sum of the figures in 87 can be divided by 3 without a remainder.

J 2, because 816 is an even number; 3, because the sum of its figures can be divided by 3 without a remainder; and also 4, because 16 can be divided by 4 without a remainder

K None of these tests of divisibility works for 41.

L 2, because 104 is an even number; also 4, because 4 in 04 can be divided by 4 without a remainder

M 2, because 444 is an even number; 3, because the sum of its figures can be divided by 3 without a remainder; and also 4, because 44 can be divided by 4 without a remainder

N 2, because 74 is an even number

O 3, because the sum of the figures in 225 can be divided by 3 without a remainder; 5, because it ends in 5

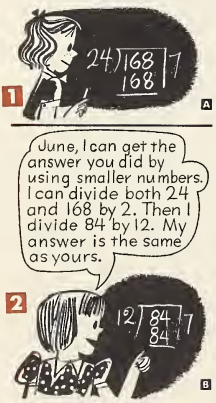
P 2, because 508 is an even number; 4, because the 8 in 08 can be divided by 4 without a remainder

(block 3)

For most of these exercises, there are two or more possible sets of dividends and divisors. Accept any correct sets. Only the quotients are given below.

A	9	C	5	E	8	G	4	I	5	K	36	M	43
B	7	D	8	F	7	H	6	J	5	L	73	N	62

Thinking straight


24 168 7
168
June, I can get the answer you did by using smaller numbers. I can divide both 24 and 168 by 2. Then I divide 84 by 12. My answer is the same as yours.
12 84 7
84
My way is easiest. I can divide both 24 and 168 by 4. Then I divide 42 by 6. I can do that in my head.

A Look at Picture A. Is June's division correct?

B In Picture B, Ann found smaller numbers that she could divide to get the same answer that June found. How did Ann get the 12? The 84?

C In Picture C, how did Nancy get the 42? The 6?

D Notice that in Picture B, Ann first divided both the 168 and the 24 by 2. In Picture C, Nancy first divided the 168 and the 24 by ■.

E Now divide 216 by 24.

F Divide both 216 and 24 by 2. Now divide 108 by 12. Is the answer the same as when you divided 216 by 24?

G Divide both 216 and 24 by 4. Now divide 54 by 6. What do you notice about the answer?

H Imagine that you are going to divide a number by another number. What might you do to the two numbers so that you can work with smaller numbers and still get the same answer?

I To use this way of dividing, you need to know what numbers can be divided by other numbers. Divide 2, 4, 6, 8, and 10 by 2. Is there a remainder when you divide? Numbers that can be divided by 2 without remainders are even numbers. All other whole numbers are odd numbers. Are numbers that end in 2, 4, 6, 8, and 0 even numbers? Why?

J Which numbers below are even numbers? Which are odd numbers?

37 114 69 490 662 75 94 43 701

136 Learning that dividing or multiplying a divisor and dividend by the same number does not change the quotient, choosing a common divisor

130

136

1 Ask the pupils to look at Picture A and verify the computation shown there.

2 After a pupil has read what Ann said in Picture B, let the class answer the related questions in Exercise B.

3 Ask a pupil to read what Nancy says in Picture C. Then call on pupils to answer the questions in Exercises C and D.

4 Let pupils use pencil and paper to divide 216 by 24, as directed in Exercise E. Call on pupils to answer Exercises F and G.

5 Be sure the pupils see that if they divide both dividend and divisor by the *same* number, they will have a smaller dividend and a smaller divisor to work with and the quotient will be the same.

6 After pupils have discussed Exercise I, let them try dividing other even numbers by 2.

7 Be sure the pupils understand that all even numbers are divisible by 2 and all odd numbers are not.

K To decide whether or not a number can be divided by 3 without a remainder, first find the sum of the figures in the number. Let us look at 318. The sum of 3, 1, and 8 is ■. Next divide this sum, 12, by 3. Since there is no remainder when 12 is divided by 3, the number 318 can also be divided by 3 without a remainder. Divide 318 by 3. 1

L Can 117 be divided by 3 without a remainder? Add 1, 1, and 7. What do you do next? Divide 117 by 3.

M Which of the numbers below can be divided by 3 without remainders?
240 95 108 412 630 115

N To decide whether or not a number can be divided by 4 without a remainder, look at just the last two figures. If this part of the number can be divided by 4, the number itself can also be divided by 4. The last two figures in 216 are 16. There is no remainder when 16 is divided by 4. The number 216 can also be divided by 4. Divide 216 by 4. 2

O Which of the numbers below can be divided by 4 without remainders?
532 119 306 136 220 424

P Any number that ends in 0 or 5 can be divided by 5. Which numbers below can be divided by 5?
212 615 87 480 335 900 3

O In Picture B, how did Ann know that she could divide both 24 and 168 by 2? 4

R What is the answer when you divide 54 by 9? Multiply both 54 and 9 by 2. Now divide 108 by 18. Is the answer the same as when you divide 54 by 9? 5

S What is the answer when you divide 72 by 8? Multiply both 72 and 8 by 4. Now divide 288 by ■. Is the answer the same as when you divide 72 by 8?

T Now multiply both 72 and 8 by 7. Divide 504 by 56. Is the answer the same as when you divide 72 by 8?

For each number below, tell whether you can divide it by 2, by 3, by 4, or by 5 without a remainder. For each number explain how you know. 6

A	82	E	735	I	87	M	444
B	231	F	253	J	816	N	74
C	700	G	532	K	41	O	225
D	590	H	946	L	104	P	508

Look at each division example below. For each one find the answer by using other numbers. 7

A	324 ÷ 36 = n	H	162 ÷ 27 = n
B	280 ÷ 40 = n	I	120 ÷ 24 = n
C	125 ÷ 25 = n	J	250 ÷ 50 = n
D	192 ÷ 24 = n	K	504 ÷ 14 = n
E	128 ÷ 16 = n	L	3066 ÷ 42 = n
F	252 ÷ 36 = n	M	1935 ÷ 45 = n
G	160 ÷ 40 = n	N	1860 ÷ 30 = n

137

137

1 Exercises K, L, and M demonstrate that a number can be divided by 3 if the sum of the figures in its numeral can be divided by 3. Call on various pupils to answer the questions in these exercises.

2 Exercise N explains that if the last two figures in a number can be divided by 4, then the whole number can. Call on pupils to answer Exercise O.

3 Have the pupils test the numbers provided in Exercise P for divisibility by 5.

4 Pupils should now know that both 24 and 168 are even numbers and so can be divided by 2.

5 Exercises R, S, and T show pupils that answers are not changed when both dividend and divisor are multiplied by the same number. Discuss the questions.

6 Treat these exercises orally.

7 Let the pupils compute on paper. Ask able pupils to use all the tests of divisibility given in this lesson.

138-139 Exploring problems

Expanded Notes are not considered necessary for this lesson.

Objectives

The child learns to solve multiplicative problem situations in which the product is known and either the size of the equal groups or the number of equal groups is unknown.

Vocabulary

New words page 139 pitcher*

Comments

Multiplicative problem situations arise in which the unknown number is not the product but is either the number of equal groups or the number in each group. This lesson gives the child some experience with such situations.

Problem A on page 138 describes a situation in which the number of equal groups is not known, but the size of the equal groups (8 doughnuts) and the numeral representing the number of doughnuts (24) are known. Children will readily see why the situation is multiplicative (the total quantity is accumulated by opening equal-sized packages and putting the doughnuts together). Consequently, the equation $n \times 8 = 24$ describes this situation.

Dramatization with objects should help to show that while the situation is multiplicative the solution depends on another kind of action. If possible, prepare in advance envelopes each containing 8 objects (buttons or pieces of oaktag or paper will do). Prepare at least 7 envelopes. Show the class the pile of envelopes, and explain that there are 8 objects in each. Ask them to imagine that each envelope is a package of 8 doughnuts. Have someone open envelopes and combine groups of 8 until there are 24 objects in all. The class can see that this is multiplicative action and that the equation must be $n \times 8 = 24$.

To find the numeral to replace *n*, the children must see that divisive action is involved. Action with the objects will lead them to see that finding the number

of equal groups when the numbers representing the total and the size of the equal groups are known is a matter of separating the total (24) into groups of the designated size (8). These groups can be put back into packages and the number of packages determined. The computational process, of course, is division. In the pupils' book divisive action is suggested by putting rings around equal groups.

Problem B describes the problem situation in which the number of equal groups and the number representing the total are known, but the size of the equal groups is unknown.

Dramatization with objects will again show why the equation $4 \times n = 48$ describes the action that produced the final grouping. Then it is possible to show that to find the numeral to replace *n* (the size of the equal groups), the 48 buns may be put back into the 4 packages, so that, finally, the 4 packages have the same number of buns in them. By computation the answer is found by dividing 48 by 4. This partitive division comes about because every time one bun is put in each of the four packages 4 buns are used. The 48 buns, in other words, are used 4 at a time.

Answers

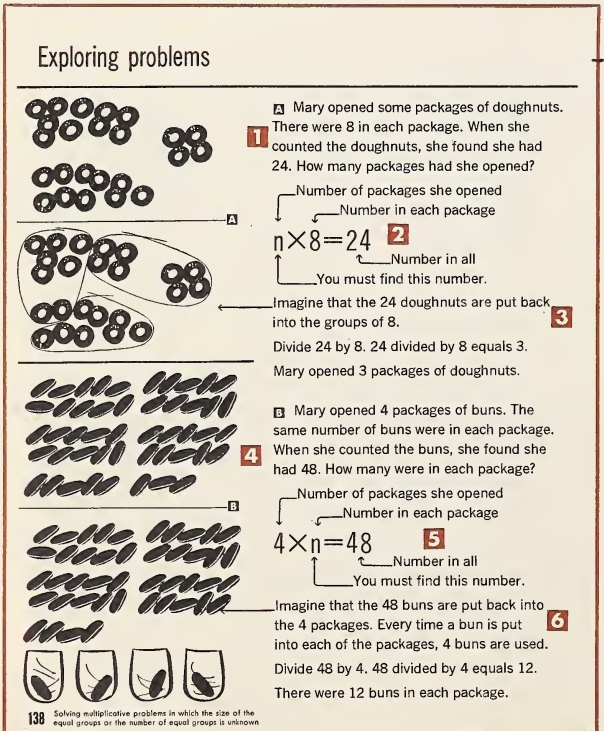
Page 139:

(block 1)

C 8	F 2
D 4	G [36]×$.05=$1.80
E 12	H 6×[$.40]=$2.40

(block 2)

A [5]×32=160	F 12	K 6
B 7×[16]=112	G 5	L 29
C 10×[35]=350	H 9	M 14
D 5	I 46	N 84
E 4	J 5	O 21


Exploring problems
A Mary opened some packages of doughnuts. There were 8 in each package. When she counted the doughnuts, she found she had 24. How many packages had she opened?
1
Number of packages she opened
Number in each package
n×8=24
2
Number in all
You must find this number.
Imagine that the 24 doughnuts are put back into the groups of 8.
3
Divide 24 by 8. 24 divided by 8 equals 3.
Mary opened 3 packages of doughnuts.
B Mary opened 4 packages of buns. The same number of buns were in each package. When she counted the buns, she found she had 48. How many were in each package?
4
Number of packages she opened
Number in each package
4×n=48
5
Number in all
You must find this number.
Imagine that the 48 buns are put back into the 4 packages. Every time a bun is put into each of the packages, 4 buns are used.
6
Divide 48 by 4. 48 divided by 4 equals 12.
There were 12 buns in each package.
138 Solving multiplicative problems in which the size of the equal groups or the number of equal groups is unknown

138

1. Have the pupils relate this problem to the first scene of Picture A.
2. Help them see that this equation fits the problem situation. Use objects to demonstrate that the action of accumulating the total quantity (24) is multiplicative.
3. Get the pupils to see that to find the numeral to replace *n*, the multiplication must be "undone." To accomplish this, the 24 doughnuts can be separated into groups of 8 and put back into packages. The rings around groups of 8 in Scene 2 suggest division.
4. Read the problem and relate it to the picture.
5. Discuss the equation and demonstrate the multiplicative action with objects.
6. Make clear that 4 buns have been put back into the packages. If this is continued, a group of 4 will be separated from the 48 each time until all the buns have been put in packages. This suggests partitive division.

C In Problem A you found what number 8 must be multiplied by to get 24. To find this, you divided 24 by ■.

D In Problem B you found what number must be multiplied by 4 to get 48. To find this, you divided 48 by ■.

E Sally emptied 5 bags of candy into a dish. Each bag held the same number of pieces. There were 60 pieces in all. How many pieces did each bag hold?

Number of groups.
Number in each group.
Number in all.

$5 \times n = 60$

You must find this number.

What number must be multiplied by 5 to get 60? Divide 60 by 5.

F Mrs. White poured juice from bottles into a pitcher. Each bottle held 24 fl. oz. Then there were 48 fl. oz. in the pitcher. How many bottles of juice had she poured out?

Number of groups.
Number of ounces in each group.
Number of ounces in all.

$n \times 24 = 48$

You must find this number.

To get 48, 24 must be multiplied by what number? Divide 48 by 24.

G For a number of days, Kathy put 5¢ a day into a bank. When she counted the money in the bank, she had $1.80. For how many days had she put 5¢ a day into the bank?

THINK $1.80 equals how many cents? By what number must you multiply 5 to get 180? Divide 180 by 5.

H For 6 days, Tony earned the same amount each day. Then he found that he had earned $2.40 in all. How much had he earned on each of the 6 days?

THINK $2.40 equals how many cents? What number must be multiplied by 6 to get 240? Divide 240 by 6.

A By what number must you multiply 32 to get 160?

B What number must be multiplied by 7 to get 112?

C What number must be multiplied by 10 to get 350?

D	$n \times 15 = 75$	J	$17 \times n = 85$
E	$21 \times n = 84$	K	$n \times 50 = 300$
F	$9 \times n = 108$	L	$33 \times n = 957$
G	$n \times 38 = 190$	M	$n \times 65 = 910$
H	$43 \times n = 387$	N	$n \times 5 = 420$
I	$n \times 7 = 322$	O	$58 \times n = 1218$

You have learned how to make new kinds of equations for problems about equal groups.

139

139

1 If the children have trouble in answering Exercise C, refer back to Picture A on page 138. If necessary, use Picture B when discussing Exercise D. You may need to use objects to clarify the ideas in these exercises.

2 Have the pupils read this problem and then discuss the equation. Use objects, if you think it necessary, before the children do any computation.

3 Discuss the equation in terms of the problem. You may want to use objects before any computation is attempted.

4 Discuss Problems G and H. Have the equations written on the chalkboard, and talk about what each part of the equation stands for. Then the class may compute to find the answers.

5 Assign Exercises A to O as written work. Tell the pupils to show all the computation on their papers. For Exercises A, B, and C, they should also show the equations.

140-142 Moving forward

Expanded Notes for this lesson are on pages 306-308.

Objectives

The child learns how to find other pairs of numbers to express a rate or comparison by multiplying or dividing both numbers whose numerals form a ratio by the same number. He learns how to reduce a ratio to an equal ratio when a term is missing in the reduced ratio.

Vocabulary

There are no new words.

Comments

Up to this time the children have learned to write and read ratios that express rates, to write and read ratios that express comparisons, and to write and read fraction numerals. On pages 132-133 the children learned to distinguish among the three types of situations. Now they are ready to begin the computational procedures for reducing ratios and fraction numerals. The word "reduce" is used when a fraction numeral or ratio is changed to an equal fraction numeral or ratio. This lesson deals with the reduction of ratios.

The children have been taught that more than one ratio can be written to express the same rate or comparison. They must now realize that when a ratio is reduced it still stands for the same rate or expresses the same comparison. All they have done is to use a new pair of numerals to express the rate or comparison.

By means of pictures on pages 112-123, the children were shown how to divide groups of objects into equal subgroups and from them find new ratios to express the rate or comparison. Although the procedures in this lesson switch to computational methods of finding equal ratios, you will still want to use pictures on the chalkboard to help the children understand that multiplying or dividing both numbers represented by the terms of a ratio accomplishes the same thing. You can also use objects to clarify the computational methods.

An important point to stress when teaching the computational procedures is that *both* numbers represented

by the terms of a ratio must be multiplied or divided by the same number. For example, if John buys 4 pencils for 10 cents, it is not the numbers 4 and 10 that are important, but the rate 4 per 10. If John buys only 2 pencils, he pays only 5 cents; if he buys 8 pencils, he pays 20 cents.

The same thing is true for comparisons. If Tim's 12 planes are being compared to Ned's 10 planes, we are concerned with the relation of 12 to 10. As long as we are concerned with the comparison of 12 to 10, the numeral in one term cannot be changed unless the numeral representing the other term is changed in the same way. Since the 12 and 10 are used together to express a comparison, to express the same comparison based on only 6 of Tim's, we should think: "12 divided by 2 equals 6. So we must also divide 10 by 2 to get the other number. It will be 5."

Answers

Page 141:

(block 2)

A 8/10, 4/5 B 3/4 C 12/10, 6/5
D 15/10, 3/2 E 1/3 F 5/3
G 3/7 H 8/2, 4/1
I to T: Innumerable ratios can be written.

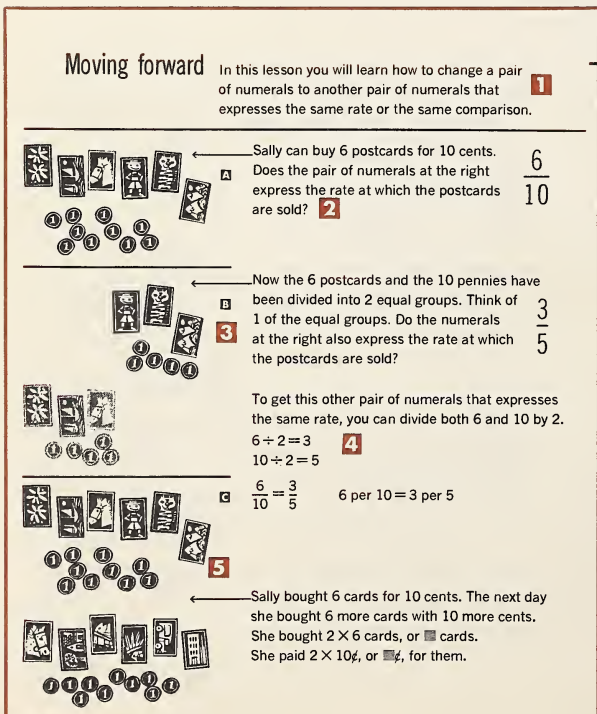
Page 142:

(block 2)

A 7	D 1	G 14	J 2	M 3	P 45
B 6	E 2	H 75	K 8	N 8	Q 45
C 15	F 10	I 3	L 8	O 3	R 46

Keeping skillful:

(block 1)	(block 2)	(block 3)
A 156	A 279	A 413
B 218	B 2411	B 3040
C 199	C 5274	C 36
D 111	D 7011	D 4891
E 301	E 7747	E 54
F 793	F 29,121	F 17 and 9 rm.
G 985	G 33,410	G 76
H 1422	H 2157	H 411 and 6 rm.
I 2392	I 13,771	I 803
J 2224	J 24,417	J 4680


Moving forward
In this lesson you will learn how to change a pair of numerals to another pair of numerals that expresses the same rate or the same comparison.
1
A
Sally can buy 6 postcards for 10 cents. Does the pair of numerals at the right express the rate at which the postcards are sold?
2
6/10
B
3
Now the 6 postcards and the 10 pennies have been divided into 2 equal groups. Think of 1 of the equal groups. Do the numerals at the right also express the rate at which the postcards are sold?
3/5
To get this other pair of numerals that expresses the same rate, you can divide both 6 and 10 by 2.
6 ÷ 2 = 3
10 ÷ 2 = 5
4
C
6/10 = 3/5
6 per 10 = 3 per 5
5
Sally bought 6 cards for 10 cents. The next day she bought 6 more cards with 10 more cents.
She bought 2 × 6 cards, or ■ cards.
She paid 2 × 10¢, or ■¢, for them.

140

1 Remind the pupils that a rate or comparison can be expressed by more than one ratio and so far they have found other ratios by using pictures or objects. Now they will find other ratios by computation.

2 Ask the pupils to read the text, then look at Picture A. Stress that 6/10 expresses the cost of cards per pennies.

3 Call on someone to explain Picture B and read the text. Stress that 3/5 expresses the rate of cards per pennies in terms of one of the equal groups.

4 Explain that to find the new ratio by computation both 6 and 10 are divided by 2, and this is the same as dividing the cards and pennies into 2 equal groups.

5 Point out that Sally bought two groups of cards like the group in Picture A. Ask, "Would she need two groups of pennies like the one in Picture A?" Have the children fill in the screens as they read.

Think of the whole group of cards Sally bought. Do the numerals at the right also express the rate at which the postcards are sold? $\frac{12}{20}$ **1**

You can get these two new numerals to express the rate 6 per 10 by multiplying both 6 and 10 by 2.

$2 \times 6 = \square$ $\quad \frac{6}{10} = \frac{\square}{\square}$

$2 \times 10 = \square$

A You can find other ratios equal to the ratio at the right. One way is to divide both 12 and 8 by the same number. 4 is a number by which both 12 and 8 can be divided without a remainder. $\frac{12}{8}$

$12 \div 4 = \square$ $\quad \frac{12}{8} = \frac{\square}{\square}$ **2**

$8 \div 4 = \square$

B By what other number can both 12 and 8 be divided?

$12 \div 2 = \square$ $\quad \frac{12}{8} = \frac{\square}{\square}$ **3**

$8 \div 2 = \square$

You can also find other ratios that are equal to the ratio 12 over 8 by multiplying both 12 and 8 by the same numbers.

C $3 \times 12 = \square$ $\quad \frac{12}{8} = \frac{\square}{\square}$ **4**

$3 \times 8 = \square$

D $5 \times 12 = \square$ $\quad \frac{12}{8} = \frac{\square}{\square}$

$5 \times 8 = \square$

E $10 \times 12 = \square$ $\quad \frac{12}{8} = \frac{\square}{\square}$

$10 \times 8 = \square$

For each ratio below, write a ratio equal to it. Use numbers smaller than the ones given. In Example A, you could use 8 and 10, or 4 and ■. **5**

A $\frac{16}{20}$ B $\frac{9}{12}$ C $\frac{24}{20}$ D $\frac{30}{20}$

E $\frac{5}{15}$ F $\frac{25}{15}$ G $\frac{9}{21}$ H $\frac{16}{4}$

For each ratio below, write another ratio equal to it. Use numbers larger than the ones given.

I $\frac{4}{3}$ J $\frac{2}{5}$ K $\frac{4}{9}$ L $\frac{3}{10}$

M $\frac{7}{8}$ N $\frac{1}{6}$ O $\frac{12}{5}$ P $\frac{9}{2}$

Q $\frac{1}{2}$ R $\frac{7}{3}$ S $\frac{5}{7}$ T $\frac{5}{1}$

A Look at the ratio below. It is expressed by the numerals 9 and 6. Then another ratio equal to it is written with other numerals. One of them, 18, has been found for you. You must find the missing numeral. **6**

$\frac{9}{6} = \frac{18}{\square}$ ← You must find this numeral.

By what number was 9 multiplied to get 18?

$\square \times 9 = 18$ $\quad \frac{9}{6} = \frac{18}{12}$

$2 \times 9 = 18$

$2 \times 6 = 12$

Now turn the page.

B $\frac{3}{4} = \frac{\square}{16}$ **1**

$\square \times 4 = 16$

$4 \times 4 = 16$ $\quad \frac{3}{4} = \frac{\square}{16}$

$4 \times 3 = 12$

C $\frac{10}{15} = \frac{2}{\square}$

By what number was 10 divided to get 2?

$10 \div \square = 2$ **2**

$10 \div 5 = 2$ $\quad \frac{10}{15} = \frac{2}{3}$

$15 \div 5 = 3$

D $\frac{35}{56} = \frac{\square}{8}$

$56 \div \square = 8$

$56 \div 7 = 8$ $\quad \frac{35}{56} = \frac{\square}{8}$ **3**

$35 \div 7 = 5$

Find the missing numeral in each example below. **4**

A $\frac{14}{16} = \frac{\square}{8}$ G $\frac{7}{4} = \frac{\square}{8}$ M $\frac{15}{9} = \frac{5}{\square}$

B $\frac{5}{3} = \frac{10}{\square}$ H $\frac{6}{25} = \frac{18}{\square}$ N $\frac{16}{30} = \frac{\square}{15}$

C $\frac{3}{4} = \frac{\square}{20}$ I $\frac{28}{21} = \frac{4}{\square}$ O $\frac{24}{36} = \frac{2}{\square}$

D $\frac{4}{12} = \frac{\square}{3}$ J $\frac{9}{18} = \frac{1}{\square}$ P $\frac{8}{15} = \frac{24}{\square}$

E $\frac{6}{1} = \frac{12}{\square}$ K $\frac{10}{16} = \frac{5}{\square}$ Q $\frac{9}{1} = \frac{\square}{5}$

F $\frac{1}{2} = \frac{5}{\square}$ L $\frac{21}{24} = \frac{7}{\square}$ R $\frac{3}{23} = \frac{6}{\square}$

When you multiply or divide both numerals in a ratio by the same number, you get another pair of numerals that expresses the same rate or the same comparison.

Keeping skillful **5**

A	$27+41+58+30=n$	A	$421+n=700$	A	$7 \times 59 = n$
B	$64+33+72+49=n$	B	$n+2693=5104$	B	$5 \times 608 = n$
C	$50+76+48+25=n$	C	$8370-n=3096$	C	$n \times 8 = 288$
D	$16+24+52+19=n$	D	$n-934=6077$	D	$67 \times 73 = n$
E	$87+45+91+78=n$	E	$15008-7261=n$	E	$n \times 15 = 810$
F	$138+429+226=n$	F	$n+4279=33400$	F	$1063 \div 62 = n$
G	$370+201+298+116=n$	G	$n-8410=25000$	G	$2964 \div n = 39$
H	$406+355+372+289=n$	H	$5729+n=7886$	H	$3705 \div 9 = n$
I	$697+700+458+537=n$	I	$40869-27098=n$	I	$4015 \div n = 5$
J	$888+483+761+92=n$	J	$73392-n=48975$	J	$45 \times 104 = n$

141

1 The rate is now expressed by a new ratio. Relate the computation to the two equal groups of cards and pennies in Picture C.

2 Show pictorially that dividing by 4 gives a ratio that equals 12/8.

3 Adapt the procedures in Note 2.

4 Have the children take turns answering. Be sure they understand that when they have multiplied, the new ratio will be equal to the original ratio.

5 Give the pupils time to work Exercises A to T independently. Point out that, since the new ratios for A to H are to use smaller numbers, they will divide. The ratios for I to T are to use larger numbers, and so they will multiply. Ask, "Why is there no limit to the ratios that can be written for Exercises I to T?"

6 Tell the class they first find whether 9 can be multiplied or divided to get 18. Then find out by what number it can be multiplied or divided. Ask, "Now what must you do with 6?"

142

1 Explain that the 4 can be multiplied by 4 to get 16. Ask, "Now what must we do with the 3?" Discuss the computation with the children.

2 Show that when one term of a ratio has been divided by a number, the other term must also be divided by the same number. Ask the pupils to supply the answers.

3 Let the children do the computation independently. Be sure they understand that they must multiply or divide by the same number that has already been used. Check each child's work. If necessary, use pictures and objects to explain how to correct any errors they make.

4 Have the children discover the number by which one term of the first ratio in each exercise was multiplied or divided. Then they must perform the same operation on the other term. Use these exercises as written work.

5 Tell the children to write each example in computational form and compute.

143-146 Moving forward

Expanded Notes for this lesson are on pages 308-311.

Objectives

The child learns how to compute to change a fraction numeral to another fraction numeral that expresses the same fraction.

Vocabulary

New words page 146 except*, form*, simplest*

Comments

The idea that different fraction numerals can be used to symbolize a fraction was introduced on pages 126-131. Now the children learn how to reduce fraction numerals computationally. Some children in your class may have realized while they were studying pages 140-142 that the computational procedures for reducing ratios also apply to fraction numerals.

When we reduce fraction numerals, we replace them by other fraction numerals that are equivalent and represent the same fraction. Make sure the children understand that the various fraction numerals are only different ways of expressing the same fraction. The fraction itself remains the same. Only the numerals are different. In other words, the children should see that $\frac{2}{3}$, $\frac{4}{6}$, and $\frac{8}{12}$ are simply different ways of writing the same fraction. The children should not think of these fraction numerals as different fractions.

To clarify this idea, think of a plane figure, part of it shaded. With only this information, you cannot tell how much of it is shaded. You can only point to the shaded portion and say that "this much" of the figure is shaded. To describe "this much," it is necessary, first, to divide the figure into equal parts. The part of the figure that is shaded then can be expressed by a fraction numeral that shows the number of equal parts (the denominator) and the number of these parts that are being considered (the numerator). Note that no matter how many or how few divisions have been made, the shaded portion (which the fraction represents) remains the same. However, many fraction numerals may be written to describe it.

Note that on page 146 (in the "Thinking straight" section) the term *simplest form* is introduced. Give special attention to this term because it is used extensively in referring to the best way to express answers involving fractions.

Answers

Page 145:

Answers are not given for Blocks 1, 3, or 4 since the answers are obvious or a great variety of answers is possible.

(block 2)

A $\frac{3}{6}$, $\frac{1}{2}$
B $\frac{9}{12}$, $\frac{6}{8}$, $\frac{3}{4}$
C $\frac{7}{8}$
D $\frac{3}{6}$, $\frac{2}{4}$, $\frac{1}{2}$
E $\frac{3}{5}$
F $\frac{2}{3}$
G $\frac{3}{4}$
H $\frac{8}{10}$, $\frac{4}{5}$
I $\frac{5}{9}$
J $\frac{5}{7}$
K $\frac{8}{20}$, $\frac{4}{10}$, $\frac{2}{5}$
L $\frac{7}{12}$
M $\frac{8}{12}$, $\frac{4}{6}$, $\frac{2}{3}$
N $\frac{6}{9}$, $\frac{2}{3}$
O $\frac{12}{14}$, $\frac{6}{7}$
P $\frac{3}{4}$

Page 146:

A 8
B 4
C 18
D 8
E 5
F 9
G 6
H 16
I 5
J 1
K 14
L 5
M 7
N 15
O 27
P 5
Q 2
R 4

Thinking straight:

(block 1)

A 2,3,6
B 2
C 2,4
D 2,5,10
E 5
F 4
G 8
H 2
I 5
J 3
K 8
L 10
M 8

(block 2)

In the answers below, when the answer is "Yes," this means that both the numerator and denominator cannot be divided by any number except 1; when the answer is "No," this means that both the numerator and the denominator can be divided by some number other than 1.

A 5; no
B 2; 2
C Yes
D Yes
E No
F Yes
G Yes
H No
I No
J Yes
K No

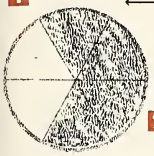
(block 3)

A $\frac{1}{3}$
B $\frac{2}{3}$
C $\frac{3}{4}$
D $\frac{3}{4}$
E $\frac{4}{5}$
F $\frac{3}{5}$
G $\frac{2}{3}$
H $\frac{1}{2}$
I $\frac{1}{3}$
J $\frac{1}{5}$
K $\frac{2}{3}$
L $\frac{4}{5}$
M $\frac{1}{2}$
N $\frac{3}{4}$
O $\frac{3}{8}$
P $\frac{7}{9}$
Q $\frac{4}{5}$
R $\frac{7}{8}$
S $\frac{5}{8}$
T $\frac{5}{6}$

Moving forward

In this lesson you will learn how to change a fraction numeral to another fraction numeral that expresses the same fraction.

1

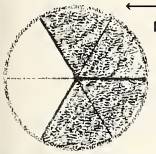


A

The piece of paper in Picture A is marked off into ▪ equal parts.
What fraction of the paper is green?

2

Now imagine that the same paper has been marked off into only 3 equal parts. Each part is $\frac{1}{3}$ of the paper.



B

How many one sixths are in each $\frac{1}{3}$?
Can you still say that $\frac{4}{6}$ of the paper is green?
Is $\frac{2}{3}$ of the paper green?

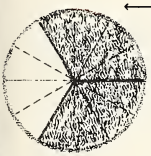
3

You can change $\frac{4}{6}$ to thirds by dividing the 6 and the 4 in $\frac{4}{6}$ by 2.

$6 \div 2 = 3$
$4 \div 2 = 2$

$\frac{4}{6} = \frac{2}{3}$

4



C

Now imagine that the same piece of paper has been marked off into 12 equal parts. Each part is $\frac{1}{12}$ of the paper. How many one twelfths are in each $\frac{1}{6}$?
Is $\frac{8}{12}$ of the circle green?
Is $\frac{4}{6}$ of the circle green?

5

You can change $\frac{4}{6}$ to twelfths by multiplying the 6 and the 4 in $\frac{4}{6}$ by 2.

$2 \times 6 = 12$
$2 \times 4 = 8$

$\frac{4}{6} = \frac{8}{12}$

$\frac{2}{3}$, $\frac{4}{6}$, and $\frac{8}{12}$ all express the fraction of the paper that is green.

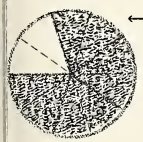
1

D

Here is a new piece of paper marked off into tenths.
What fraction of this paper is brown?

2

Now imagine that the same paper has been marked off into 5 equal parts. Each part is $\frac{\square}{\square}$ of the paper.



E

$\frac{\square}{10}$ of the paper is brown. $\frac{\square}{5}$ of the paper is brown.

3

You can change $\frac{8}{10}$ to fifths by dividing both 10 and 8 by 2.

$10 \div 2 = \square$
$8 \div 2 = \square$

$\frac{8}{10} = \frac{\square}{\square}$

You have divided both the denominator and the numerator of $\frac{8}{10}$ by 2 to get another fraction numeral that expresses the same fraction.

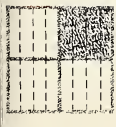
4

F

This square piece of paper is marked off into ~~~.
What fraction of this square is green?

5

The square has now been marked off into 16 equal parts.



G

$\frac{\square}{4}$ of the square is green. $\frac{\square}{16}$ of the square is green.

6

You can change $\frac{1}{4}$ to sixteenths by multiplying both 4 and 1 by 4.

$4 \times 4 = \square$
$4 \times 1 = \square$

$\frac{1}{4} = \frac{\square}{\square}$

You have multiplied both the denominator and the numerator of $\frac{1}{4}$ by 4 to get another fraction numeral that expresses the same fraction.

143

1. Have the children note that the shaded portion of the circular piece of paper is the same in all three pictures. Have them observe that the light, the heavy, and the broken lines mark off different fractional parts.
2. In Picture A, point out that the portion that is shaded green is the part we are thinking about.
3. Emphasize that now that the paper is marked off in thirds, each third is found to contain two sixths. Show how this fact is used in computationally reducing $\frac{4}{6}$ to $\frac{2}{3}$.
4. Have the children note that the shaded portion is now divided into still smaller parts. Every one sixth is made up of two twelfths.
5. Explain that it takes two twelfths to make one sixth, and it is possible to change the larger parts (one sixths) to the smaller parts (one twelfths). The fraction numeral to represent this change can be found by multiplying the numerator and denominator of $\frac{4}{6}$ by 2.

144

1. Ask the children what fraction tells how much of the paper is shaded.
2. Have the children use Picture E to find the missing numerals. Be sure they note that each fifth is made up of two tenths.
3. Get the pupils to explain why, in changing the fraction numeral $\frac{8}{10}$ to fifths, both numerator and denominator must be divided by 2 [because there are two tenths in one fifth]. Have them note that the new fraction numeral is $\frac{4}{5}$. The fraction numeral has been changed, but the shaded part of the paper remains the same.
4. Have the children tell what fraction numeral represents the shaded part of the paper.
5. Have the class note the smaller subdivisions of the square. Get them to observe that each one fourth contains 4 of these smaller subdivisions, or four sixteenths.
6. Be sure the children see why the numerator and denominator are each multiplied by 4.

A You can find many fraction numerals that are equal to $\frac{12}{16}$. One way to find such a numeral is to divide the denominator and the numerator by the same number. 16 and 12 can both be divided by 4.

$16 \div 4 = \square$ $\quad \frac{12}{16} = \frac{\square}{\square}$ 1
$12 \div 4 = \square$

B By what other number can both 16 and 12 be divided?

$16 \div 2 = \square$ $\quad \frac{12}{16} = \frac{\square}{\square}$
$12 \div 2 = \square$

You can also find other fraction numerals equal to $\frac{12}{16}$ by multiplying both the denominator and the numerator by the same number.

C $2 \times 16 = \square$ $\quad \frac{12}{16} = \frac{\square}{\square}$
$2 \times 12 = \square$

D $3 \times 16 = \square$ $\quad \frac{12}{16} = \frac{\square}{\square}$
$3 \times 12 = \square$

E $5 \times 16 = \square$ $\quad \frac{12}{16} = \frac{\square}{\square}$ 2
$5 \times 12 = \square$

For each example below, write a fraction numeral that is equal to the fraction numeral given, but use a smaller denominator. For Example A you could write $\frac{3}{6}$. You could also write $\frac{\square}{\square}$.

A	$\frac{9}{18}$	E	$\frac{6}{10}$	I	$\frac{15}{27}$	M	$\frac{16}{24}$				
B	$\frac{18}{24}$ 3	F	$\frac{10}{15}$	J	$\frac{25}{35}$	N	$\frac{18}{27}$				
C	$\frac{14}{16}$	G	$\frac{9}{12}$	K	$\frac{16}{40}$	O	$\frac{24}{28}$				
D	$\frac{6}{12}$	H	$\frac{16}{20}$	L	$\frac{21}{36}$	P	$\frac{15}{20}$				

Now write a fraction numeral that is equal to the fraction numeral given, but use a larger denominator.

A	$\frac{2}{5}$ 3	D	$\frac{5}{8}$	G	$\frac{1}{3}$	J	$\frac{3}{10}$				
B	$\frac{5}{6}$	E	$\frac{4}{9}$	H	$\frac{3}{4}$	K	$\frac{11}{12}$				
C	$\frac{1}{4}$	F	$\frac{2}{7}$	I	$\frac{1}{2}$	L	$\frac{9}{10}$				

A The same fraction is written in two ways below. First it is written as $\frac{3}{8}$. Then it is written with the denominator 16, but you must find the missing numerator.

$\frac{3}{8} = \frac{\square}{16}$ ← You must find this number. 4

By what number was 8 multiplied to get 16?

$2 \times 8 = 16$ $\quad \frac{3}{8} = \frac{6}{16}$
$2 \times 3 = 6$

B $\frac{9}{15} = \frac{\square}{5}$

By what number was 15 divided to get 5?

$15 \div 3 = 5$ $\quad \frac{9}{15} = \frac{3}{5}$ 4
$9 \div 3 = 3$

C $\frac{1}{4} = \frac{5}{\square}$

$\square \times 1 = 5$
$5 \times 1 = 5$ $\quad \frac{1}{4} = \frac{5}{\square}$ 4
$5 \times 4 = 20$

D $\frac{14}{20} = \frac{7}{\square}$

$14 \div \square = 7$ $\quad \frac{14}{20} = \frac{7}{\square}$ 4
$20 \div \square = \square$

A	$\frac{4}{5} = \frac{\square}{10}$ 1	D	$\frac{1}{2} = \frac{\square}{16}$	G	$\frac{2}{3} = \frac{\square}{9}$	J	$\frac{5}{15} = \frac{\square}{3}$	M	$\frac{21}{24} = \frac{\square}{8}$	P	$\frac{12}{15} = \frac{4}{\square}$
B	$\frac{9}{12} = \frac{3}{\square}$	E	$\frac{4}{20} = \frac{1}{\square}$	H	$\frac{5}{8} = \frac{10}{\square}$	K	$\frac{1}{2} = \frac{7}{\square}$	N	$\frac{3}{4} = \frac{\square}{20}$	Q	$\frac{1}{8} = \frac{\square}{16}$
C	$\frac{1}{6} = \frac{3}{\square}$	F	$\frac{3}{8} = \frac{\square}{24}$	I	$\frac{2}{10} = \frac{1}{\square}$	L	$\frac{20}{24} = \frac{\square}{6}$	O	$\frac{2}{5} = \frac{6}{\square}$	R	$\frac{12}{21} = \frac{\square}{7}$

When you multiply or divide the denominator and the numerator of a fraction numeral by the same number, you get another fraction numeral that expresses the same fraction.

Thinking straight

A Both 12 and 18 can be divided by three numbers besides 1. What are the numbers? 2

B Both 10 and 16 can be divided by ■.

C Both 12 and 16 can be divided by ■ and ■.

D Both 20 and 30 can be divided by ■, ■, and ■.

What is the largest number by which each of these pairs of numbers can be divided without a remainder?

E	10, 15	H	10, 18	K	8, 16
F	20, 24	I	15, 20	L	10, 20
G	24, 32	J	9, 21	M	16, 24

A $\frac{15}{20} = \frac{3}{4}$. By what number were both 15 and 20 divided? Is there any number besides 1 by which both 3 and 4 can be divided without a remainder?

B $\frac{20}{24} = \frac{10}{12}$. By what number were 20 and 24 divided? By what number can 10 and 12 also be divided?

C $\frac{10}{12} = \frac{5}{6}$. Because 5 and 6 cannot be divided without a remainder by any number except 1, we say that $\frac{5}{6}$ is in simplest form. Is $\frac{3}{7}$ in simplest form? How do you know? 3

Which fraction numerals below are in simplest form? How do you know?

D	$\frac{4}{5}$	F	$\frac{3}{10}$	H	$\frac{4}{6}$	J	$\frac{1}{8}$
E	$\frac{8}{12}$	G	$\frac{11}{16}$	I	$\frac{6}{15}$	K	$\frac{9}{12}$

Change to simplest form.

A	$\frac{5}{15}$	F	$\frac{6}{10}$	K	$\frac{16}{24}$	P	$\frac{14}{18}$ 4
B	$\frac{4}{6}$	G	$\frac{12}{18}$	L	$\frac{12}{15}$	Q	$\frac{24}{30}$
C	$\frac{9}{12}$	H	$\frac{6}{12}$	M	$\frac{10}{20}$	R	$\frac{21}{24}$
D	$\frac{12}{16}$	I	$\frac{3}{9}$	N	$\frac{18}{24}$	S	$\frac{10}{16}$
E	$\frac{16}{20}$	J	$\frac{4}{20}$	O	$\frac{6}{16}$	T	$\frac{25}{30}$

145

1 Get the pupils to explain the two ways they have studied to change fraction numerals [by multiplication and division]. If necessary, use diagrams to demonstrate how $\frac{12}{16}$ can be reduced to $\frac{3}{4}$, $\frac{6}{8}$, $\frac{24}{32}$, etc.

2 Have the children generalize that when reduction is by division, the number of such possible reductions is limited, but that any number of reductions by multiplication can be performed.

3 Allow the children to do Exercises A to P and A to L independently, but be prepared to give help where it is needed. Do not expect all the pupils to find all the possible equivalents.

4 Get the children to see that the number to use when multiplying or dividing can be found by inspection. Discuss each exercise and demonstrate with diagrams if necessary.

146

1 Have the children work independently to find the missing numerals. Let them verify their work and discuss exercises that were troublesome.

2 Exercises A to M should lead to the generalization that certain pairs of numbers have various common divisors.

3 Note the introduction of the term *simplest form*. Be sure the children understand what this term means, since it will be used frequently from now on.

4 Have the children work Problems A to T independently. Discuss the answers.

147 Thinking straight

Expanded Notes are not considered necessary for this lesson.

Objectives

The child becomes acquainted with the meaning of profit and loss.

Vocabulary

New words **page 147** flour*, charge*, profit, loss

Comments

Although this lesson is not intended as an introduction to the principles of merchandising, some consideration of these principles is unavoidable in a discussion of profit and loss resulting from a sale. In the activity described on page 147, pupils have only the cost of materials to consider as expenses. It should be pointed out, however, that in the business world other expenses, such as the cost of labor, equipment, etc., would have to be taken into consideration. In the business world, articles that are slow to sell may subsequently be sold at a reduced price. The examples in this lesson apply to a special situation in which everything must be sold in one day.

By questions and discussion, lead the children to see that all the money received for things they have made and sold does not represent a profit if any expense entered into the making of whatever was sold. Let them discover that to make a profit the selling price of the article must be more than the cost of producing it.

If you discuss how to price articles, you might ask if it is better to sell 5 articles at a profit of 25¢ on each article than to sell 20 of the same articles at a profit of only 10¢ apiece. You can also point out that in setting a price one must consider not only the *desired* profit, but also a *reasonable* profit. Questions like these should be considered: "How many people are likely to want to buy? How much does the article usually cost?"

When the pupils are ready to work on the problems, warn them that some of the problems will use information and answers from previous problems on this page. Caution the pupils to preserve work and answers as they go. When the problems have been worked, discuss the method of work and the answers.

In discussing Problem J, you can mention that since the boys still had 7 birdhouses left they might be able to make up their loss by selling them at a reduced price to people who did not attend the sale. The children should be able to explain why, although the boys lost money on the birdhouses, the fifth grade as a whole had a profit on the fair.

Answers

A $\$.72+\$.70+\$.46+\$.92+\$2.10=[\$4.90]$
$\$.32+\$.40+\$1.40+\$.34+\$.88+\$.26=[\$3.60]$
$\$.70+\$.30+\$.60+\$.34+\$.28+\$.23=[\$2.45]$

B $\$4.90+\$3.60+\$2.45=[\$10.95]$

C $(\$9.50)-\$4.90=[\$4.60]$

D $(\$10.00)-\$3.60=[\$6.40]$

E $(\$6.00)-\$2.45=[\$3.55]$

F $\$9.50+(\$9.50)+(\$5.20)=[24.20]$

G $\$24.20-\$10.95=[\$13.25]$

H $\$11.52\div[\$.96]=12$

I $5\times\$2.25=[\$11.25]$

J $\$11.52-\$11.25=[\$.27]$

K $\$13.25-\$.27=[\$12.98]$

Thinking straight

A To raise money, the fifth-grade girls helped their mothers bake cakes and cookies for a school fair. The cost of the materials they used is shown above. What was the cost of all the materials for cakes? For cookies? For cupcakes? 1

B Find the total cost of the materials for cakes, cookies, and cupcakes. 2

C The girls had to decide how much to charge for the cakes, cookies, and cupcakes. They wanted to set a price so that they would have some money left over after the materials were paid for. The money left over would be their profit. They decided to sell the cakes at 95¢ apiece. What profit did they expect to make on the 10 cakes? 3

D They priced the cookies to sell at 50¢ a dozen. Find the profit they expected to make on the 20 doz. cookies. 3

E They decided to sell the cupcakes at 5¢ apiece. What profit did they expect to make on all the cupcakes? 3

F The girls sold all the cakes, 19 doz. cookies, and 104 cupcakes. How much money did they get in all? 4

G Find the girls' total profit. 5

H The fifth-grade boys made 12 birdhouses. They spent $11.52 for wood, paint, and other materials. What was the average cost per birdhouse? 6

I The boys sold 5 of the birdhouses at $2.25 apiece. How much money did they get for the 5 birdhouses?

J Since the boys got less money for the 5 birdhouses that they sold than it cost to make all the birdhouses, they had a loss instead of a profit. How much was the loss?

K How much profit did the fifth-grade boys and girls make in all?

Introduction to idea of profit and loss 147

147

1 Let the pupils figure these costs. Tell them to keep the answers.

2 The children should write the answers from Problem A in a column under the heading "Cost of Materials." Have them add these costs.

3 Watch for pupils who have difficulty with this problem. Help them with their thinking by asking questions.

4 Have the pupils write the amounts received from the sale of cakes, cookies, and cupcakes in a column headed "Money Received from Sale." Have them record the total.

5 When the pupils have found the total profit (by subtracting the total in the "Cost of Materials" column from the total in the "Money Received" column), let them compare it with the expected profit.

6 Have the pupils find these answers.

148-150 Moving forward

Expanded Notes for this lesson are on pages 311-313.

Objectives

The child reviews his knowledge of improper fractions and mixed numbers, which were introduced in the previous grade. He also learns some new terminology.

Vocabulary

New words page 148 disk, proper*; page 149 mixed*

Comments

The child knows that he can use a fraction numeral composed of a pair of numerals to express a part or "fraction" of a whole thing. This lesson reminds him that it is possible to use a pair of numerals to express a quantity equal to or larger than the quantity that 1 represents. He learns to use the term "improper fraction" to express such situations. The pictures and accompanying text show him that some of these pairs of numerals may represent whole numbers ($\frac{9}{9}=1$), and other pairs may represent a whole number plus a fraction ($\frac{7}{5}=1\frac{2}{5}$). The sum in this case is called a mixed number. It is important that he understand that an improper fraction and a mixed number may represent the same quantity. The computational procedure for changing from an improper fraction to a whole number or a mixed number is taught in the next lesson.

By the time the child has finished this lesson, he should be able to distinguish among proper fractions, improper fractions, and mixed numbers and be able to write the numerals representing those numbers. He should also have observed that the numerator of a proper fraction is smaller than the denominator, and that the numerator of an improper fraction is larger than or equal to the denominator.

Answers

Page 150:

(block 1)

Proper fractions: A, D, E, G, H

Improper fractions: J, K, M, N, P

Mixed numbers: Q, T, U, W

(block 3)

A $1\frac{9}{10}$	E $\frac{2}{2}$	I $3\frac{1}{2}$	M $\frac{11}{5}$
B $\frac{8}{8}$	F $7\frac{5}{12}$	J $\frac{12}{12}$	N $\frac{1}{8}$
C $6\frac{1}{3}$	G $1\frac{3}{16}$	K $\frac{9}{15}$	O $8\frac{11}{12}$
D $\frac{9}{4}$	H $\frac{4}{4}$	L $7\frac{2}{3}$	P $\frac{14}{10}$

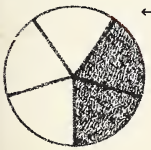
(block 4)

A $2\frac{1}{2}$ and $\frac{5}{2}$	D $2\frac{3}{12}$ and $\frac{27}{12}$
B $1\frac{2}{3}$ and $\frac{5}{3}$	E $3\frac{4}{5}$ and $\frac{19}{5}$
C $1\frac{5}{8}$ and $\frac{13}{8}$	F $1\frac{7}{16}$ and $\frac{23}{16}$

Moving forward

This lesson will explain how fraction numerals can be used to show one whole thing or more than one whole thing.

1 **A**



You can use the numerals 2 and ▩ to tell what fraction of the paper disk is red.

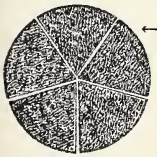
What does the 5 show? ← $\frac{2}{5}$ → What does the 2 show? **2**

two fifths

Is $\frac{2}{5}$ less than 1?

A fraction numeral like $\frac{2}{5}$, that stands for less than a whole thing, or less than 1, is a proper fraction. **3**

B



Each disk has been cut into 5 equal parts. Each of these parts is $\frac{1}{5}$ of a whole disk. How many of these one fifths are red? **4**

You can use the numerals 7 and 5 to tell what fraction of these disks is red.

What does the 5 show? ← $\frac{7}{5}$ → What does the 7 show? **5**

seven fifths

Is $\frac{7}{5}$ more than 1?

A fraction numeral like $\frac{7}{5}$, that stands for more than a whole thing, or more than 1, is an improper fraction. **6**

148

1. The pupils should read the first sentence and study Picture A.
2. Have pupils look at the symbol $\frac{2}{5}$ and the text, using Picture A to answer the questions about the numerator and the denominator.
3. Have the pupils read this text. Discuss the term *proper fraction*. Ask for other examples of a proper fraction. Be sure pupils understand that any proper fraction is less than 1.
4. Have the pupils read these three lines and relate them to Picture B.
5. Ask the pupils to look at the symbol $\frac{7}{5}$ and read the text. Be sure they understand that the denominator, 5, shows that each disk was cut into 5 equal parts, or fifths.
6. Picture B should make it easy for pupils to see that $\frac{7}{5}$ is more than 1. Tell why $\frac{7}{5}$ is called an improper fraction. Have pupils give other examples of an improper fraction. Tell pupils that they will learn more about improper fractions on the next page.

C You also can write the numeral shown below to tell how much of the disks is red.

1

The 1 shows 1 whole disk. → $1\frac{2}{5}$ ← The 2 shows you are thinking of 2 of the equal parts.

The 5 shows that the other disk was cut into 5 equal parts.

2

one and two fifths

$1\frac{2}{5}$ is a mixed number. It is made up of a whole number, 1, and a fraction, $\frac{2}{5}$. Does a mixed number stand for more than one whole thing?

You can see that the red parts of the disks in Pictures B and C are equal.

$\frac{7}{5} = 1\frac{2}{5}$

The cake has been cut into ▪ equal parts. Each part is $\frac{1}{▪}$ of the cake. How much of the cake is on the plate? 3

D You can write a fraction numeral to tell how much of the cake is on the plate.

4

What does this 9 show? → $\frac{9}{9}$ ← What does this 9 show?

nine ninths

Is $\frac{9}{9}$ equal to 1?

A fraction like $\frac{9}{9}$, that stands for a whole thing, or 1, is also an improper fraction.

$\frac{9}{9} = 1$

Which numerals stand for proper fractions? 1

A	B	C	D	E	F	G	H
$\frac{1}{4}$	$\frac{6}{6}$	$\frac{5}{3}$	$\frac{2}{7}$	$\frac{9}{10}$	$\frac{12}{10}$	$\frac{3}{5}$	$\frac{10}{16}$

Which numerals stand for improper fractions?

I	J	K	L	M	N	O	P
$\frac{3}{8}$	$\frac{7}{4}$	$\frac{2}{2}$	$\frac{4}{6}$	$\frac{19}{16}$	$\frac{13}{10}$	$\frac{5}{9}$	$\frac{12}{12}$

Which numerals stand for mixed numbers?

Q	R	S	T	U	V	W	X
$2\frac{2}{3}$	6	$\frac{5}{8}$	$1\frac{7}{10}$	$4\frac{1}{5}$	$\frac{8}{9}$	$7\frac{3}{8}$	$\frac{5}{6}$

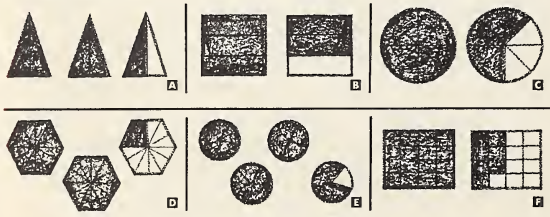
A Write 5 proper fractions.
B Write 5 improper fractions. 2
C Write 5 mixed numbers.

Write the numerals.

A one and nine tenths 3
B eight eighths

C six and one third
D nine fourths
E two halves
F seven and five twelfths
G one and three sixteenths
H four fourths
I three and one half
J twelve twelfths
K nine fifteenths
L seven and two thirds
M eleven fifths
N one eighth
O eight and eleven twelfths
P fourteen tenths

For each picture below, write two numerals that tell how much of the pieces of paper is green. You can write $2\frac{1}{2}$ and $\frac{5}{2}$ for Picture A. 4



Now you should be able to read and write any fraction numeral or mixed number.

149

1 Have the pupils compare Picture C with Picture B. The only difference is that in Picture C the fifths have been put back together so that the whole disks can be more easily seen. Explain to the pupils that they are going to learn another way to express more than one whole thing.

2 Have the pupils read what is said about each of the numbers in the mixed number $1\frac{2}{5}$. Ask them to find in the picture the quantity that each numeral represents. Discuss the term *mixed number*. Pupils should see that a mixed number stands for more than one whole thing. Discuss the fact that $\frac{7}{5}$ and $1\frac{2}{5}$ are equal.

3 Have the pupils read these three lines and relate them to Picture D.

4 Tell the pupils to use the picture to answer the questions about $\frac{9}{9}$. Stress the fact that fractions like $\frac{9}{9}$, $\frac{3}{3}$, $\frac{6}{6}$, etc., are also improper fractions.

150

1 Discuss Exercises A to X. Be sure the pupils can explain why the numerals express proper fractions, improper fractions, or mixed numbers. Get the pupils to observe that the numerator of a proper fraction is smaller than the denominator, and that the numerator of an improper fraction is equal to or larger than the denominator.

2 Ask the pupils to write Exercises A to C on their papers. Afterwards, let some of the children put their work on the chalkboard so that the rest of the class can discuss it.

3 Ask the pupils to write the letters A to P on their papers and after each letter write the appropriate fraction numeral or mixed number.

4 Be sure that the pupils write two numerals for each picture. Discuss the answers afterwards.

151-153 Moving forward

Expanded Notes for this lesson are on pages 313-315.

Objectives

The child learns how and why division can be used to change an improper fraction to a mixed number or to a whole number.

Vocabulary

There are no new words.

Comments

In the last lesson the children learned that an improper fraction and a mixed number can represent the same value, but they did not learn how to change one to the other.

In this lesson the children learn the computational procedure for changing an improper fraction to a mixed number or a whole number. After addition of fractions has been studied, the process for changing mixed numbers to improper fractions will be taught.

No attempt has been made to give the children a rule to memorize for use in changing improper fractions to mixed numbers or whole numbers. The authors believe that children should understand the thinking behind the computation they do, and so pictures are provided that illustrate the action involved, thereby explaining why the process of division is used.

In connection with this lesson children also begin to get a background for using the remainder in division to form a fraction. A real treatment of this, of course, comes later.

Answers

Page 153:

(block 1)

A 6 B 23 by 10 C Ones D 8

(block 2)

A $6\frac{1}{2}$	F 1	K $6\frac{1}{3}$	P 2	U 7
B 4	G $1\frac{1}{2}$ or $1\frac{3}{6}$	L $3\frac{1}{2}$ or $3\frac{3}{6}$	Q 1	V 7
C 2	H 3	M 2	R 5	W 1
D $1\frac{3}{8}$	I $8\frac{2}{5}$	N $1\frac{1}{32}$	S 12	X 9
E $4\frac{1}{2}$ or $4\frac{4}{8}$	J $8\frac{2}{7}$	O $1\frac{1}{8}$ or $1\frac{2}{16}$	T $3\frac{1}{5}$	

Moving forward

You will now learn how to change any improper fraction to an equal mixed number or whole number.

A

Each piece of pie in Picture A is $\frac{1}{4}$ of a whole pie. There are 14 fourths.

The 4 shows that each pie was cut into 4 equal parts.

$\frac{14}{4}$

The 14 shows that there are 14 equal parts.

B

Now the fourths are being arranged as whole pies.

This 4 shows that each pie was divided into 4 equal parts.

$\frac{4}{4}$

This 4 shows that 4 fourths are used to make one whole.

It takes 4 fourths to make 1 whole. $\frac{4}{4} = 1$.

C

The 14 fourths have been arranged as 3 whole pies and 2 fourths of another pie.

To find how many wholes, or ones, there are in 14 fourths, divide the 14 by the 4.

$14 \div 4 = 3$ and 2 remainder. There are 3 ones, and 2 fourths are left over.

Now turn the page.

151

1 Tell pupils that these pieces of pie came from a number of equal-sized pies. Ask the pupils to use the text to find what part of a pie each of these pieces is. Then have someone count the fourths to verify that there are 14.

2 Be sure pupils understand that $\frac{14}{4}$ is an improper fraction and what each numeral means.

3 Tell the pupils to use the picture to find the number of fourths that make 1 pie.

4 Discuss the numerals in $\frac{4}{4}$.

5 Tell the children that we want to find how many whole pies and how many extra fourths these 14 fourths make. Tell them we can do this by separating the whole group into groups of 4 fourths and then putting the 4 fourths together to make whole pies. They can see that $3\frac{2}{4}$ pies were made.

6 Point out that when 14 fourths are divided into groups of 4 fourths the answer is the same as when 14 is divided by 4.

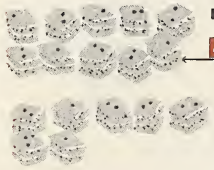
The 14 fourths equal 3 pies and 2 fourths of another pie.

$\frac{14}{4}$ = 3 ones and 2 fourths

The 3 stands for the three wholes, or 3 ones. The 4 shows that each pie was divided into 4 equal parts. The 2 stands for the two equal parts that are left over.

$3\frac{2}{4}$

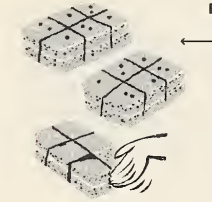
$\frac{14}{4} = 3\frac{2}{4}$. $3\frac{2}{4}$ in simplest form is $3\frac{1}{2}$.



Each piece of cake in Picture E is $\frac{1}{6}$ of a whole cake. There are ■ sixths.

The 6 shows that each cake was cut into ■ equal parts. The 17 shows that there are ■ equal parts.

$\frac{17}{6}$

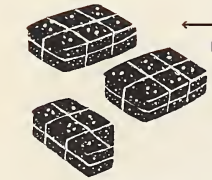


The 17 sixths have been arranged as ■ whole cakes and ■ sixths of another cake.

It takes 6 sixths to make one whole. $\frac{6}{6}$ = ■.

To find how many wholes, or ones, there are in 17 sixths, divide ■ by 6.

17 ÷ 6 = 2 and ■ remainder. There are ■ whole cakes, and ■ pieces are left over.



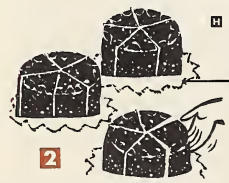
The 17 sixths equal 2 cakes and 5 sixths of another cake.

$\frac{17}{6}$ = 2 ones and ■ sixths

The 2 stands for the two wholes, or 2 ones. The 5 stands for the 5 equal parts that are left over. The 6 shows that each cake was cut into 6 equal parts.

$2\frac{5}{6}$

$\frac{17}{6} = 2\frac{■}{6}$



15 fifths have been arranged as ■ whole cakes. It takes 5 fifths to make 1 whole. $\frac{5}{5}$ = ■.

To find how many wholes, or ones, there are in 15 fifths, divide ■ by ■.

15 ÷ 5 = ■ $\frac{15}{5} = 3$

4 A To change $\frac{18}{6}$ to a whole number or a mixed number, divide 18 by ■.

B To change $\frac{23}{10}$ to a whole number or a mixed number, divide ■ by ■.

C When you divide to change $\frac{17}{5}$ to a whole number or a mixed number, does the 3 you get stand for ones or for fifths?

D When you change $\frac{19}{8}$ to a mixed number, the denominator of the fraction is ■.

5 Write each improper fraction below as a whole number or a mixed number.

A	B	C	D	E	F
$\frac{13}{2}$	$\frac{16}{4}$	$\frac{12}{6}$	$\frac{11}{8}$	$\frac{36}{8}$	$\frac{6}{6}$
G	H	I	J	K	L
$\frac{9}{6}$	$\frac{24}{8}$	$\frac{42}{5}$	$\frac{58}{7}$	$\frac{19}{3}$	$\frac{21}{6}$
M	N	O	P	Q	R
$\frac{24}{12}$	$\frac{33}{32}$	$\frac{18}{16}$	$\frac{16}{8}$	$\frac{12}{12}$	$\frac{25}{5}$
S	T	U	V	W	X
$\frac{48}{4}$	$\frac{16}{9}$	$\frac{21}{3}$	$\frac{56}{8}$	$\frac{16}{16}$	$\frac{18}{2}$

Now you should be able to change any improper fraction to an equal whole number or mixed number.

152

1 The children should see that these pies are the same as those in Picture C.

2 Discuss the fact that they have found that $\frac{14}{4} = 3\frac{2}{4}$. Point out that they have changed an improper fraction to a mixed number. Be sure pupils understand that the 3 stands for 3 wholes, or ones, that the 4 shows that each pie was cut into 4 equal parts, and that the 2 shows that 2 equal parts are left over.

3 Discuss why we can write: $3\frac{2}{4} = 3\frac{1}{2}$.

4 Have pupils read the first statement at the right of Picture E. Tell them to count the pieces in Picture E to find how many sixths there are.

5 Ask someone to complete the explanations for the numbers in the improper fraction.

6 Pupils must understand that the 17 pieces in Picture E were put in groups of 6 because 6 sixths equal 1.

7 Lead the pupils to see that the number of ones in $\frac{17}{6}$ can be found by dividing 17 by 6.

153

1 Let a pupil read the explanations for the numbers in the mixed number and relate the mixed number to the picture. The pupils must understand that they have changed the improper fraction $\frac{17}{6}$ to the mixed number $2\frac{5}{6}$ and that this was done by dividing 17 by 6.

2 Ask the class to imagine that the pieces of cake shown are not put together. Then let someone count to find the number of equal pieces there are in each cake and also the total number of fifths.

3 Discuss the fact that to find the number of wholes in 15 fifths, 15 can be divided by 5. Point out that an improper fraction may also be expressed as either a mixed number or a whole number.

4 Discuss Exercises A to D.

5 Assign these exercises as written work.

154-155 Using arithmetic

Expanded Notes are not considered necessary for this lesson.

Objectives

The child practices solving problems of the types taught to this point.

Vocabulary

New words **page 154** camping*, slept*, tents*, bacon*, cocoa, traveled*, equipment; **page 155** canteen, aid*, knapsack

Comments

The children should continue to make an equation for each problem. They should work the problems independently first, because such independent work is more interesting for them and also because it provides you with a good way of finding weaknesses they may have.

After pupils have worked the problems, supply equations and answers and let them verify their work. Find out which, if any, problems caused a great deal of trouble, and which caused only slight trouble. Then devote a few minutes to problems that only one or two children missed. Let the children help each other with these problems. After this, as much time as necessary should be spent discussing any problems that many in the class found difficult.

Answers

A $(10)\div 2=[5]$

B $5\times 5=[25]$

C $5\times \$14.75=[\$73.75]$

D $(\$1.16)+(\$1.06)+(\$.69)=[\$2.91]$

E $(\$.45)+(\$.63)+\$.43=[\$1.51]$

F $(\$1.77)+(\$.50)=[\$2.27]$

G $(\$1.54)+(\$.48)=[\$2.02]$

H $\$2.91+\$1.51+\$2.27+\$2.02=[\$8.71]$

I $(40)-20=[20]$

J $(30)\div[3]=10$

K $30-[23]=7$

L $87\div[29]=3$

M $24-(21)=[3]$

N $\$2.75+\$.49+\$1.15=[\$4.39]$

O $(\$5.50)-\$4.39=[\$1.11]$

P $\$1.47+[\$1.53]=(\$3.00)$

Q $[\$20.00]-\$16.50=\$3.50$

R $[20]\times\$.15=\$3.00$ or $\$3.00\div\$.15=[20]$

S $[\$2.60]+\$.85=\$3.45$

T $\$5.00-(\$4.70)=[\$.30]$

U $\$2.95-\$1.50=[\$1.45]$

Using arithmetic 1

A When Tony's father and Paul's father took 8 boys on an overnight camping trip, all of them slept in tents. Each tent was large enough for 2 persons. How many tents did they have?

B Each tent weighed 5 lb. How much did all the tents weigh?

C Paul's father said that the tents were worth $14.75 each. How much were all the tents worth?

D For the trip, Tony bought 2 lb. of bacon at 58¢ per pound, 2 doz. eggs at 53¢ per dozen, and 3 cans of baked beans at 23¢ per can. How much did all these groceries cost?

E Don bought groceries, too. He bought 3 lb. of bananas at 15¢ per pound, 3 lb. of peaches at 21¢ per pound, and 1 doz. oranges for 43¢. How much in all did Don spend for groceries?

F Bill bought 3 lb. of hot dogs at 59¢ per pound and 2 doz. rolls at 25¢ per dozen. How much did he spend?

G Jim bought the milk for the trip. He bought 2 gal. at 77¢ per gallon and 2 qt. at 24¢ per quart. How much did the milk cost?

H How much did all the food for the trip cost?

I The boys drank 20 cups of milk for supper and used the rest of the milk the next morning to make cocoa. How many cups of milk did they use to make cocoa?

THINK How many cups of milk did Jim buy for the trip?

J Paul's mother had made 18 yellow cupcakes and 12 chocolate cupcakes for the trip. She had made an average of how many cupcakes per person?

K After supper 7 cupcakes were left. How many cupcakes had been eaten?

L By the time the camping party got home, they had driven 87 mi. The round trip had taken about 3 hr. The party had traveled an average of about how many miles per hour?

M The camping party spent about 18 hr. in the woods. They were away from home how many more or how many fewer hours than 1 day?

THINK First add 18 to 3. Why?

N Don spent $2.75 for a cooking kit, 49¢ for a matchbox, and $1.15 for a compass. He spent how much in all for this camping equipment?

154

1 Assign these problems and O to U on the following page as written work. Caution the pupils that for some of the problems they will have to use information from a previous problem. Instruct them to write the equation for each problem with the letter *n* in the proper place first, then to compute, find the answer, and, finally, rewrite the equation with the answer in place of the letter *n*. Remind them, too, that some of the problems involve at least two steps in solving them. For such problems, tell them to write "master" equations first, using both screens (or squares) and the letter *n*.

N Bill spent $2.50 for a canteen, $1.75 for an eating kit, and $1.25 for a first-aid kit. He spent how much more for camping equipment than Don?

O Tony has $1.47. He wants to buy a first-aid kit and an eating kit like Bill's. How much more money does he need?

P Paul's brother used some of his savings to buy a sleeping bag. After he had spent $16.50 for the sleeping bag, he had $3.50 of his savings left. How much money had he saved?

Q Paul is saving 15¢ per week to buy a large canteen that costs $3. How long will it take Paul to save enough money to buy the canteen?

S George earned 85¢. Then he had enough money to buy a knapsack that cost $3.45. How much money did he have before he earned the 85¢?

T Jack had $5. He bought a knapsack like George's and a first-aid kit like Bill's. How much money did he have left?

U Dick sold an old pair of field glasses to Jim for $1.50. Dick had paid $2.95 for the field glasses. He sold them for how much less than he had paid for them?

1

Checking up

Test 1
Think of these numerals as either fraction numerals or as ratios. In each example, find the missing numeral.

A $\frac{3}{1}=\frac{15}{\square}$
B $\frac{10}{14}=\frac{\square}{7}$
C $\frac{2}{3}=\frac{\square}{24}$
D $\frac{20}{25}=\frac{4}{\square}$
E $\frac{1}{7}=\frac{\square}{21}$
F $\frac{5}{8}=\frac{15}{\square}$
G $\frac{6}{5}=\frac{\square}{30}$
H $\frac{18}{12}=\frac{6}{\square}$
I $\frac{20}{15}=\frac{\square}{3}$
J $\frac{5}{11}=\frac{45}{\square}$
K $\frac{28}{4}=\frac{7}{\square}$
L $\frac{3}{39}=\frac{\square}{13}$
M $\frac{15}{3}=\frac{5}{\square}$
N $\frac{42}{4}=\frac{\square}{2}$
O $\frac{7}{8}=\frac{\square}{32}$
P $\frac{10}{15}=\frac{20}{\square}$

Test 2
Change each of these fraction numerals to simplest form.

A $\frac{4}{6}$
B $\frac{3}{9}$
C $\frac{2}{4}$
D $\frac{6}{10}$
E $\frac{6}{8}$
F $\frac{8}{18}$
G $\frac{10}{20}$
H $\frac{8}{12}$
I $\frac{6}{16}$
J $\frac{10}{15}$
K $\frac{8}{24}$
L $\frac{18}{20}$
M $\frac{6}{15}$
N $\frac{8}{10}$
O $\frac{6}{12}$
P $\frac{20}{24}$
Q $\frac{18}{30}$
R $\frac{12}{14}$

Test 3
Express each of these improper fractions as a mixed number or as a whole number.

A $\frac{29}{4}$
B $\frac{18}{6}$
C $\frac{25}{3}$
D $\frac{19}{5}$
E $\frac{10}{10}$
F $\frac{24}{16}$
G $\frac{11}{2}$
H $\frac{16}{4}$
I $\frac{39}{3}$
J $\frac{8}{8}$
K $\frac{42}{6}$
L $\frac{13}{4}$
M $\frac{28}{6}$
N $\frac{63}{8}$
O $\frac{42}{12}$
P $\frac{27}{5}$

End-of-block tests on reduction of fractions and ratios 155

155 Checking up

Expanded Notes are not considered necessary for this lesson.

Objectives

The child tests his skill in changing ratios and fraction numerals to equal ratios and fraction numerals, in reducing fraction numerals to simplest form, and in expressing improper fractions as mixed numbers or whole numbers.

Vocabulary

There are no new words.

Comments

The three tests in this lesson can be used to determine the children's understanding of and skill in the reduction of fraction numerals and ratios. Any weakness revealed indicates that a review of the lessons on changing ratios (pages 140-142), changing fraction numerals (pages 143-146), reducing to simplest form (page 146), and changing improper fraction numerals to mixed numbers (pages 151-153) is necessary.

155

1 After these problems have been solved, provide answers so that the children may verify their own work. Then find out which problems caused difficulty and discuss them with the class.

Have the children work independently while they write the answers for each test. Then discuss the tests with the whole class. Note that in discussing Test 1, the children can read the paired numerals as expressing fractions, rates, or comparisons. Or, since they do not really know what the paired numerals represent, they may use the phrase "3 over 1 equals 15 over 5" (for Example A).

Answers

Test 1				Test 2				Test 3			
A	15/5	J	45/99	A	$\frac{2}{3}$	J	$\frac{2}{3}$	A	$7\frac{1}{4}$	J	1
B	5/7	K	7/1	B	$\frac{1}{3}$	K	$\frac{1}{3}$	B	3	K	7
C	16/24	L	1/13	C	$\frac{1}{2}$	L	$\frac{9}{10}$	C	$9\frac{1}{3}$	L	$3\frac{1}{4}$
D	4/5	M	5/1	D	$\frac{3}{5}$	M	$\frac{2}{5}$	D	$3\frac{4}{5}$	M	$4\frac{2}{3}, 4\frac{4}{6}$
E	3/21	N	21/2	E	$\frac{3}{4}$	N	$\frac{4}{5}$	E	1	N	$7\frac{7}{8}$
F	15/24	O	28/32	F	$\frac{4}{9}$	O	$\frac{1}{2}$	F	$1\frac{1}{2}, 1\frac{8}{16}$	O	$3\frac{1}{2}, 3\frac{6}{12}$
G	36/30	P	20/30	G	$\frac{1}{2}$	P	$\frac{5}{6}$	G	$5\frac{1}{2}$	P	$5\frac{2}{5}$
H	6/4			H	$\frac{2}{3}$	Q	$\frac{3}{5}$	H	4		
I	4/3			I	$\frac{3}{8}$	R	$\frac{6}{7}$	I	13		

O Bill spent $2.50 for a canteen, $1.75 for an eating kit, and $1.25 for a first-aid kit. He spent how much more for camping equipment than Don?

P Tony has $1.47. He wants to buy a first-aid kit and an eating kit like Bill's. How much more money does he need?

Q Paul's brother used some of his savings to buy a sleeping bag. After he had spent $16.50 for the sleeping bag, he had $3.50 of his savings left. How much money had he saved?

R Paul is saving 15¢ per week to buy a large canteen that costs $3. How long will it take Paul to save enough money to buy the canteen?

S George earned 85¢. Then he had enough money to buy a knapsack that cost $3.45. How much money did he have before he earned the 85¢?

T Jack had $5. He bought a knapsack like George's and a first-aid kit like Bill's. How much money did he have left?

U Dick sold an old pair of field glasses to Jim for $1.50. Dick had paid $2.95 for the field glasses. He sold them for how much less than he had paid for them?

Checking up

Test 1 **1**

Think of these numerals as either fraction numerals or as ratios. In each example, find the missing numeral.

A $\frac{3}{1}=\frac{15}{\square}$ I $\frac{20}{15}=\frac{\square}{3}$

B $\frac{10}{14}=\frac{\square}{7}$ J $\frac{5}{11}=\frac{45}{\square}$

C $\frac{2}{3}=\frac{\square}{24}$ K $\frac{28}{4}=\frac{7}{\square}$

D $\frac{20}{25}=\frac{4}{\square}$ L $\frac{3}{39}=\frac{\square}{13}$

E $\frac{1}{7}=\frac{\square}{21}$ M $\frac{15}{3}=\frac{5}{\square}$

F $\frac{5}{8}=\frac{15}{\square}$ N $\frac{42}{4}=\frac{\square}{2}$

G $\frac{6}{5}=\frac{\square}{30}$ O $\frac{7}{8}=\frac{\square}{32}$

H $\frac{18}{12}=\frac{6}{\square}$ P $\frac{10}{15}=\frac{20}{\square}$

Test 2

Change each of these fraction numerals to simplest form. **2**

A $\frac{4}{6}$ J $\frac{10}{15}$

B $\frac{3}{9}$ K $\frac{8}{24}$

C $\frac{2}{4}$ L $\frac{18}{20}$

D $\frac{6}{10}$ M $\frac{6}{15}$

E $\frac{6}{8}$ N $\frac{8}{10}$

F $\frac{8}{18}$ O $\frac{6}{12}$

G $\frac{10}{20}$ P $\frac{20}{24}$

H $\frac{8}{12}$ Q $\frac{18}{30}$

I $\frac{6}{16}$ R $\frac{12}{14}$

Test 3

Express each of these improper fractions as a mixed number or as a whole number. **2**

A $\frac{29}{4}$ I $\frac{39}{3}$

B $\frac{18}{6}$ J $\frac{8}{8}$

C $\frac{28}{3}$ K $\frac{42}{6}$

D $\frac{19}{5}$ L $\frac{13}{4}$

E $\frac{10}{10}$ M $\frac{28}{6}$

F $\frac{24}{16}$ N $\frac{63}{8}$

G $\frac{11}{2}$ O $\frac{42}{12}$

H $\frac{16}{4}$ P $\frac{27}{5}$ **3**

End-of-block tests on reduction of fractions and ratios 155

155

1 Direct the children either to rewrite each equation with the numeral that replaces the screen or to write the number pair with a numeral replacing the screen. Instruct them to label each response with its letter.

2 Direct each child to write the new fraction numeral with its labeling letter.

3 Discuss the work when the children have finished. Individual difficulties can be gone over separately at another time. Any reteaching needed by the whole class should be done at this time.

156 Thinking straight

Expanded Notes for this lesson are on pages 315-316.

Objectives

The child practices adding and subtracting mentally by using short cuts.

Vocabulary

New words page 156 Parker*

Comments

When adding and subtracting with pencil and paper, it is customary to start with the ones' column and work through the tens to the hundreds, etc., but in mental calculation the opposite approach is more convenient—that is, the hundreds or tens are computed first, and the ones last. This approach is not entirely strange to the children, since it is the method they use in counting money—beginning with dollars and ending with cents.

There are no "rules" in mental calculation. Do not, then, expect the children to use any one method alone. Rather, encourage them to use as many different methods as possible and to try to become ingenious in inventing other methods. The children should learn to change the numbers to the nearest manageable unit. The nearest "easily manageable unit" in a particular situation will not always be the same for each child. The majority of children will find numbers whose numerals end in zero most convenient, but a few children may find those whose numerals end in "5" also convenient. It might be helpful to begin the lesson with oral practice in rounding numbers upward and downward to the nearest 5, 10, 50, 75, 100, etc.

Remind the children that, in subtracting, if the same amount is either added to or subtracted from *both* numbers before computing, the answer will remain unaffected, since the difference between the two numbers has not been changed.

In working with the exercises in the textbook, no child should be required to try work beyond his capacity. If he needs to make notations on paper to aid

his memory, encourage him to do so. For example, in adding the amounts 156 and 149, he might round off each of them to 150 and write the sum 300 on paper. He should then be able to add 6 and subtract 1 mentally.

In connection with this page, you may want to discuss the value of mental calculation with the class. Pupils who sell newspapers probably can cite many instances in which the ability to calculate mentally is valuable. Other pupils may think of examples in science or the social studies where the ability to compute mentally is very useful. To promote interest in mental calculation, experiences might be provided in the classroom by devoting a few minutes now and then either at the beginning or at the end of the arithmetic period.

Answers

(block 1)

A When she changed 28¢ to 20¢ and 79¢ to 70¢ for easy adding, she did not add the ones. Then she added the ones: 8¢+9¢=17¢. Other ways to add 79 to 28: 79+20=99, 99+8=107; 80+30=110, 110−3=107

B When she changed 156 to 150, she ignored the 6 ones, which she had to add later. When she changed 149 to 150, she added 1 more one than she should have, so she had to subtract it to find the correct answer.

C 260; 260+50=310; Subtract 5

D When she changed 37 to 32 and subtracted it from 62, the 32 was 5 less than the amount she should have subtracted. Other ways to subtract 37 from 62: 67−37=30, 30−5=25; 62−40=22, 22+3=25

E She subtracted 2 more than she should have from a number that was 4 less than the right number. So she needed to add 6.

F Because you need to subtract 24 more than 204 in order to subtract 228 in all

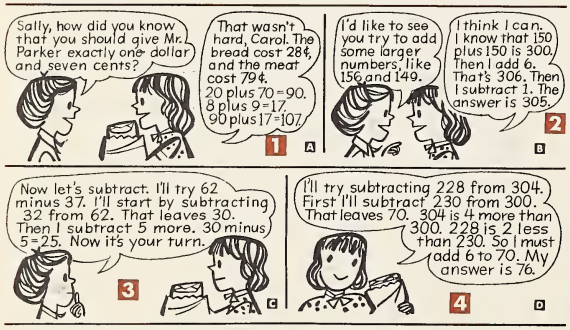
(block 2)

A 76	E 387
B 69	F 500
C 50	G 485
D 121	H 850

(block 3)

A 33	E 42
B 34	F 62
C 64	G 334
D 16	H 342

Thinking straight


Sally, how did you know that you should give Mr. Parker exactly one dollar and seven cents?
That wasn't hard, Carol. The bread cost 28¢, and the meat cost 79¢. 20 plus 70=90. 8 plus 9=17. 90 plus 17=107.
1
A
I'd like to see you try to add some larger numbers, like 156 and 149.
I think I can. I know that 150 plus 150 is 300. Then I add 6. That's 306. Then I subtract 1. The answer is 305.
2
B
Now let's subtract. I'll try 62 minus 37. I'll start by subtracting 32 from 62. That leaves 30. Then I subtract 5 more. 30 minus 5=25. Now it's your turn.
3
C
I'll try subtracting 228 from 304. First I'll subtract 230 from 300. That leaves 70. 304 is 4 more than 300. 228 is 2 less than 230. So I must add 6 to 70. My answer is 76.
4
D

A In Picture A, why did Sally add 17 to 90? Think of another way to add 79 to 28.

B In Picture B, why did Sally add 6 to 300 and then subtract 1?

C Another way to add 149 to 156 is to begin by adding 100 to 160. The sum is [illegible]. Then add 50 to 260. What would you do next?

D In Picture C, why did Carol subtract 5? Think of another way to subtract 37 from 62.

E In Picture D, why did Sally have to add 6 to 70?

F Here is another way to subtract 228 from 304 in your head. Begin by subtracting 204 from 304. Why do you then have to subtract 24 from 100?

5 Add in your head.

A 42+34=n	E 132+255=n
B 19+50=n	F 307+193=n
C 24+26=n	G 289+196=n
D 73+48=n	H 475+375=n

Subtract in your head.

A 50−17=n	E 194−152=n
B 83−49=n	F 201−139=n
C 95−31=n	G 758−424=n
D 42−26=n	H 617−275=n

156

1 Let two girls use Picture A and take the parts of Sally and Carol. The rest of the class should act as a participating audience to check what is being said—mentally, if they can, or by making penciled notes of intermediate stages. Use Exercise A in connection with this picture.

2 Proceed in the same way to dramatize Picture B. Discuss Exercises B and C in connection with this picture.

3 Let two girls dramatize Picture C. Ask another pupil to answer Exercise D in connection with it.

4 Follow the same procedure with a different girl for Picture D and use Exercises E and F with this picture.

5 Let pairs of children or individual pupils work out these exercises. The thinking should be verbalized so that the class can follow it. Let the class decide if they have chosen good ways to do the exercises.

157 Thinking straight

Expanded Notes are not considered necessary for this lesson.

Objectives

The child increases his understanding of area and perimeter.

Vocabulary

New words page 157 covering*, fit*

Comments

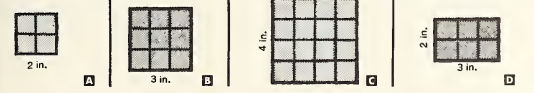
In this lesson, although pupils do not learn the computational method for finding areas (this will come a little later), they do meet some new ideas. For example, the children will see how area changes in relation to a change in the dimensions of a square. They see that if 1 inch is added to each side, the area is increased. And they learn informally what happens to the area when the dimensions of a square are doubled.

Although children are not taught the method for computing the area of a triangle in this grade, background for future understanding is provided. They see that when two congruent right triangles are put together, a rectangle results. They learn informally that the area of one of these triangles is $\frac{1}{2}$ that of the rectangle. Eventually, when they meet a formula for the area of a triangle, such as $A=\frac{1}{2}ba$, they will find it more meaningful. They also learn informally how to find the perimeter of a rectangle when they know the dimensions of the congruent triangles from which it is formed.

Answers

A Rectangle
B Rectangle D
C [A] 4 sq. in.; [B] 9 sq. in.; [C] 16 sq. in.; [D] 6 sq. in.
D 4
E 2 in.; 8 in.
F [B] 12 in.; [C] 16 in.; [D] 10 in.
G Triangles
H Yes; yes
I 12 in.
J Rectangle
K 3 in.; 4 in.; 3 in.; 4 in.; 5 in.
L 14 in.
M 12 sq. in.
N 6 sq. in.

Thinking straight



1 A Pictures A, B, C, and D show polygons. Each polygon is a ——.

B Which rectangle is not a square?

2 C Imagine that the little squares covering each polygon shown above are square inches. What area is shown in each picture?

3 D How many squares the same size as Square A would fit on Square C?

4 E What is the length of each side of Square A? The perimeter of Square A is ☐ in.

F Find the perimeters of Polygons B, C, and D.

5 G The polygons in Picture E below are ——.

6 H Are the triangles in Picture E the same shape? Are they the same size?

6 I Find the perimeter of one of the triangles in Picture E.

6 J Pictures F and G below show what happens when the triangles in Picture E are put together. What kind of polygon do they form?

7 K How long is Side v of Polygon G? Side w? Side x? Side y? Line z?

L The perimeter of Rectangle G is ☐ in.

M What is the area of Rectangle G?

8 N By using the pictures below, you can find the area of one of the triangles in Picture E. Try it.

157

157

1 Discuss how we know that Pictures A, B, and C are squares. Lead pupils to see that the sides of Square B are 1 inch longer than those of Square A and that those of Square C are 2 inches longer.

2 Have the pupils count the inch squares in Polygons A to D to find the areas.

3 Help the pupils determine that 4 squares with the area of Square A would fit on Square C, although the sides of Square C are only 2 inches longer.

4 Discuss how to find perimeters and have the class answer Exercises E and F.

5 Use Exercises G, H, and I with this picture.

6 Point out that each triangle has a right angle and the 2 equal triangles make a rectangle.

7 Get the pupils to relate the sides of Polygon G to those in the triangles in Picture E. Then have them answer Exercises L and M.

8 Lead the pupils to see that the area of one triangle is half that of Rectangle G.

158-159 Moving forward

Exponded Notes for this lesson are on pages 317-318.

Objectives

The child learns how to use fraction numerals to indicate positions on a number line. The child sees that equivalent fraction numerals are different names for the same fraction because they represent the same point on the number line. He uses the number line to compare fractions.

Vocabulary

There are no new words.

Comments

The number lines on pages 158 and 159 provide the child with a quick visual device for comparing one fraction with another. (There is no computational reduction of fraction numerals in this lesson.)

In discussing these pages with the class, continue to make it clear that more than one fraction numeral can represent the same fraction. Thus, for example, the fraction numerals $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, etc., can all be used to describe one half of a pie.

The number line on page 158 is a horizontal number line, reading from left to right. Children should not get the idea that this is the only position such a line may take. For this reason, a vertical number line is shown on page 159. The scale on a number line is arbitrary.

In extending the number system to include fractions, the child now learns that a fraction numeral can serve as a name for a position in a number sequence, just as a whole number does. The number line is a device for clarifying this idea. The fraction numerals correspond to points on the line itself, just as whole numbers can be made to correspond to points on a number line.

The number line is also useful in helping children understand the meaning of equivalent fraction numerals, since it is easy to see that equivalent fraction numerals all correspond to the same point on the number line. Moreover, the number line provides a method for comparing fraction numerals by observing the distances from zero of the points which the fraction numerals represent. Thus, since on the number line $\frac{3}{4}$ is farther from zero than $\frac{7}{16}$, it is clear that $\frac{3}{4}$ is larger than $\frac{7}{16}$.

Each child will need a ruler and a sheet of paper at least 11 in. long for Exercises U and W on page 159. Exercise W is intended for able students. They should be able to use the half-inch marks on their rulers. (Suggestions for helping the slow pupil in making his own number line will be found in the Expanded Notes for this lesson.)

Answers

A 0 (zero)

B [E, I, M]

C [C, E, G, I, K, M, O, Q]

D [E, I, M]

E [E, I, M, Q]

F [I]

G These fraction numerals are at the same point on the number line; therefore, they are equal.

H [G, $\frac{3}{8}$]

I [M, $\frac{6}{8}$, $\frac{12}{16}$]

J $\frac{3}{4}$

K $\frac{1}{2}$

L They stand for parts or fractions of the line. They stand for twelfths, sixths, fourths, thirds, or halves of the line.

M They are all at the same point on the number line and the same distance from 0.

N [J, $\frac{3}{4}$]

O [I, $\frac{4}{6}$, $\frac{8}{12}$]

P $\frac{7}{12}$ is closer to 0 than $\frac{2}{3}$ on the line and is therefore less than $\frac{2}{3}$.

Q $\frac{1}{3}$ is less than $\frac{5}{12}$ because the fraction numeral $\frac{1}{3}$ is not as far from 0 as the fraction numeral $\frac{5}{12}$.

R Less

S More

T By showing that $\frac{6}{12}$ belongs to the same point as $\frac{1}{2}$; $\frac{8}{12}$ belongs to the same point as $\frac{2}{3}$; and $\frac{9}{12}$ belongs to the same point as $\frac{3}{4}$.

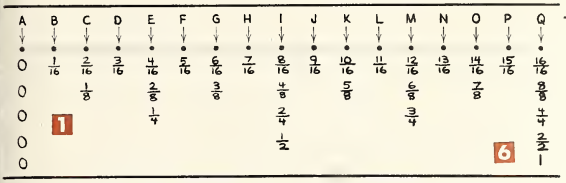
U 10 parts; 0

V 10, 5, 2; no; yes

W You can also use tenths, fifths, fourths, and halves.

Moving forward

A number line can be used to show whether one fraction is larger or smaller than other fractions.



A The picture above shows a number line. This number line ends with the number 1. It begins with what number?

B Arrows A, B, C, D, and so on show points that divide the line into equal parts. The fraction numerals below the points stand for parts of the line. Beginning with 0, read the fraction numerals that have 16 as the denominator. What point shows $\frac{4}{16}$ of the number line? $\frac{8}{16}$ of the number line? $\frac{12}{16}$ of the number line?

C Read the fraction numerals that have 8 as the denominator. What points show these parts of the number line?

D What point shows $\frac{2}{8}$ of the number line? $\frac{6}{8}$ of the line? $\frac{8}{8}$ of the line?

E Read the fraction numerals that have 4 as the denominator. What points show these parts of the number line?

F One point shows $\frac{1}{2}$, $\frac{2}{4}$, $\frac{4}{8}$, and $\frac{8}{16}$ of the number line. Name this point.

G The fraction numerals $\frac{8}{16}$, $\frac{4}{8}$, $\frac{2}{4}$, and $\frac{1}{2}$ stand for the same point on the number line. How does this show that the fraction numerals are equal?

H What point shows $\frac{6}{16}$ of the number line? What other fraction numeral stands for the same part of the number line? $\frac{6}{16} = \frac{\square}{8}$

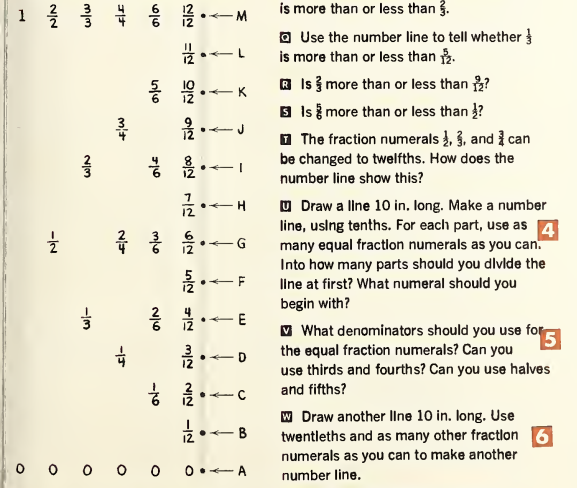
I What point shows $\frac{3}{4}$ of the number line? What other fraction numerals stand for the same part of the number line? $\frac{3}{4}$ equals what other fraction numerals?

J Which is more, $\frac{3}{4}$ of the number line or $\frac{5}{8}$ of the number line?

K Which is less, $\frac{1}{2}$ of the number line or $\frac{9}{16}$ of the number line?

L Look at the number line below. What do the fraction numerals at the left of the line stand for?

M How does this number line show that $\frac{6}{12}$, $\frac{3}{6}$, $\frac{2}{4}$, and $\frac{1}{2}$ are all names for the same part of the number line?



N What point shows $\frac{9}{12}$ of this number line? What other fraction numeral is equal to $\frac{9}{12}$?

O What point shows $\frac{2}{3}$ of the number line? What other fraction numerals equal $\frac{2}{3}$?

P Use the number line to tell whether $\frac{7}{12}$ is more than or less than $\frac{2}{3}$.

Q Use the number line to tell whether $\frac{1}{3}$ is more than or less than $\frac{5}{12}$.

R Is $\frac{2}{3}$ more than or less than $\frac{9}{12}$?

S Is $\frac{3}{8}$ more than or less than $\frac{1}{2}$?

T The fraction numerals $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ can be changed to twelfths. How does the number line show this?

U Draw a line 10 in. long. Make a number line, using tenths. For each part, use as many equal fraction numerals as you can. Into how many parts should you divide the line at first? What numeral should you begin with?

V What denominators should you use for the equal fraction numerals? Can you use thirds and fourths? Can you use halves and fifths?

W Draw another line 10 in. long. Use twentieths and as many other fraction numerals as you can to make another number line.

Now you know that you can compare fractions by arranging fraction numerals on a number line.

158

1. Direct attention to the number line and have someone answer the question in Exercise A.
2. Point out that the dots are all an equal distance apart, and that some points on the line have more than one fraction numeral below them. Then let pupils answer the questions.
3. Let the pupils take turns answering the questions in Exercises C to I.
4. Ask such questions as: "Which point is farther from zero, Point M or Point K? Does $\frac{3}{4}$ go with Point M on the number line? What other fraction numerals go with Point M? What fraction numerals go with Point K? Is $\frac{12}{16}$ more than $\frac{10}{16}$? How does the number line show that $\frac{3}{4}$ is more than $\frac{5}{8}$?"
5. Use the same method you used in Exercise J.
6. You might now ask, "Could we make this line go beyond 1? If we made a line from 0 to 2 and divided the distance from 1 to 2 into 16 equal parts, what numerals would we put on the first point after 1?" [$\frac{17}{16}$, $1\frac{1}{16}$]

159

1. Have a group discussion about the differences and similarities between this number line and the one on page 158.
2. Have the pupils note that all these fraction numerals go with Point G on the number line.
3. Pupils should be able to answer Exercises N to T by using the number line.
4. Let the pupil discover that he must divide his line into 10 equal parts. He should know that zero is assigned first to a point on the line.
5. The pupil should remember that tenths can be reduced to fifths and halves. Ask questions such as: "Where will you put $\frac{1}{5}$? Where will you write $\frac{2}{5}$? What other fraction numeral can you write for $\frac{6}{10}$?"
6. Help pupils who attempt this exercise by asking questions such as those suggested for the previous exercise.

160-162 Moving forward

Expanded Notes for this lesson are on pages 318-320.

Objectives

The child learns to compare fractions expressed by different denominators by replacing the fraction numerals with equivalent fraction numerals that have a common denominator.

Vocabulary

New words page 160 dotted; page 161 common*

Comments

Earlier lessons that provide readiness for the work in this lesson are: the number line (pages 158-159); reduction of fractions (pages 143-146); learning that multiplying or dividing both divisor and dividend by the same number does not change the quotient (pages 136-137); finding what to multiply by to get a known product (pages 138-139). All of this preceding work is related to finding a common denominator.

Before pupils can compute to find a common denominator, they must understand what the term means. This understanding can best be brought about by using a comparison situation. It must be demonstrated that to find how much larger one fraction is than another, the fraction numerals must be replaced by, or "changed to," equivalent fraction numerals that have the same denominator. The children must see that if they want to compare $\frac{3}{4}$ of a pie with $\frac{2}{3}$ of an equal-sized pie, they cannot make this comparison precisely until they think of the $\frac{3}{4}$ pie and the $\frac{2}{3}$ pie as twelfths of a pie.

On page 160 the pupils are asked to compare $\frac{2}{3}$ of one piece of string with $\frac{3}{5}$ of another piece of string of the same length. They learn that they must think of these fractional parts of the pieces of string as expressed by fractions with the same denominator; that is, they must find other fraction numerals to express these quantities. Picture B shows that $\frac{3}{5}$ can be thought of as $\frac{6}{10}$, but that $\frac{2}{3}$ cannot be expressed in tenths. Picture C shows that $\frac{3}{5}$ can be thought of as $\frac{9}{15}$ and that $\frac{2}{3}$ can be thought of as $\frac{10}{15}$. Now it can be determined that $\frac{2}{3}$ of one piece of string is $\frac{1}{15}$ longer than $\frac{3}{5}$ of the other piece of string.

Page 161 shows how unequal fractional parts of equal-sized rectangles can be compared. Now the children are ready to learn how to compare different fractions computationally.

On page 162, be sure the pupils understand that sometimes the larger denominator can be used as the common denominator. Also emphasize the idea that it usually is easier to compare fractions when the smallest possible common denominator is used.

No formal rules for finding a common denominator need be given in this lesson.

Answers

(block 1)

A Yes, $\frac{3}{5}$ of the brown string is shorter than $\frac{2}{3}$ of the red string. No.

B No; You cannot tell how much shorter $\frac{3}{5}$ of the string is than $\frac{2}{3}$ of the string because the strings are divided into fifths and thirds only, and there are no smaller parts to show how much larger one fraction is than another.

C No; There are 6 tenths in $\frac{3}{5}$, and you can tell that there are more than 6 tenths but fewer than 7 tenths in $\frac{2}{3}$. But you still cannot tell how much shorter $\frac{3}{5}$ is than $\frac{2}{3}$.

D $\frac{3}{5}=\frac{9}{15}$; $\frac{2}{3}=\frac{10}{15}$; $\frac{3}{5}$ of the string is $\frac{1}{15}$ shorter than $\frac{2}{3}$ of the string.

E Yes, $\frac{6}{10}$

F Yes, $\frac{10}{15}$; yes, $\frac{9}{15}$

G 15

H $\frac{3}{4}$; $\frac{5}{6}$

I No

J No

K Yes; yes

L $\frac{10}{12}$; $\frac{9}{12}$; larger

M No; $\frac{3}{4}$ cannot be expressed as eighteenths because 18 is not divisible by 4.

N Yes; Both $\frac{3}{4}$ and $\frac{5}{6}$ can be expressed as twenty-fourths because 24 is divisible by both 4 and 6.

O $\frac{5}{6}=\frac{10}{12}$, $\frac{3}{4}=\frac{9}{12}$; $\frac{5}{6}=\frac{20}{24}$, $\frac{3}{4}=\frac{18}{24}$; $\frac{5}{6}=\frac{30}{36}$, $\frac{3}{4}=\frac{27}{36}$; $\frac{5}{6}=\frac{40}{48}$, $\frac{3}{4}=\frac{36}{48}$

P 12

Q No; $\frac{3}{4}$ cannot be expressed as tenths because 10 is not divisible by 4.

R Yes; $\frac{14}{20}$; $\frac{15}{20}$

S $\frac{14}{20}$, $\frac{15}{20}$; $\frac{3}{4}$ is larger.

T $\frac{5}{6}$; yes; It can be divided by both 12 and 6. $\frac{10}{12}$
U No; no
V Yes; $\frac{20}{24}$; $\frac{21}{24}$; $\frac{7}{8}$

(block 2)

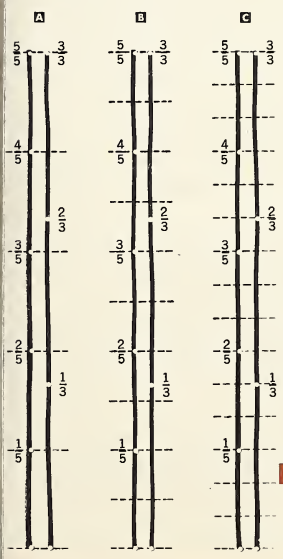
A $\frac{10}{15}, \frac{12}{15}, \frac{2}{3}$
B $\frac{2}{8}, \frac{3}{8}, \frac{1}{4}$
C $\frac{12}{16}, \frac{11}{16}, \frac{11}{16}$
D $\frac{10}{16}, \frac{9}{16}, \frac{9}{16}$
E $\frac{4}{9}, \frac{3}{9}, \frac{1}{3}$
F $\frac{14}{24}, \frac{9}{24}, \frac{3}{8}$
G $\frac{5}{30}, \frac{9}{30}, \frac{1}{6}$
H $\frac{7}{8}, \frac{6}{8}, \frac{3}{4}$
I $\frac{7}{10}, \frac{4}{10}, \frac{2}{5}$
J $\frac{8}{16}, \frac{7}{16}, \frac{7}{16}$
K $\frac{11}{12}, \frac{9}{12}, \frac{3}{4}$
L $\frac{20}{30}, \frac{27}{30}, \frac{2}{3}$
M $\frac{9}{18}, \frac{8}{18}, \frac{4}{9}$
N $\frac{15}{24}, \frac{16}{24}, \frac{5}{8}$
O $\frac{15}{18}, \frac{14}{18}, \frac{7}{9}$
P $\frac{8}{12}, \frac{7}{12}, \frac{7}{12}$

Keeping skillful:

(block 1)	(block 2)	(block 3)
A 12,487	A 3780	A 26,536
B 151	B $52.75	B $1.93
C $13.47	C 6425	C 68 and 19 rm.
D 16,304	D 2397	D $261.95
E 1394	E 190	E 102,432
F $124.63	F $23.28	F $.92
G 592	G 1133	G 117 and 40 rm.
H $127.48	H $15.34	H 36
	I 376	I $955.57

Moving forward

Now you will learn what denominator to use to find out which of two fractions is larger or smaller.



A Picture A shows two strings that are the same length. The brown string is divided into fifths. The red string is divided into thirds. Can you tell from the picture whether $\frac{3}{5}$ of the brown string is longer or shorter than $\frac{2}{3}$ of the red string? Can you tell how much longer or shorter it is? **1**

B Notice in Picture A that the dotted lines divide both strings into fifths. Use the picture to count the number of fifths in $\frac{2}{3}$. Now can you tell how much shorter $\frac{3}{5}$ is than $\frac{2}{3}$? Why or why not? **2**

C The same strings are shown in Picture B. The dotted lines show tenths. Count the tenths in $\frac{3}{5}$. Count the tenths in $\frac{2}{3}$. Now can you tell how much shorter $\frac{3}{5}$ is than $\frac{2}{3}$? Why or why not? **3**

D Picture C shows the same strings again. The dotted lines show fifteenths. Count the fifteenths in $\frac{3}{5}$. **4** Count the fifteenths in $\frac{2}{3}$. Now you should be able to tell how much shorter $\frac{3}{5}$ of the string is than $\frac{2}{3}$ of the string.

160

1 Have the pupils read Exercise A. Have them note the brown string marked in fifths and the red string marked in thirds in Picture A. Pupils should see that $\frac{3}{5}$ is less than $\frac{2}{3}$, but that it is not easy to tell exactly how much less.

2 Have the pupils read Exercise B and use Picture A to answer the questions. The children should realize that it is difficult to express $\frac{2}{3}$ as fifths since a dotted line does not go through the $\frac{2}{3}$ point mark on the red string.

3 Let the pupils read this and note the division of the strings into tenths in Picture B. They should see that the points for $\frac{3}{5}$ and $\frac{6}{10}$ are opposite and that $\frac{2}{3}$ is between $\frac{6}{10}$ and $\frac{7}{10}$

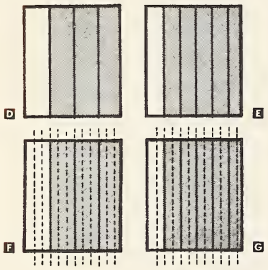
4 The pupils should observe the division into fifteenths in Picture C and that the points for $\frac{3}{5}$ and $\frac{9}{15}$ are opposite; and similarly for $\frac{2}{3}$ and $\frac{10}{15}$. They can now see that $\frac{3}{5}$ is $\frac{1}{15}$ less than $\frac{2}{3}$.

E In Picture A you cannot tell how many fifths there are in $\frac{2}{3}$. In Picture B you cannot tell how many tenths there are in $\frac{2}{3}$. Can you tell how many tenths there are in $\frac{3}{5}$?

F Now look at Picture C. Can you tell how many fifteenths there are in $\frac{2}{3}$? In $\frac{3}{5}$?

G Before you try to say how much smaller $\frac{3}{5}$ is than $\frac{2}{3}$, you change both fraction numerals to other fraction numerals with the denominator ■. 15 is the common denominator because $\frac{3}{5}$ and $\frac{2}{3}$ can each be expressed as a number of fifteenths.

H Rectangles D, E, F, and G are all the same size. ■/4 of Rectangle D is red. ■/6 of Rectangle E is red.



I Now find whether $\frac{5}{6}$ is larger or smaller than $\frac{3}{4}$. You have learned that you can compare fractions when the fraction numerals you use have the same denominator. Do $\frac{3}{4}$ and $\frac{5}{6}$ have the same denominator?

J You should find a common denominator for $\frac{3}{4}$ and $\frac{5}{6}$. $\frac{5}{6}$ has the larger denominator. Can you express $\frac{3}{4}$ as a number of sixths? 6 is not a common denominator for $\frac{3}{4}$ and $\frac{5}{6}$.

K Pictures F and G show Rectangles D and E marked off in twelfths. Can you tell how many twelfths there are in $\frac{3}{4}$? In $\frac{5}{6}$? 12 is a common denominator for $\frac{3}{4}$ and $\frac{5}{6}$.

L $\frac{5}{6}=\frac{■}{12}$. $\frac{3}{4}=\frac{■}{12}$. Is $\frac{5}{6}$ larger or smaller than $\frac{3}{4}$?

M You can express $\frac{5}{6}$ as $\frac{15}{18}$. Could you use 18 as a common denominator when you compare $\frac{5}{6}$ and $\frac{3}{4}$? Why?

N Could you use 24 as a common denominator when you compare $\frac{5}{6}$ and $\frac{3}{4}$? Why?

O There are many denominators, such as 12, 24, 36, and 48, that you can use when comparing $\frac{5}{6}$ and $\frac{3}{4}$. 12, 24, 36, and 48 can all be divided by both 6 and 4 without a remainder. Use each of these four denominators to compare $\frac{5}{6}$ and $\frac{3}{4}$.

Now turn the page.

P When you compare fractions, it is usually easier to use the smallest common denominator. What is the smallest common denominator for $\frac{5}{6}$ and $\frac{3}{4}$?

Q Which is larger, $\frac{3}{4}$ or $\frac{7}{10}$? You should find a common denominator for $\frac{3}{4}$ and $\frac{7}{10}$. Should you use the larger denominator, 10, as a common denominator? Why or why not?

R Try 20 as a common denominator. Can 20 be divided by both 10 and 4? Can you tell how many twentieths there are in $\frac{7}{10}$? In $\frac{3}{4}$?

S $\frac{7}{10}=\frac{■}{20}$. $\frac{3}{4}=\frac{■}{20}$. Which is larger, $\frac{3}{4}$ or $\frac{7}{10}$?

T Which is smaller, $\frac{5}{6}$ or $\frac{11}{12}$? Can you use the larger denominator, 12, as a common denominator? Why or why not? $\frac{5}{6}=\frac{■}{12}$. Which is smaller, $\frac{5}{6}$ or $\frac{11}{12}$?

U Which is larger, $\frac{5}{6}$ or $\frac{7}{8}$? Should you use 8 as a common denominator? Should you use 16?

V Try 24 as a common denominator. Can 24 be divided by both 6 and 8? $\frac{5}{6}=\frac{■}{24}$. $\frac{7}{8}=\frac{■}{24}$. Which is larger, $\frac{5}{6}$ or $\frac{7}{8}$?

For each pair of fraction numerals below, change the numerals so that they have a common denominator. Then for each pair decide which expresses the smaller fraction.

A	$\frac{2}{3}, \frac{4}{5}$	E	$\frac{4}{9}, \frac{1}{3}$	I	$\frac{7}{10}, \frac{2}{5}$	M	$\frac{1}{2}, \frac{4}{9}$
B	$\frac{1}{4}, \frac{3}{8}$	F	$\frac{7}{12}, \frac{3}{8}$	J	$\frac{1}{3}, \frac{1}{16}$	N	$\frac{5}{8}, \frac{2}{3}$
C	$\frac{3}{4}, \frac{11}{16}$	G	$\frac{1}{6}, \frac{3}{10}$	K	$\frac{11}{12}, \frac{3}{4}$	O	$\frac{5}{6}, \frac{7}{9}$
D	$\frac{5}{8}, \frac{9}{16}$	H	$\frac{7}{8}, \frac{3}{4}$	L	$\frac{2}{3}, \frac{9}{10}$	P	$\frac{2}{3}, \frac{7}{12}$

Now you should be able to find the larger or smaller of two fractions that have different denominators.

Keeping skillful

Find the sum.

A	8723, 604, 3160	A	$n+5320=9100$	A	$248\times107=n$
B	48, 17, 23, 6, 57	B	$\$80.11-\$27.36=n$	B	$\$110.01\div n=57$
C	\$6.44, \$5.03, \$.98, \$1.02	C	$n-402=6023$	C	$5731\div84=n$
D	9046, 7258	D	$5843+n=8240$	D	$65\times\$4.03=n$
E	55, 493, 782, 64	E	$290-n=100$	E	$776\times132=n$
F	\$20.00, \$11.85, \$92.78	F	$\$52.08+n=\$75.36$	F	$\$45.08\div n=49$
G	46, 19, 87, 402, 38	G	$4310-3177=n$	G	$5071\div43=n$
H	\$7.40, \$32.96, \$87.12	H	$n-\$9.94=\$5.40$	H	$2844\div n=79$
		I	$n+625=1001$	I	$803\times\$1.19=n$

161

1 Use Exercises E, F, and G to help the pupils verbalize what they discovered on page 160. The children must see that before $\frac{3}{5}$ and $\frac{2}{3}$ can be compared, the numerals must be changed to equivalent fraction numerals with the same denominator.

2 Have the children use Rectangles D and E in finding the answers for Exercises H and I.

3 Ask them to recall how to change a fraction numeral to an equivalent fraction numeral. (Multiply or divide both terms by the same number.) They should see that $\frac{3}{4}$ cannot be changed to sixths, since 6 is not divisible by 4.

4 Have the pupils use Pictures F and G to find the number of twelfths in $\frac{3}{4}$ and $\frac{5}{6}$.

5 Let the pupils use Pictures F and G to find the answers, or let them compute.

6 Let the children explain why $\frac{3}{4}$ cannot be expressed as eighteenths.

7 Have the pupils explain why the denominators mentioned in Exercises N and O can be used.

162

1 Be sure the children understand why it is best to use the smallest common denominator. [It saves work; fewer mistakes are made.]

2 First get the pupils to see why 10 is not a common denominator. [10 is not divisible by the denominator 4.] Then suggest that they try a larger number. Suggest that they multiply 10 by 2 and see if the product is divisible by both 10 and 4. Then have the pupils change $\frac{7}{10}$ and $\frac{3}{4}$ to twentieths to find which is larger.

3 Pupils should see that the larger denominator, 12, can be divided by 6, so they can change $\frac{5}{6}$ to twelfths and then compare.

4 Develop the idea that pupils should first examine the larger denominator. Then they should try multiples of it until they find one that can be used.

5 Assign these exercises as written work.

6 Assign these exercises as written work in a separate lesson.

163-164 Using arithmetic

Expanded Notes are not considered necessary for this lesson.

Objectives

The child solves problems of the types he has studied thus far in the program. In the "Keeping skillful" he extends his knowledge of multiplication to numbers which have four-figure numerals. He also practices solving different types of multiplicative equations.

Vocabulary

New words page 163 California*, women*, through*, Fort Laramie, oxen*, mules, coffee, Sunday, gunpowder, lead*; page 164 reached*

Comments

It is always advisable to let children work a set of problems independently before there is any general discussion. Then the child is obliged to think through each problem for himself, and you can more easily detect any problems that cause difficulty. Always require the children to write an equation for each problem and use the letter *n* to hold a place for the unknown number. When the answer has been found, the equation should be rewritten with the answer supplied in place of *n*.

Warn the pupils that there are some problems that require more than one step in their solution and also some problems that make use of information or answers from previous problems. Remind the pupils that when a problem has more than one step, they are to write the master equation first, using *n* for the answer they are to find and a screen (▤) for the missing information needed before the master equation can be solved. Then they should write the necessary computation and, finally, the master equation with numerals to replace *n* and the screen.

The "Keeping skillful" exercises should be treated as a separate assignment. Examples A and B introduce multiplication using four-figure numerals. Be sure the pupils understand that multiplying thousands is the same in principle as multiplying hundreds or tens.

Answers

Using arithmetic:

A 37,570+825+1126=[39,521]
B 39,521−30,032=[9489]
C 31,502+22,878+7650+5754=[67,784]
D 4×(365)=[1460] or (600)+(600)+(100)+(160)=[1460]
E [245]−95=150
F [8]+17=25
G 6×$65=[$390]
H ($900)−$390=[$510] or 6×($85)=[$510]
I (119)÷[17]=7
J 25×(15)=[375]
K 126÷[18]=7
L 2500÷18=[138] and 16 rm., about 139 days
M 59−[6]=53
N 150−[133]=17
O $1275÷[$75]=17 or 17×[$75]=$1275
P 7×$1.50=[$10.50]
Q $10.00÷[$.83]=12 and 4¢ rm., about $.83

Keeping skillful:

(block 1)

C 8742	H $85.38	M 76,342	R $229.60
D 10,660	I $157.50	N 223,182	S $962.50
E 7483	J $295.50	O 446,772	T $1567.88
F 19,890	K $153.78	P 236,529	U $1544.16
G 40,056	L $282.12	Q 103,040	V $1410.00

(block 2)

A 22,968	D 233,685	G 21,297	J 62,775
B 450	E $3.67	H 208	
C 692	F 938	I 507	

Using arithmetic 1

A The fifth graders were reading about the people who went to California in 1849 and 1850 to look for gold. 37,570 men, 825 women, and 1126 children traveled through Fort Laramie on their way to California in the first 6 months of 1850. How many persons traveled through Fort Laramie in the 6 months?

B A total of 30,032 persons traveled through Fort Laramie in the year 1849. How many more people traveled through the town in the first 6 months of 1850 than in all of 1849?

C In 1850 the travelers had with them 31,502 oxen, 22,878 horses, 7650 mules, and 5754 cows. They had how many animals in all with them?

D Before the Cook family started for California, they planned to take with them as food for each person 150 lb. of flour, 150 lb. of bacon, 25 lb. of coffee, and 40 lb. of sugar. How many pounds of food did the Cooks plan to take with them for their family of four?

E Jim Burns sold 95 lb. of his flour. He still had 150 lb. for himself. How much flour had he to begin with?

F Jack Bell bought 17 lb. of coffee. Then he had the 25 lb. of coffee that he needed for himself. How many pounds had he to begin with?

G Mr. Cook bought 6 oxen for $65 apiece. How much did he pay for all of the oxen?

H He hoped to sell the oxen for $150 apiece when he got to California. He hoped to get how much more for the 6 oxen than he had paid for them?

I The Cook family joined a train of 14 wagons to make the trip. The first week the wagon train traveled 21 mi. on Monday, 19 mi. on Tuesday, 16 mi. on Wednesday, 18 mi. on Thursday, 14 mi. on Friday, 15 mi. on Saturday, and 16 mi. on Sunday. The first week the wagon train traveled an average of how many miles per day?

J Each man took 5 lb. of gunpowder and 10 lb. of lead. How much did all the gunpowder and lead weigh that the 25 men in the wagon train took?

K One week the wagon train traveled 126 mi. This was an average of how many miles per day?

L Many wagon trains traveled a distance of 2500 mi. If they traveled at the average speed given in Problem K, about how many days did it take them to get to California?

M Of the 59 persons who started out with the Cooks' wagon train, only 53 reached California. The rest died or turned back. How many persons died or turned back?

N Near the end of the trip the Cook family had only 17 lb. of bacon left for each person. How many pounds of each person's supply of bacon had been eaten?

O In California Jim Burns found 17 oz. of gold in 6 days. He was paid $1275 for it. How much was he paid for 1 oz. of the gold?

P In California it cost $1.50 to have one shirt washed and ironed. How much did Jim pay to have 7 shirts washed and ironed?

Q Eggs cost $10 a dozen. About how much did 1 egg cost? 1

Keeping skillful 2

A $6 \times 3209 = n$

$$\begin{array}{r} 3209 \\ \times\ \ 6 \\ \hline 19254 \end{array}$$

The 19 means thousands because you multiplied 3000 by 6. Why is 19 written here and not 18?

B $27 \times \$18.70 = n$

$$\begin{array}{r} \$18.70 \\ \times\ \ 27 \\ \hline 130\,90 \\ 374\,00 \\ \hline \$504.90 \end{array}$$

Why are a point and a dollar sign written in the answer?

C $2 \times 4371 = n$
D $2665 \times 4 = n$
E $7 \times 1069 = n$
F $3978 \times 5 = n$
G $8 \times 5007 = n$
H $3 \times \$28.46 = n$
I $9 \times \$17.50 = n$
J $6 \times \$49.25 = n$
K $2 \times \$76.89 = n$
L $3 \times \$94.04 = n$

M $38 \times 2009 = n$
N $54 \times 4133 = n$
O $7206 \times 62 = n$
P $41 \times 5769 = n$
Q $4480 \times 23 = n$
R $16 \times \$14.35 = n$
S $22 \times \$43.75 = n$
T $76 \times \$20.63 = n$
U $48 \times \$32.17 = n$
V $25 \times \$56.40 = n$

A $88 \times 261 = n$
B $n \times 73 = 32850$
C $9 \times n = 6228$
D $405 \times 577 = n$
E $6 \times n = \$22.02$
F $n \times 52 = 48776$
G $93 \times 229 = n$
H $n \times 8 = 1664$
I $46 \times n = 23322$
J $81 \times 775 = n$

163

1 Ask the pupils to work Problems A to Q (M to Q are on page 164). Direct them first to write an equation with the letter *n* to hold a place for the unknown number (or, in the case of two-step problems, the letter *n* and a screen). Next they are to write any computation necessary to find the answers, and finally they are to rewrite the equation with the answer in it. Have them label the problems with the identifying letters used in the book.

164

1 When the pupils have completed this problem set, provide them with answers, and let them verify their work. Discuss thoroughly any problems that have caused trouble.

2 Discuss Examples A and B. The pupils should see that multiplying thousands is no different from multiplying hundreds or tens. Then assign Exercises C to V and A to J as written work. Provide the pupils with answers and let them verify their work.

165-168 Moving forward

Expanded Notes for this lesson are on pages 321-323.

Objectives

The child learns that fractional parts of things can be combined or separated just as whole things can be combined or separated. He learns that fractions can be added or subtracted if the numerals representing them have a common denominator.

Vocabulary

New words page 165 combined, possible*; page 168 contains*

Comments

This lesson extends additive and subtractive ideas to fractions; actual computation will be taught in subsequent lessons. Here the pupils are shown that addition and subtraction of fractions involve the same basic ideas of combining and separating as were involved in addition and subtraction of whole numbers. But the pupils will also discover in this lesson that some new techniques are needed to get the answer in its simplest form. They should see that they cannot proceed to add or subtract unless the fraction numerals have a common denominator.

The exercises at the bottom of page 168 provide the pupils with practice in finding common denominators for fraction numerals as preparation for addition and subtraction of fractions. The children are not expected to add or subtract the fractions, but only to find the common denominators they would use. If you want the able pupils to change the fraction numerals to equivalent fraction numerals with common denominators, you may let them.

Answers

Page 168:

Do not insist that the pupils give the smallest common denominator. Accept any correct denominator.

A 4	E 6	I 10	M 12	Q 16	U 24
B 10	F 12	J 12	N 15	R 24	V 20
C 6	G 12	K 16	O 8	S 10	W 30
D 10	H 8	L 8	P 12	T 9	X 30

Moving forward

In this lesson you will see that fractions of things can be combined or separated. You will also learn when it is possible to add or subtract fractions.

1

$\frac{1}{2}$ of one pie has been put with $\frac{1}{2}$ of another pie of the same size. Do you know what numeral to use to tell how much pie there is in all? Why or why not?

Picture A shows that $\frac{1}{2}+\frac{1}{2}=1$.

2

$\frac{1}{8}$ of one cake has been put with $\frac{1}{8}$ of another cake of the same size. Do you know what numeral to use to tell how much cake there is in all? Why or why not?

3

Picture C shows a way to find out how much cake there is in Picture B.

Imagine cutting the $\frac{1}{2}$ cake into equal parts, so that each equal part is $\frac{1}{8}$ of a whole cake.

How many one eighths are there in $\frac{1}{2}$?

4

Picture D shows that you can use $\frac{5}{8}$ to tell how much cake there is in all.

The picture shows that $\frac{1}{2}+\frac{1}{8}=\frac{5}{8}$.

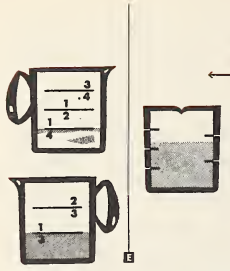
165

1 Using the text and both scenes of Picture A, the pupils should have no difficulty seeing that $\frac{1}{2}+\frac{1}{2}=1$.

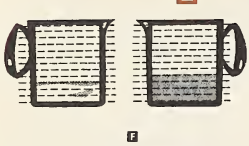
2 Have the pupils read the text. Then let them identify the $\frac{1}{2}$ of a cake and the $\frac{1}{8}$ of a cake in the first scene of Picture B and note that when the fractions of a cake are combined in Scene 2, they do not know how to tell how much cake there is.

3 Remind the class that in a previous lesson, when they were comparing one fraction with another, they had to change the fraction numerals to equivalent fraction numerals so that the two fraction numerals had a common denominator. The pupils should see that they can change the one half to eighths and that, to do so, they imagine the whole cake has been cut into eighths. Then they can see that $\frac{1}{2}$ of a cake is the same as $\frac{4}{8}$ of a cake.

4 Using Picture D, the pupils can count the total number of eighths in $\frac{1}{2}+\frac{1}{8}$.

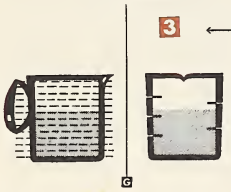


1 $\frac{1}{3}$ cup of juice has been combined with $\frac{1}{4}$ cup of water. Do you know what numeral to use to tell what fraction of a cup of liquid there is in all? Why or why not?



2 In Picture F you imagine that both cups have been marked to show twelfths. You think of twelfths because 12 can be divided by both ▩ and ▩.

How many one twelfths are there in $\frac{1}{4}$?
How many one twelfths are there in $\frac{1}{3}$?

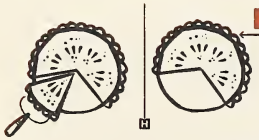


3 There is what fraction of a cup of liquid when the juice and water are combined?

The pictures show that $\frac{1}{4}+\frac{1}{3}=\frac{7}{12}$.

If the cups had been marked in eighths, would you know how much liquid there is? Why or why not?

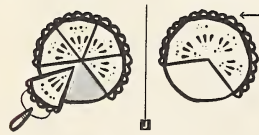
166



1 $\frac{1}{6}$ of a pie has been taken from $\frac{5}{6}$ of a pie. Can you tell what fraction of the pie is left? Why or why not?

Imagine cutting the $\frac{5}{6}$ of a pie into equal parts, so that each equal part is $\frac{1}{6}$ of the whole pie. How many equal parts will there be?

2



Now the picture shows that you can use $\frac{4}{6}$ to tell what fraction of the pie is left after $\frac{1}{6}$ is taken away.

The pictures show that $\frac{5}{6}-\frac{1}{6}=\frac{4}{6}$.

3

$\frac{1}{3}$ of a candy bar is broken off a whole candy bar. Can you tell what fraction of the bar is left?

4

5 Think of the whole candy bar as marked off into 3 equal parts.
When $\frac{1}{3}$ is taken away, what fraction is left?

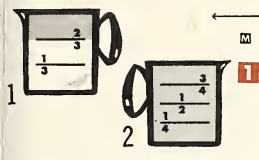
The pictures show that $1-\frac{1}{3}=\frac{2}{3}$.

166

1. Have the pupils read the text and relate it to Scenes 1 and 2 of Picture E. Be sure they see that the customary markings for thirds and fourths on measuring cups will not tell them just how much liquid is in the cup in Scene 2.
2. Discuss with the pupils why it is a good idea to imagine that the cups have now been marked off in twelfths. Using Picture F, they should be able to count to find the number of twelfths in $\frac{1}{4}$ and $\frac{1}{3}$.
3. Explain to the pupils that Scene 1 of Picture G shows the liquid combined in a cup marked off in twelfths, and they can count the total number of twelfths. The second cup is the same as the one in Scene 2 of Picture E, so the pupils can compare and see that the same amount of liquid is shown.

167

1. Pupils should use Picture H to answer the questions in the accompanying text.
2. Point out to the class that, since they are dealing only with sixths, they already have a common denominator and simply need to find out how many sixths they had at first and how many sixths were left.
3. Have pupils use Picture J to verify the statements made in the text opposite it.
4. Treat Picture K and its text as you did Picture H.
5. Pupils should use Picture L to answer the question in the text. Point out that Scene 2 of Picture L shows the same part of a candy bar [$\frac{2}{3}$] as Scene 2 of Picture K.



Cup 1 contains $\frac{2}{3}$ cup of milk. Cup 2 is empty. If you pour $\frac{1}{2}$ cup of milk from Cup 1 into Cup 2, how much milk will be left in Cup 1?

M 1

Imagine that both cups have been marked to show sixths. You think of sixths because 6 can be divided by both ▦ and ▥.

2

N How many one sixths are there in $\frac{2}{3}$?
How many one sixths are there in $\frac{1}{2}$?

When you have poured 3 sixths of a cup into Cup 2, what fraction of a cup of milk is left in Cup 1?

3 The pictures show that $\frac{2}{3}-\frac{1}{2}=\frac{1}{6}$.

Now you see that you can add fractions or subtract one fraction from another. But first you must make sure that the fraction numerals have a common denominator. What denominator would you use in adding or subtracting the fractions in each of these exercises? 4

A $\frac{3}{4}, \frac{1}{2}$	E $\frac{5}{6}, \frac{2}{3}$	I $\frac{3}{5}, \frac{1}{2}$	M $\frac{1}{4}, \frac{1}{6}$	Q $\frac{5}{8}, \frac{3}{16}$	U $\frac{7}{8}, \frac{1}{3}$
B $\frac{9}{10}, \frac{1}{2}$	F $\frac{3}{4}, \frac{1}{3}$	J $\frac{11}{12}, \frac{2}{3}$	N $\frac{4}{5}, \frac{2}{3}$	R $\frac{7}{8}, \frac{1}{6}$	V $\frac{1}{4}, \frac{1}{5}$
C $\frac{1}{2}, \frac{1}{3}$	G $\frac{5}{12}, \frac{1}{4}$	K $\frac{9}{16}, \frac{1}{2}$	O $\frac{3}{4}, \frac{3}{8}$	S $\frac{2}{5}, \frac{1}{10}$	W $\frac{7}{10}, \frac{1}{3}$
D $\frac{9}{10}, \frac{3}{10}$	H $\frac{1}{2}, \frac{3}{8}$	L $\frac{7}{8}, \frac{5}{8}$	P $\frac{1}{6}, \frac{1}{12}$	T $\frac{5}{9}, \frac{1}{3}$	X $\frac{5}{6}, \frac{2}{5}$

Now you know that when you add or subtract fractions, the denominators must be alike. Often you will have to change the fraction numerals.

168

168

1 Have pupils read the text beside Picture M. Help them understand that they are to imagine that they pour enough milk from Cup 1 to reach the $\frac{1}{2}$ mark on Cup 2.

2 Discuss why it is a good idea to imagine that both cups have been marked in sixths.

3 Have pupils count in Cup 1 the number of sixths in $\frac{2}{3}$. Then have them use Cup 2 to find the number of sixths in $\frac{1}{2}$. They should then see that the $\frac{1}{6}$ cup of milk left in Cup 1 means that $\frac{4}{6}-\frac{3}{6}=\frac{1}{6}$ or $\frac{2}{3}-\frac{1}{2}=\frac{1}{6}$.

4 Assign these exercises as written work. Do not ask the children to rewrite the fraction numerals with common denominators. Ask them just to write the numeral they would use as a common denominator for each pair. Do not require the pupils to use the smallest common denominator.

169 Keeping skillful

Expanded Notes are not considered necessary for this lesson.

Objectives

The child practices for skill in choosing a common denominator for fraction numerals. He also practices computing with whole numbers.

Vocabulary

There are no new words.

Answers

(block 1)

A No; no; yes

B $\frac{9}{16}, \frac{12}{16}, \frac{6}{16}; \frac{3}{8}; \frac{9}{16}; \frac{3}{4}$

C $\frac{5}{6}, \frac{2}{3}, \frac{5}{8}, \frac{7}{12}$; no; no; because it is the next largest number divisible by 12

D $\frac{11}{12}, \frac{9}{12}, \frac{8}{12}; \frac{2}{3}, \frac{3}{4}, \frac{11}{12}$

E $\frac{16}{20}, \frac{15}{20}, \frac{10}{20}; \frac{1}{2}, \frac{3}{4}, \frac{4}{5}$

F $\frac{10}{12}, \frac{7}{12}, \frac{8}{12}; \frac{7}{12}, \frac{2}{3}, \frac{5}{6}$

G $\frac{14}{24}, \frac{15}{24}, \frac{18}{24}; \frac{7}{12}, \frac{5}{8}, \frac{3}{4}$

H $\frac{9}{10}, \frac{8}{10}, \frac{5}{10}; \frac{1}{2}, \frac{4}{5}, \frac{9}{10}$

I $\frac{15}{20}, \frac{12}{20}, \frac{14}{20}; \frac{3}{5}, \frac{7}{10}, \frac{3}{4}$

J $\frac{20}{24}, \frac{16}{24}, \frac{15}{24}, \frac{18}{24}; \frac{5}{8}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}$

K $\frac{8}{12}, \frac{10}{12}, \frac{9}{12}, \frac{9}{12}; \frac{2}{3}, \frac{9}{12}, \frac{3}{4}, \frac{5}{6}$ or $\frac{2}{3}, \frac{3}{4}, \frac{9}{12}, \frac{5}{6}$

(block 2)

A 12,000 lb.	F 3 bu.	J 730 da.	N 72 qt.
B 40 fl. oz.	G 72 pt.	K 17 bu.	O 7 min.
C 2 yr.	H 8 fl. oz.	L 7 lb.	P 224 da.
D 144 in.	I 6 hr.	M 39 yd.	Q 30 pt.
E 8 pk.			

(block 3)

A 11,774	G 58	M 98	
B 42	H 42 and 21 rm.	N 69	
C 9	I 80,522	O 303 and 48 rm.	
D 8	J 36	P 107	
E 65 and 40 rm.	K 11	Q 63	
F 17	L 56,304		

(block 4)

A 12,951	F 33,021	J 6011	N 6366
B 2379	G 594	K 1459	O 6700
C 6398	H 936	L 766	P 1243
D 7946	I 23,123	M 47,077	Q 191
E 405			

Keeping skillful

1

A To arrange $\frac{9}{16}$, $\frac{3}{4}$, and $\frac{3}{8}$ in the order of size, beginning with the smallest, you should first change them to fraction numerals with a common denominator. Can you write all of them as fourths? As eighths? As sixteenths?

B Change the three fraction numerals to sixteenths. Which fraction is the smallest? Which is next in size? Which is the largest?

2

C Arrange $\frac{5}{8}$, $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{7}{12}$ in the order of size. Begin with the largest. Should you use 8 as a common denominator? Should you use 3 or 6 or 12 as a common denominator? Since 12 is the largest of the four denominators, try 24 for the common denominator. Why?

3

Write the fraction numerals in each exercise below with a common denominator. Then write them in the order of size of the fractions. In each exercise begin with the smallest.

D $\frac{11}{12}, \frac{3}{4}, \frac{2}{3}$

E $\frac{4}{5}, \frac{3}{4}, \frac{1}{2}$

F $\frac{5}{6}, \frac{7}{12}, \frac{2}{3}$

G $\frac{7}{12}, \frac{5}{8}, \frac{3}{4}$

H $\frac{9}{10}, \frac{4}{5}, \frac{1}{2}$

I $\frac{3}{4}, \frac{3}{5}, \frac{7}{10}$

J $\frac{5}{6}, \frac{2}{3}, \frac{5}{8}, \frac{3}{4}$

K $\frac{2}{3}, \frac{5}{8}, \frac{9}{12}, \frac{3}{4}$

4

A 6 T. = ■ lb.
B ■ fl. oz. = 5 c.
C 104 wk. = ■ yr.
D 4 yd. = ■ in.
E 64 qt. = ■ pk.
F 96 qt. = ■ bu.
G ■ pt. = 9 gal.
H 16 tbs. = ■ fl. oz.
I ■ hr. = 360 min.
J 2 yr. = ■ da.
K ■ bu. = 68 pk.
L 112 oz. = ■ lb.
M ■ yd. = 117 ft.
N 18 gal. = ■ qt.
O ■ min. = 420 sec.
P 32 wk. = ■ da.
Q ■ pt. = 15 qt.

5

A $29 \times 406 = n$
B $1512 \div n = 36$
C $n \times 16 = 144$
D $57 \times n = 456$
E $5435 \div 83 = n$
F $1683 \div n = 99$
G $n \times 36 = 2088$
H $2709 \div 64 = n$
I $38 \times 2119 = n$
J $85 \times n = 3060$
K $814 \div n = 74$
L $184 \times 306 = n$
M $1764 \div 18 = n$
N $n \times 25 = 1725$
O $24591 \div 81 = n$
P $15 \times n = 1605$
Q $3024 \div n = 48$

5

A $3684 + 9267 = n$
B $7000 - n = 4621$
C $n + 625 = 7023$
D $n - 1536 = 6410$
E $831 - 426 = n$
F $n + 32090 = 65111$
G $689 + n = 1283$
H $n - 402 = 534$
I $74230 - 51107 = n$
J $2975 + 3036 = n$
K $9462 - n = 8003$
L $2274 + n = 3040$
M $68472 - 21395 = n$
N $n + 7916 = 14282$
O $n - 5006 = 1694$
P $777 + n = 2020$
Q $1100 - n = 909$

169

169

1. Discuss Exercises A and B with the pupils, letting them use paper and pencil to change the fraction numerals to equivalent fraction numerals with the same denominator, if necessary.
2. Discuss Exercise C with the class. Then have pupils arrange the fractions in order of size by writing the fraction numerals in the proper order on their papers.
3. Assign Exercises D to K as written work. Discuss the exercises afterwards with the children.
4. Direct pupils to write the answer on paper opposite its identifying letter.
5. Pupils should copy the numerals in computational form and find the answers. Direct them to label each exercise.

170 Thinking straight

Expanded Notes are not considered necessary for this lesson.

Objectives

The child makes generalizations regarding fractions that enable him to compare certain fractions without changing the numerals in the denominators.

Vocabulary

New words page 170 certain*, become*

Comments

By now some of the children are comparing some fractions by purely mental methods. The lesson on the number line (pages 158-159) helped them to compare certain fractions without finding common denominators for their numerals. And if the able pupils made the wall panel suggested in the Expanded Notes for the lesson on improper fractions and mixed numbers (pages 148-150), they will have a good idea of the relative size of fractions, whole numbers, and mixed numbers based on halves, thirds, and fourths. In the fourth-grade book of this program, pupils were shown how to compare fractions with numerators of 1. They learned that, for a given numerator, the larger the denominator, the smaller is the fraction that is represented. This concept is retaught in this lesson.

Answers

(block 1)

A Larger; smaller

B Rectangle F is marked into more equal parts than Rectangle B. So each equal part ($\frac{1}{3}$) in Rectangle B is larger than each equal part ($\frac{1}{7}$) in Rectangle F.

C Larger; smaller

D Exercise B established that $\frac{1}{3}$ is larger than $\frac{1}{7}$. Therefore, the remaining part ($\frac{2}{3}$) in Rectangle B must be smaller than the remaining part ($\frac{6}{7}$) in Rectangle F.

E Larger; smaller

F $\frac{6}{7}$ indicates 6 out of 7 parts, or almost the whole thing; $\frac{1}{3}$ indicates 1 out of 3 parts, or only a small portion of the whole thing.

G $\frac{5}{4}$; $\frac{4}{5}$; $\frac{4}{5}$

H $\frac{3}{2}$; $\frac{5}{6}$

(block 2)

A $\frac{1}{5}$
B $\frac{1}{7}$
C $\frac{3}{4}$
D $\frac{1}{4}$
E $\frac{1}{3}$
F $\frac{6}{7}$
G $\frac{1}{4}$
H $\frac{5}{7}$
I $\frac{1}{4}$
J $\frac{8}{9}$
K $\frac{1}{8}$
L $\frac{11}{12}$
M $\frac{1}{3}$
N $\frac{3}{4}$
O $\frac{1}{2}$
P $\frac{2}{3}$

(block 3)

A $\frac{7}{10}$
B $\frac{5}{6}$
C $\frac{11}{12}$
D $\frac{3}{4}$
E $\frac{3}{4}$
F $\frac{5}{6}$
G $\frac{3}{16}$
H $\frac{16}{15}$
I $\frac{7}{8}$
J $\frac{4}{9}$
K $\frac{3}{4}$
L $\frac{11}{12}$
M $\frac{4}{15}$
N $\frac{8}{7}$
O $\frac{5}{6}$
P $\frac{2}{3}$

Thinking straight

1

2 A The pictures above will help you decide whether certain fractions are larger or smaller than other fractions. Start with Picture A. As you go from picture to picture, does the red part of each become smaller or larger? Does the white part become smaller or larger?

B How do Pictures B and F show that $\frac{1}{3}$ is larger than $\frac{1}{7}$?

C Is $\frac{1}{8}$ larger or smaller than $\frac{1}{7}$? Is $\frac{1}{6}$ larger or smaller than $\frac{1}{4}$?

3 D How do Pictures B and F also show that $\frac{2}{3}$ is smaller than $\frac{6}{7}$?

E Is $\frac{6}{7}$ larger or smaller than $\frac{3}{4}$? Is $\frac{2}{3}$ larger or smaller than $\frac{5}{6}$?

F Sometimes two fractions are so different in size that you can tell which is larger just by looking at the fraction numerals. For example, to decide which is larger, $\frac{6}{7}$ or $\frac{1}{3}$, you should not have to change the denominators. Why?

4 G Suppose you wanted to compare $\frac{4}{5}$ with $\frac{3}{4}$. Which fraction is more than 1? Which fraction is less than 1? Which fraction is the smaller?

H Which is larger, $\frac{2}{3}$ or $\frac{3}{2}$? Which is smaller, $\frac{2}{3}$ or $\frac{5}{6}$?

5 For each pair of fractions below, try to decide which is the smaller. Do not change the denominators.

A	$\frac{1}{5}, \frac{1}{2}$	E	$\frac{1}{3}, \frac{5}{6}$	I	$\frac{1}{4}, \frac{7}{9}$	M	$\frac{1}{2}, \frac{1}{3}$
B	$\frac{1}{3}, \frac{1}{7}$	F	$\frac{2}{6}, \frac{6}{7}$	J	$\frac{8}{9}, \frac{8}{8}$	N	$\frac{3}{4}, \frac{6}{5}$
C	$\frac{3}{4}, \frac{3}{5}$	G	$\frac{11}{12}, \frac{1}{4}$	K	$\frac{1}{4}, \frac{1}{8}$	O	$\frac{6}{7}, \frac{1}{2}$
D	$\frac{2}{3}, \frac{1}{4}$	H	$\frac{5}{7}, \frac{7}{5}$	L	$\frac{11}{12}, \frac{12}{11}$	P	$\frac{2}{3}, \frac{3}{4}$

6 For each pair of fractions below, decide which is the larger. Try first to decide without changing the denominators. If you cannot decide, change the denominators.

A	$\frac{2}{5}, \frac{7}{10}$	E	$\frac{3}{4}, \frac{11}{20}$	I	$\frac{7}{8}, \frac{11}{16}$	M	$\frac{4}{15}, \frac{1}{5}$
B	$\frac{1}{4}, \frac{5}{6}$	F	$\frac{5}{8}, \frac{5}{6}$	J	$\frac{1}{3}, \frac{4}{9}$	N	$\frac{7}{8}, \frac{8}{7}$
C	$\frac{11}{12}, \frac{5}{6}$	G	$\frac{3}{16}, \frac{1}{8}$	K	$\frac{7}{12}, \frac{3}{4}$	O	$\frac{5}{5}, \frac{5}{6}$
D	$\frac{5}{16}, \frac{3}{4}$	H	$\frac{15}{16}, \frac{16}{15}$	L	$\frac{9}{10}, \frac{11}{12}$	P	$\frac{1}{5}, \frac{2}{3}$

170

1 Get the pupils to observe that Rectangles A to F are all the same size but that the divisions within the rectangles vary. Be sure they observe that only one part is white in each, and that there are more red parts as they go from left to right.

2 When the pupils answer Exercises A to C, they should begin to realize that when the numerator is 1, the larger the denominator, the smaller the fraction is.

3 For Exercises D to F, have the pupils justify their answers in their own words.

4 If necessary for Exercises G and H, draw diagrams to demonstrate that the improper fractions represent more than 1 whole.

5 Use Exercises A to P for oral work. Pupils may refer to Pictures A to F or draw diagrams if they need help.

6 You may use this last set of exercises as either oral or written work. Be sure the pupils can justify their decisions.

171-176 Learning how

Expanded Notes for this lesson are on pages 323-326.

Objectives

The child learns to add proper fractions.

Vocabulary

New word page 176 recipe

Comments

The children have had careful preparation for the addition of fractions. They have learned that fractions can be added and the resulting sum named if the fraction numerals have the same denominator. They have had much experience with changing a fraction numeral to an equivalent fraction numeral with a specified denominator. They have learned what a common denominator is and have practiced finding a common denominator for two or more fraction numerals that have different denominators. They have learned to change an improper fraction to a mixed number or a whole number. The children should now be ready to add proper fractions.

Notice that the first example in the lesson is $\frac{1}{2} + \frac{1}{3}$ This example requires the child, right at the start, to find a common denominator and change both fraction numerals before he proceeds to add. He sees the need for a common denominator and the importance of the denominator much more clearly than the child who begins by adding eighths "as if they were apples" and who is, therefore, quite likely to ignore the denominator almost completely.

Notice, too, that the second example in the SEE step has three addends. Once a child has learned to add two fractions, he should have no special difficulty with adding three fractions.

Do not stress using the smallest common denominator. Let the children feel comfortable about using any common denominator that will work. Also, do not insist that the children reduce proper fraction numerals in their answers to simplest form. They can, however, be reasonably expected to reduce improper fractions to mixed numbers. What is important is that the children learn how to add two or more fractions. Refinements in choosing a denominator and expressing an answer require much experience.

The addition of proper fractions is first shown in the horizontal computational form because that is a more convenient form in which to add proper fractions than is the vertical computational form. The vertical computational form is shown as an alternative on pages 175-176. This is done for two reasons. The first is that some teachers may prefer to have the children put their work on their papers in vertical form, and the second is that later on, when mixed numbers are added, the vertical computational form will be more convenient. The practice exercises in the book will continue to be shown in equation form. The child should know how to use each form, as well as have the experience of writing his work in whatever computational form is convenient.

The four-step teaching method is used in this lesson. On pages 175-176 the vertical computational form is introduced, and examples with a variety of denominators and answer forms are included. You will want to make sure the children understand thoroughly all the situations used in the TRY step, so that they will be ready to work the exercises in the DO step. These exercises may be used in several ways. You may or may not want all the children to do both blocks of problems. The second block may be used by the children who need additional practice, or it may be used as a follow-up in the future. You may have the children use the horizontal computational form with the first block of exercises and the vertical form with the second block. Or, you may let the child decide for himself which computational form he will use for each problem. At the end of this lesson the children should be able to use either computational form with ease.

Answers

Page 176:

(block 1)

A $1\frac{1}{2}$ or $1\frac{2}{4}$	D 1	G $1\frac{1}{2}$	J 1	M $1\frac{7}{15}$
B $\frac{11}{12}$	E $1\frac{1}{16}$	H $\frac{5}{6}$	K $2\frac{5}{12}$	N $\frac{7}{24}$
C $\frac{2}{3}$	F $1\frac{1}{5}$	I $\frac{7}{8}$	L $\frac{1}{4}$	

(block 2)

A $\frac{13}{24}$	D $1\frac{3}{5}$	G $1\frac{1}{3}$	J $1\frac{1}{4}$
B $\frac{2}{3}$	E $1\frac{17}{24}$	H $\frac{17}{20}$	K $1\frac{1}{12}$
C $\frac{19}{20}$	F $2\frac{5}{16}$	I $1\frac{17}{24}$	L $2\frac{9}{16}$

Learning how

In this lesson you will learn how to add fractions.

see

A

Mrs. Page gave $\frac{1}{2}$ of a cake to one friend and $\frac{1}{3}$ of the same cake to another friend. How much of the cake did she give to these two friends? **1**

$\frac{1}{2}+\frac{1}{3}=\square$ You must find this number.

B

Put $\frac{1}{3}$ of the cake with $\frac{1}{2}$ of the cake. You do not know what fraction of a whole cake she gave away. **2**

You will add $\frac{1}{3}$ to $\frac{1}{2}$. You must change the fraction numerals before you add.

C

Imagine cutting each of the fractions of the cake into equal pieces so that each piece is one sixth of the whole cake. **3**

Then the $\frac{1}{2}$ of the cake would be cut into 3 pieces, or 3 sixths of the whole cake. The $\frac{1}{3}$ of the cake would be cut into 2 pieces, or 2 sixths of the whole cake.

Before you add $\frac{1}{2}$ to $\frac{1}{3}$, find a common denominator. 6 is the smallest common denominator.

$\frac{1}{2}=\frac{3}{6}$ $\frac{1}{3}=\frac{2}{6}$ **4**

$\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}$

Now turn the page.

1

D

Now put the 2 sixths of the cake with the 3 sixths. There are 5 sixths.

Add the numerators to find how many sixths there are in all. There are 5 sixths.

$\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$ ← Write 5 as the numerator of the answer. You are adding sixths. So write 6 as the denominator.

2

Mrs. Page gave $\frac{5}{6}$ of a cake to her friends.

$\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$

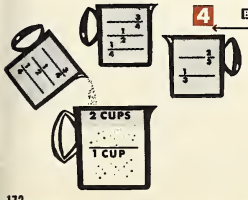
see

A

Mrs. Page used $\frac{3}{4}$ cup of sugar for a pie, $\frac{1}{3}$ cup of sugar for cookies, and $\frac{1}{2}$ cup of sugar for coffee cake. She used how much sugar in all? **3**

$\frac{3}{4}+\frac{1}{3}+\frac{1}{2}=\square$ You must find this number.

4



B

Put $\frac{3}{4}$ cup of sugar, $\frac{1}{3}$ cup of sugar, and $\frac{1}{2}$ cup of sugar into a larger measuring glass. You do not know what numeral to use to tell how much sugar she used.

You will add $\frac{3}{4}$, $\frac{1}{3}$, and $\frac{1}{2}$. You must change the fraction numerals before you add.

171

1 Have the pupils read the problem and the equation and relate the fraction numerals to the picture. Ask them whether they think Mrs. Page gave a whole cake away.

2 Have the pupils read the text and look at Picture B. They should observe that the two portions are put together and we cannot tell exactly how much cake there is in all. They should remember from a previous lesson that $\frac{1}{3}$ and $\frac{1}{2}$ should be expressed by fraction numerals with a common denominator before they are added.

3 Picture C shows the $\frac{1}{2}$ cake as a part of the reconstructed whole cake cut into 6 equal parts. The reconstructed cake is shown again with $\frac{1}{3}$ of the cake in place. It should be easy for pupils to see how many one sixths are in each of the two portions of cake. Discuss why 6 was chosen as the common denominator.

4 Be sure all pupils can express $\frac{1}{2}$ and $\frac{1}{3}$ by fraction numerals with the denominator 6.

172

1 Explain that, since both portions of cake have now been cut into sixths and put together, counting the one sixths tells how many sixths Mrs. Page gave away.

2 Have the pupils read the equation in large type and the text around it. They should see that adding the numerators gives the same answer as counting the parts in Picture D.

3 Call on someone to read the problem and the equation and find the amounts of sugar in Picture A.

4 Let a pupil explain what has been done in the picture. Ask, "Can we tell by looking at the large container just how much sugar is in it? What must be done to the fraction numerals before we add?"

C

Picture C shows that you can think of the measuring cups as marked to show twelfths.

1

Now you can think of the $\frac{3}{4}$ cup of sugar as $\frac{9}{12}$ cup. You can think of the $\frac{1}{3}$ cup as $\frac{4}{12}$ cup and of the $\frac{1}{2}$ cup as $\frac{6}{12}$ cup.

Before you add $\frac{3}{4}$, $\frac{1}{3}$, and $\frac{1}{2}$, find a common denominator. 12 is the smallest common denominator.

2

$\frac{3}{4}=\frac{\square}{12}$ $\frac{1}{3}=\frac{\square}{12}$ $\frac{1}{2}=\frac{\square}{12}$

$\frac{3}{4}+\frac{1}{3}+\frac{1}{2}=\frac{9}{12}+\frac{4}{12}+\frac{6}{12}$

D

Put $\frac{9}{12}$ cup, $\frac{4}{12}$ cup, and $\frac{6}{12}$ cup together.

3

4

Add the numerators. There are 19 twelfths.

$\frac{9}{12}+\frac{4}{12}+\frac{6}{12}=\frac{19}{12}$

Write 19 as the numerator of the answer. You are adding twelfths. Write 12 as the denominator.

E

5

Mrs. Page used $\frac{19}{12}$ cups of sugar. She used more than one whole cup.

Mrs. Page used 1 whole cup and $\frac{7}{12}$ of another cup of sugar.

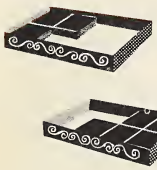
$\frac{19}{12}=1\frac{7}{12}$

Mrs. Page used $1\frac{7}{12}$ cups of sugar.

$\frac{3}{4}+\frac{1}{3}+\frac{1}{2}=1\frac{7}{12}$

think

A



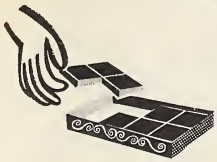
1

When Mary was cleaning a cupboard, she found one package with $\frac{3}{8}$ of a bar of chocolate in it. She found another package with $\frac{5}{8}$ of a bar in it. She said, "Altogether we have as much chocolate as there is in one whole bar." Was she right?

$\frac{5}{8}+\frac{3}{8}=\square$ You must find this number.

Put the $\frac{3}{8}$ of a bar with the $\frac{\square}{8}$ of a bar.

B



To add $\frac{3}{8}$ to $\frac{5}{8}$, do you have to change the denominators? Why?

2

What do you do with the numerators?

$\frac{5}{8}+\frac{3}{8}=\frac{8}{8}$

Why is 8 written in the numerator? Why is 8 also written in the denominator?

3

C

4

Mary had $\frac{8}{8}$ of a bar of chocolate.

Did she have as much chocolate as there is in one whole bar?

$\frac{8}{8}=1$

Was Mary right?

$\frac{5}{8}+\frac{3}{8}=1$

173

1 Have the pupils count the number of parts into which each cup has been marked. Then let them count how many twelfths are in $\frac{3}{4}$, in $\frac{1}{2}$, and in $\frac{1}{3}$.

2 Have pupils read the text. Discuss why 12 was chosen as the common denominator. If necessary, review the procedure for reducing $\frac{3}{4}$, $\frac{1}{3}$, and $\frac{1}{2}$ to twelfths.

3 Explain that since each fractional part of a cup of sugar is now expressed in twelfths, these amounts when put together will show how much sugar Mrs. Page used altogether. Note that the total amount is more than one cup. Each small division marked on the large 2-cup container represents $\frac{1}{12}$ of 1 standard cup.

4 Let someone explain why the numerators were added and not the denominators.

5 Have the pupils read the text. Get them to observe that Picture E shows 1 whole cup, or 12 twelfths, and $\frac{7}{12}$ of another cup.

174

1 Have the pupils read the problem. Ask: "Do the fractions in the problem correspond to the pieces of chocolate in the picture? What does the equation tell us we are going to find?"

2 Have several pupils read the questions in the text and supply the answers.

3 Be sure the meaning of the numerator and the denominator in $\frac{8}{8}$ is clear.

4 Picture C and previous work with improper fractions should make it easy for pupils to see that the answer is a whole number.

try

A $\frac{1}{2}+\frac{2}{5}=n$

Use 10 for the denominator when you add. Why?

$\frac{5}{10}+\frac{8}{10}=n$ **1**

$\frac{5}{10}+\frac{8}{10}=\frac{13}{10}$ or $1\frac{3}{10}$

How do you get the 13 in $\frac{13}{10}$?

How do you get $1\frac{3}{10}$?

Here is another way you can write $\frac{1}{2}$ and $\frac{4}{5}$ when you add.

$\frac{1}{2}=\frac{5}{10}$

$\frac{4}{5}=\frac{8}{10}$

$\frac{13}{10}$ or $1\frac{3}{10}$

You may show your work in the way you like better.

B $\frac{1}{8}+\frac{3}{4}=n$

What common denominator do you use?

Why do you not change $\frac{1}{8}$?

$\frac{1}{8}+\frac{6}{8}=n$ **1**

$\frac{1}{8}+\frac{6}{8}=\frac{7}{8}$

Is the answer in simplest form?

Here is another way you can write $\frac{1}{8}$ and $\frac{3}{4}$ when you add.

$\frac{1}{8}=\frac{1}{8}$

$\frac{3}{4}=\frac{6}{8}$

$\frac{7}{8}$

C $\frac{9}{10}+\frac{7}{10}=n$

Why do you not change the fraction numerals?

$\frac{9}{10}+\frac{7}{10}=\frac{16}{10}$ or $1\frac{6}{10}$ or $1\frac{3}{5}$ **2**

How do you know that $1\frac{3}{5}$ is the simplest form for $\frac{16}{10}$? **3**

You can also write your work for Example C in this way.

$\frac{9}{10}$

$\frac{7}{10}$

$\frac{16}{10}$ or $1\frac{6}{10}$ or $1\frac{3}{5}$ **4**

D $\frac{1}{4}+\frac{7}{12}=n$

What common denominator do you use?

Why do you not change $\frac{7}{12}$?

$\frac{3}{12}+\frac{7}{12}=n$

$\frac{3}{12}+\frac{7}{12}=\frac{10}{12}$ or $\frac{5}{6}$

How do you know that $\frac{5}{6}$ is the simplest form for $\frac{10}{12}$?

You can also write your work for Example D in this way.

$\frac{1}{4}=\frac{3}{12}$

$\frac{7}{12}=\frac{7}{12}$

$\frac{10}{12}$ or $\frac{5}{6}$

E $\frac{3}{8}+\frac{15}{16}+\frac{11}{16}=n$

What common denominator do you use?

$\frac{6}{16}+\frac{15}{16}+\frac{11}{16}=n$

$\frac{6}{16}+\frac{15}{16}+\frac{11}{16}=\frac{32}{16}$ or 2

How do you get the 32 in $\frac{32}{16}$? How do you know that 2 is another way to write $\frac{32}{16}$?

$\frac{3}{8}=\frac{6}{16}$

$\frac{15}{16}=\frac{15}{16}$

$\frac{11}{16}=\frac{11}{16}$

$\frac{32}{16}$ or 2

F $\frac{7}{8}+\frac{1}{3}+\frac{11}{12}=n$

Why do you not use 12 for the common denominator?

$\frac{21}{24}+\frac{8}{24}+\frac{22}{24}=n$

$\frac{21}{24}+\frac{8}{24}+\frac{22}{24}=\frac{51}{24}$ or $2\frac{3}{24}$ or $2\frac{1}{8}$

$\frac{7}{8}=\frac{21}{24}$

$\frac{1}{3}=\frac{8}{24}$

$\frac{11}{12}=\frac{22}{24}$

$\frac{51}{24}$ or $2\frac{3}{24}$ or $2\frac{1}{8}$ **1**

---

do

A A cooky recipe calls for $\frac{3}{4}$ cup of brown sugar and $\frac{3}{4}$ cup of white sugar. How much sugar in all does this recipe call for?

$\frac{3}{4}+\frac{3}{4}=n$ **1**

B Mrs. Fisher used $\frac{1}{4}$ cup of lemon juice and $\frac{2}{3}$ cup of orange juice to make an orange cream pie. How much juice in all did she use for the pie?

$\frac{1}{4}+\frac{2}{3}=n$

Show your answers in simplest form.

C $\frac{1}{3}+\frac{1}{3}=n$

D $\frac{7}{8}+\frac{1}{8}=n$

E $\frac{3}{16}+\frac{3}{8}+\frac{1}{2}=n$

F $\frac{9}{10}+\frac{3}{10}=n$

G $\frac{1}{4}+\frac{1}{2}+\frac{3}{4}=n$

H $\frac{2}{3}+\frac{1}{6}=n$

I $\frac{5}{8}+\frac{1}{4}=n$

J $\frac{3}{10}+\frac{1}{5}+\frac{1}{2}=n$

K $\frac{11}{12}+\frac{2}{3}+\frac{5}{6}=n$

L $\frac{1}{6}+\frac{1}{12}=n$

M $\frac{2}{3}+\frac{4}{6}=n$

N $\frac{1}{6}+\frac{1}{8}=n$

A $\frac{3}{8}+\frac{1}{6}=n$

B $\frac{7}{12}+\frac{1}{12}=n$ **2**

C $\frac{1}{5}+\frac{3}{4}=n$

D $\frac{2}{3}+\frac{1}{3}+\frac{3}{8}=n$

E $\frac{7}{8}+\frac{5}{6}=n$

F $\frac{11}{16}+\frac{3}{4}+\frac{7}{8}=n$

G $\frac{5}{6}+\frac{1}{2}=n$

H $\frac{3}{4}+\frac{1}{10}=n$

I $\frac{5}{6}+\frac{1}{2}+\frac{3}{8}=n$

J $\frac{1}{2}+\frac{3}{4}=n$

K $\frac{1}{4}+\frac{1}{6}+\frac{2}{3}=n$

L $\frac{15}{16}+\frac{7}{8}+\frac{3}{4}=n$

Now you should know how to add fractions and give the answers in different forms.

175

1. Examples A and B should be discussed carefully with the class. Compare the vertical computational form with the horizontal form. If you want your class to use one form rather than the other, tell them.
2. Note the reduction of the improper fraction.
3. Review what is meant by the simplest form of a fraction.
4. Have the pupils copy Examples C to F (E and F are on page 176) on their papers and find the answers. Then have them compare their work and answers with the computation in the book.

176

1. When the work in the TRY step has been completed, have the pupils work Problems A and B and Exercises C to N independently. Ask them to write the exercises in either horizontal or vertical computational form (as you prefer) and show each step of the computation. Encourage able pupils to give their answers in the simplest form, but do not penalize them for failing to do so. Check each pupil's work carefully, and make sure he understands how to correct any errors he has made. This is his first experience with a rather complicated technique, and extra help on your part now may avoid much trouble later.
2. Use Exercises A to L at a later date or for pupils who need more practice.

177-182 Learning how

Exponded Notes for this lesson ore on poges 326-327.

Objectives

The child learns how to add mixed numbers and fractions.

Vocabulary

New words page 179 cheese*

Comments

When the children learned to add whole numbers in the third-grade book of this program, pictures were used to show them what was taking place at every step. In this lesson, pictures are used in the same manner to guide the pupils' thinking when adding mixed numbers. Earlier, the first problem involving the addition of whole numbers introduced "carrying," and now the first problem on adding mixed numbers is a situation where, when the fractions are added, the sum is first expressed by an improper fraction numeral. Children thus learn in the very beginning to change such a result to a mixed number and add the whole number to the other whole numbers.

The fraction numerals in this problem have different denominators, making it necessary also to find a common denominator. The method used in presenting addition of mixed numbers avoids unnecessary rewriting. Note that when the fractions are added and the answer is expressed by an improper fraction numeral, the pupils are not required to write this numeral (see page 178). Instead they immediately reduce it and add the whole number in with the other whole numbers. Some of the children may be able to change to a mixed number mentally. Others may have to do the computation on scratch paper.

You may wish to review briefly the idea that to make a whole from eighths, 8 eighths are needed; from sixths, 6 sixths are needed and so on.

Do not insist that the children always work with the least common denominator. Let them use any common denominator they find is easy and meaningful. Do not insist, either, that they always reduce a fraction numeral to simplest form. However, you should insist that answers expressed by improper fraction numerals be reduced to mixed numbers.

The four-step teaching method is used to present addition of mixed numbers. The SEE step, with full explanations and pictures, appears on pages 177 and 178. The THINK step, with pictures and questions to guide the children's thinking, appears on pages 179-181. The TRY step, also on page 181, provides problems and examples without illustrations, but with computation and answers for final comparison. On page 182 in the DO step there are problems and exercises to work without help. Exercises A to H in the last column are provided for pupils who need extra work.

Answers

Page 182:

(block 1)		(block 2)
A $1\frac{7}{8}$	N $22\frac{1}{3}$ or $22\frac{2}{6}$	A $10\frac{1}{16}$
B $4\frac{11}{12}$	O 28	B $15\frac{2}{3}$ or $15\frac{4}{6}$
C $10\frac{1}{5}$ or $10\frac{2}{10}$	P $20\frac{1}{10}$	C $28\frac{1}{24}$
D $6\frac{11}{12}$	Q $71\frac{2}{15}$	D $31\frac{1}{2}$ or $31\frac{2}{4}$
E $5\frac{11}{16}$	R $5\frac{1}{12}$	E $8\frac{14}{15}$
F $4\frac{3}{5}$ or $4\frac{6}{10}$	S $12\frac{3}{5}$	F $8\frac{3}{4}$ or $8\frac{12}{16}$
G $6\frac{7}{12}$	T $19\frac{3}{5}$ or $19\frac{6}{10}$	G $73\frac{3}{20}$
H $2\frac{2}{3}$ or $2\frac{6}{9}$	U $7\frac{5}{6}$ or $7\frac{10}{12}$	H $22\frac{17}{24}$
I $10\frac{3}{16}$	V $26\frac{5}{12}$	
J $15\frac{1}{2}$ or $15\frac{3}{6}$	W $4\frac{3}{4}$ or $4\frac{6}{8}$	
K $39\frac{1}{2}$ or $39\frac{6}{12}$	X $10\frac{3}{20}$	
L $49\frac{7}{24}$	Y $11\frac{1}{16}$	
M $33\frac{5}{8}$	Z $12\frac{2}{5}$ or $12\frac{6}{15}$	

Keeping skillful:

(block 1)	(block 2)	(block 3)
A 223	A 13	A 3145
B 4321	B 42 and 12 rm.	B 1938
C 47,504	C 87	C 1846
D 2355	D 677,692	D 1386
E 761	E 63	E 2112
F 42,087	F 5	F 6162
G 209	G 77	G 10,985
H 4616	H 48	H 2448
I 3103	I 37	I 487
J 8237	J 43,960	J 3167
K 1269	K 11 and 81 rm.	K 30,753
	L 95	L 5230

Learning how

Now you will learn how to add mixed numbers and fractions.

see

A

Mrs. Cook used $6\frac{3}{4}$ apples for one pie and $4\frac{1}{2}$ apples for a smaller pie. How many apples did she use for both pies? 1

$6\frac{3}{4}+4\frac{1}{2}=n$ You must find this number.

B

Before you put the $\frac{1}{2}$ of an apple with the $\frac{3}{4}$ of an apple, think of the $\frac{1}{2}$ of an apple as $\frac{2}{4}$ of an apple.

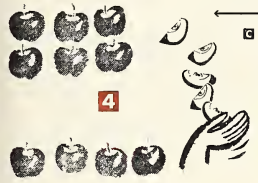
2

First you will add $\frac{1}{2}$ to $\frac{3}{4}$.

$6\frac{3}{4}=6\frac{3}{4}$ 3

$4\frac{1}{2}=4\frac{2}{4}$

Before you add, change the fraction numerals so that they will have a common denominator.



C

Now put the $\frac{2}{4}$ apple with the $\frac{3}{4}$ apple. There are $\frac{5}{4}$ apples.

4

The sum of $\frac{3}{4}$ and $\frac{2}{4}$ is $\frac{5}{4}$.

$6\frac{3}{4}=6\frac{3}{4}$ 5

$4\frac{1}{2}=4\frac{2}{4}$

$\frac{5}{4}$ apples are more than 1 apple.

Now turn the page.

D

Think of the fourths as put together to make as many whole apples as possible. There is 1 whole new apple. $\frac{1}{4}$ apple is left over.

1

Change the sum $\frac{5}{4}$ to $1\frac{1}{4}$.

$6\frac{3}{4}=6\frac{5}{4}$

$4\frac{1}{2}=4\frac{2}{4}$

$\frac{1}{4}$ Write $\frac{1}{4}$ here.

E

Think of the new apple as being put with the other whole apples.

2

1 Write 1 in the ones' column to show that there is 1 more whole.

$6\frac{3}{4}=6\frac{3}{4}$

$4\frac{1}{2}=4\frac{2}{4}$

$\frac{1}{4}$

F

Put the whole apples together. There are 11 whole apples.

3

1 Add the ones. The sum is 11.

$6\frac{3}{4}=6\frac{3}{4}$ 4

$4\frac{1}{2}=4\frac{2}{4}$

$11\frac{1}{4}$ Write 11 here.

G

Mrs. Cook used $11\frac{1}{4}$ apples for the two pies.

5

$6\frac{3}{4}+4\frac{1}{2}=11\frac{1}{4}$

177

1 Ask the children to read the problem and then look at Picture A. Help them see that because the situation is additive some way must be found to combine the fractions of apples as well as the whole apples and express the total. Have a pupil explain the equation and relate it to the picture.

2 Have the children note in Picture B that the one half apple is now changed to 2 fourths apple. Be sure they understand why this change must be made.

3 Discuss how the symbolization shows what has taken place.

4 Have the pupils note in Picture C that the fractions of apples are combined first. Let someone tell how many fourths there are in all.

5 Let the pupils explain how they know that the sum of $\frac{3}{4}$ and $\frac{2}{4}$ is $\frac{5}{4}$. Ask someone to tell how he knows that $\frac{5}{4}$ apples are more than 1 apple.

178

1 Help the pupils relate the text to Picture D to see why $\frac{5}{4}$ can be changed to $1\frac{1}{4}$. Be sure the pupils note where the $\frac{1}{4}$ is written.

2 In Picture E, get the pupils to explain why the 1 new whole apple is being put with the other whole apples. Then point out that in the computation the 1 is put with the other whole numbers. Get someone to explain why this is done.

3 In Picture F, get the pupils to note that all the whole apples are being put together. Ask someone to give the total number of whole apples.

4 Have the children add the column of whole numbers and observe where the 11 is written.

5 Direct the pupils to read the text and refer to the problem and equation on page 177. Ask if the answer seems reasonable.

In his store Mr. Parker had some cheese in 1-pound packages. One day he sold $2\frac{7}{8}$ lb. The next day he sold $\frac{3}{4}$ lb. The third day he sold $3\frac{1}{2}$ lb. How many pounds of cheese did he sell in the three days?

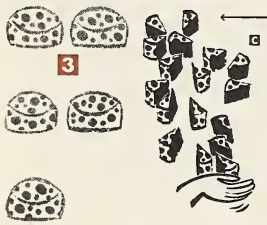
$2\frac{7}{8}+\frac{3}{4}+3\frac{1}{2}=n$ — You must find this number.

What will you put together first? Think of $\frac{3}{4}$ lb. as $\frac{\square}{8}$ lb. and $\frac{1}{2}$ lb. as $\frac{\square}{8}$ lb. Why do you think of eighths?

What will you add first?

Why do you change the fraction numerals to eighths?

$2\frac{7}{8}=2\frac{7}{8}$
$\frac{3}{4}=\frac{6}{8}$
$3\frac{1}{2}=3\frac{4}{8}$



Now what do you put together? There are how many eighths?

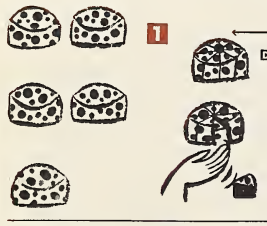
The sum of the fractions is $\frac{\square}{8}$.

$2\frac{7}{8}=2\frac{7}{8}$
$\frac{3}{4}=\frac{6}{8}$
$3\frac{1}{2}=3\frac{4}{8}$

4

$\frac{17}{8}$ pounds are more than 1 pound.

Now turn the page.



There are ▩ new whole pounds. $\frac{\square}{8}$ of a pound is left over.

Change the sum $\frac{17}{8}$, to ▩$\frac{\square}{8}$.

$2\frac{7}{8}=2\frac{7}{8}$
$\frac{3}{4}=\frac{6}{8}$
$3\frac{1}{2}=3\frac{4}{8}$ 2
$\frac{1}{8}$ ← Why is $\frac{1}{8}$ written here?

Put the ▩ new pounds with the other whole pounds.

Why is 2 written here?

$2\frac{7}{8}=2\frac{7}{8}$ 3
$\frac{3}{4}=\frac{6}{8}$
$3\frac{1}{2}=3\frac{4}{8}$
$\frac{1}{8}$

Now what do you put together? There are ▩ whole pounds.

What do you add now? The sum is ▩.

$2\frac{7}{8}=2\frac{7}{8}$
$\frac{3}{4}=\frac{6}{8}$
$3\frac{1}{2}=3\frac{4}{8}$
$7\frac{1}{8}$

Why do you write 7 here?

179

1 Have the pupils identify in Picture A the three groups of cheeses mentioned in the problem. Ask someone to relate the equation to the picture and to the problem.

2 Guide the pupils to see why the fractions of pounds of cheese should be put together first. Get them to explain why they should think of these pieces as cut into eighths. Talk about why the fraction numerals are changed to equivalent fraction numerals with a common denominator. Discuss why 8 should be used as the common denominator.

3 Let the pupils discuss the combining of the eighths in Picture C, and ask them how many eighths there are in all.

4 Discuss how the computation must be done. Discuss why the numerators of the fraction numerals must be added. Remind the children that $\frac{17}{8}$ must be kept in mind (or recorded).

180

1 The pupils should explain in Picture D where the two new whole pounds came from. They should also explain why $\frac{1}{8}$ of a pound of cheese is left over.

2 Discuss the $\frac{1}{8}$ in the computation and its position in the answer.

3 Discuss what is to be done with the 2 that represents the 2 new whole pounds of cheese. The pupils should observe that what is being done with the 2 in the computation corresponds to what is being done with the 2 new cheeses in Picture E.

4 Have someone explain why the whole cheeses are put together in Picture F. Ask another pupil to explain why the 7 is written where it is in the answer.

G

How much cheese in all did Mr. Parker sell?

$2\frac{6}{8} + \frac{3}{4} + 3\frac{1}{2} = n$

1

try

A Mrs. Fisher bought $2\frac{2}{3}$ yd. of cloth to make a dress for Jane, $2\frac{5}{8}$ yd. to make a dress for Kathy, and $\frac{3}{4}$ yd. to make an apron for herself. She bought how much cloth in all?

$2\frac{2}{3} + 2\frac{5}{8} + \frac{3}{4} = n$

2

$$2\frac{2}{3} = 2\frac{16}{24}$$
$$2\frac{5}{8} = 2\frac{15}{24}$$
$$\frac{3}{4} = \frac{18}{24}$$
$$6\frac{1}{24}$$

2 ← Either remember this 2 or write it.

B $5\frac{1}{4} + 8\frac{7}{16} = n$

3

$5\frac{1}{4} = 5\frac{4}{16}$
$8\frac{7}{16} = 8\frac{7}{16}$
$13\frac{1}{16}$

C $2\frac{3}{8} + 6\frac{2}{3} = n$

4

$2\frac{3}{8} = 2\frac{9}{24}$
$6\frac{2}{3} = 6\frac{16}{24}$
$9\frac{1}{24}$

Why do you write 9 here instead of 8?

D $1\frac{11}{12} + 3\frac{3}{4} = n$

5

$1\frac{11}{12} = 1\frac{11}{12}$
$3\frac{3}{4} = 3\frac{9}{12}$
$5\frac{8}{12}$ or $5\frac{2}{3}$

E $7\frac{9}{10} + \frac{4}{5} + 1\frac{17}{20} = n$

$7\frac{9}{10} = 7\frac{18}{20}$
$\frac{4}{5} = \frac{16}{20}$
$1\frac{17}{20} = 1\frac{17}{20}$
$10\frac{11}{20}$

F $8\frac{1}{4} + \frac{5}{6} + 5\frac{11}{12} = n$

$8\frac{1}{4} = 8\frac{3}{12}$
$\frac{5}{6} = \frac{10}{12}$
$5\frac{11}{12} = 5\frac{11}{12}$
15

Why is there no fraction numeral in the answer?

6

G $12\frac{1}{8} + 7\frac{5}{8} = n$

5

$12\frac{1}{8}$
$7\frac{5}{8}$
$19\frac{6}{8}$ or $19\frac{3}{4}$

do

A When Andy made sails for his model sailboat, he used a piece of cloth $1\frac{1}{4}$ yd. long and another piece $\frac{5}{8}$ yd. long. How many yards of cloth did he use for the sails?

$1\frac{1}{4} + \frac{5}{8} = n$

1

B Mrs. Fisher used $3\frac{1}{2}$ cups of milk to make tomato soup, $\frac{2}{3}$ cup of milk to make a cake, and $\frac{3}{4}$ cup of milk to make bread. How many cups of milk did she use in all?

$3\frac{1}{2} + \frac{2}{3} + \frac{3}{4} = n$

G $4\frac{1}{10} + 6\frac{1}{10} = n$	K $10\frac{7}{12} + 28\frac{1}{12} = n$	S $8\frac{2}{6} + 3\frac{5}{8} + \frac{4}{8} = n$	A $9\frac{1}{4} + 13\frac{1}{16} = n$
D $6\frac{2}{3} + \frac{1}{4} = n$	L $32\frac{1}{4} + 17\frac{1}{8} = n$	T $6\frac{1}{10} + 12\frac{4}{5} + \frac{7}{10} = n$	B $3\frac{5}{6} + 11\frac{5}{6} = n$
E $3\frac{9}{16} + 2\frac{1}{8} = n$	M $11\frac{7}{8} + 21\frac{3}{4} = n$	U $4\frac{1}{6} + 3\frac{5}{12} + \frac{1}{4} = n$	C $22\frac{3}{4} + 5\frac{3}{8} = n$
F $1\frac{1}{2} + 3\frac{1}{10} = n$	N $13\frac{1}{2} + 8\frac{5}{8} = n$	V $10\frac{1}{4} + 15\frac{1}{2} + \frac{2}{3} = n$	D $18\frac{1}{4} + 13\frac{1}{4} = n$
G $1\frac{5}{8} + 4\frac{3}{4} = n$	O $15\frac{2}{3} + 12\frac{1}{4} = n$	W $1\frac{3}{8} + 2\frac{1}{2} + \frac{7}{8} = n$	E $6\frac{1}{2} + 2\frac{5}{8} = n$
H $\frac{5}{9} + 2\frac{1}{9} = n$	P $12\frac{3}{10} + 7\frac{5}{8} = n$	X $\frac{3}{4} + 5\frac{9}{10} + 3\frac{1}{2} = n$	F $8\frac{7}{16} + \frac{5}{16} = n$
I $7\frac{3}{8} + 21\frac{3}{8} = n$	Q $27\frac{1}{4} + 43\frac{5}{8} = n$	Y $1\frac{1}{16} + 2\frac{5}{8} + 7\frac{1}{4} = n$	G $30\frac{5}{8} + 42\frac{3}{4} = n$
J $5\frac{2}{3} + 9\frac{5}{6} = n$	R $\frac{5}{12} + 4\frac{3}{8} = n$	Z $5\frac{1}{3} + 11\frac{1}{15} + 6\frac{1}{3} = n$	H $12\frac{5}{8} + 9\frac{7}{8} = n$

Now you should be able to add mixed numbers and fractions.

2

Keeping skillful

Find the sum.

3

A 28, 37, 19, 42, 97	A $n \times 84 = 1092$	A $n + 1692 = 4837$
B 107, 3162, 843, 209	B $558 \div 13 = n$	B $10365 - 8427 = n$
C 11029, 36475	C $43 \times n = 3741$	C $n - 762 = 1084$
D 623, 590, 1142	D $703 \times 964 = n$	D $748 + n = 2134$
E 18, 6, 307, 422, 8	E $1575 \div n = 25$	E $6230 - n = 4118$
F 9684, 32000, 403	F $96 \times n = 480$	F $2764 + 3398 = n$
G 31, 58, 12, 22, 86	G $1463 \div 19 = n$	G $n - 4863 = 6122$
H 1047, 265, 3304	H $62 \times n = 2976$	H $n + 806 = 3254$
I 940, 801, 639, 723	I $n \times 54 = 1998$	I $606 - n = 119$
J 4862, 3157, 218	J $56 \times 785 = n$	J $1150 + n = 4317$
K 44, 89, 7, 529, 600	K $983 \div 82 = n$	K $87602 - 56849 = n$
	L $14 \times n = 1330$	L $396 + 4834 = n$

181

1. Ask the pupils to see if the problem on page 179 has been solved.
2. Assign Examples A to G as individual written work. Tell the pupils to write the numerals in the vertical form. Tell them that when they have completed their work, they can use the work in the textbook to verify their selection of common denominators, the changing of fraction numerals, and the final answers.
3. Have the pupils observe that when the fractions are added in Example B, the sum is a proper fraction in simplest form.
4. Point out that in Example C it is not necessary to write 1 in the whole-number column. It can be remembered. Some pupils, however, may continue to write such numerals.
5. Point out that the proper fraction is reduced to simplest form. Ask someone to explain why such reduction is desirable.
6. Have a pupil explain why the answer is not a mixed number.

182

1. When the children work Exercises A to Z, remind them to use the vertical form for their computation. Also remind them that sometimes they will not need to change the denominators, and in other cases they can use the larger denominator for the common denominator.
2. When the pupils have completed the work, let them put the solutions on the chalkboard. Then discuss the exercises that caused the most difficulty.
3. These three blocks of work may be used separately, or they may be used together whenever such practice seems to be required.

183-184 Using arithmetic

Expanded Notes are not considered necessary for this lesson.

Objectives

The child solves problems that require the addition of fractions and mixed numbers. In the "Thinking straight" lesson he learns a short cut in adding fractions.

Vocabulary

New words page 183 hill*, Ruth, wear*

Comments

In this problem set, the children meet addition of fractions in two types of problem-solving situations. One type is described by the familiar additive equation ($2\frac{1}{2}+3\frac{1}{4}=n$). The other type is described by equations such as $n-2\frac{1}{2}=3\frac{1}{4}$. The children have had enough experience by now with this second kind of problem to know that the answer is found by addition, but it might be wise to have some class discussion about Problem A before the set of problems is assigned.

The "Thinking straight" lesson on page 184 shows the child that he can often make an exercise on adding fractions easier for himself by adding fractions to make 1 or by making other fractions that are more convenient to add than the fractions in the exercise.

Answers

A $[1\frac{7}{8}]-\frac{5}{8}=1\frac{1}{4}$

B $3\frac{9}{10}+2\frac{1}{2}=[6\frac{2}{5}]$; $[6\frac{4}{10}]$

C $1\frac{3}{4}+1\frac{5}{8}=[3\frac{3}{8}]$

D $[37]-10\frac{1}{2}=26\frac{1}{2}$

E $3\frac{1}{4}+\frac{1}{2}=[3\frac{3}{4}]$ or $\frac{1}{2}+3\frac{1}{4}=[3\frac{3}{4}]$

F $18\frac{1}{4}+18\frac{1}{4}+19\frac{2}{3}=[56\frac{1}{6}]$; $[56\frac{2}{12}]$

G $[65]-2\frac{7}{10}=62\frac{3}{10}$

H $26\frac{4}{10}+14\frac{6}{10}=[41]$

I $\frac{3}{4}+1+1\frac{1}{2}=[3\frac{1}{4}]$

J $2+1\frac{3}{8}+\frac{5}{8}=[4]$

K $1\frac{1}{3}+\frac{7}{8}=[2\frac{5}{24}]$ or $\frac{7}{8}+1\frac{1}{3}=[2\frac{5}{24}]$

L $1\frac{3}{4}+1\frac{2}{3}=[3\frac{5}{12}]$ or $1\frac{2}{3}+1\frac{3}{4}=[3\frac{5}{12}]$

M $99\frac{4}{10}+\frac{8}{10}=[100\frac{2}{10}]$ (Accept $100\frac{1}{5}°$ as correct, but explain why $100\frac{2}{10}°$ is a better answer.)

N $[63\frac{3}{4}]-1\frac{1}{2}=62\frac{1}{4}$

O $[6\frac{5}{6}]-4\frac{1}{2}=2\frac{1}{3}$

P $\frac{1}{2}+1\frac{1}{2}+1\frac{3}{4}=[3\frac{3}{4}]$

Thinking straight:

(block 2)

A $2\frac{1}{2}$ B 2 C $2\frac{1}{6}$ D $1\frac{1}{4}$ E $1\frac{3}{10}$ F 12

Using arithmetic

A Susan's mother gave her a piece of cloth. Susan used $\frac{5}{8}$ of a yard to make an apron. Then she found that she had $1\frac{1}{4}$ yd. left. How much cloth had her mother given her?

1

Amount to begin with.

Amount Susan used.

$n-\frac{5}{8}=1\frac{1}{4}$ ← Amount left.

You must find this number.

Add $\frac{5}{8}$ to $1\frac{1}{4}$.

B One morning Jim rode $3\frac{9}{10}$ mi. on his bicycle. In the afternoon he rode $2\frac{1}{2}$ mi. How far did he ride that day?

2

C Nancy grew $1\frac{3}{4}$ in. during the first six months of the year. During the last six months she grew $1\frac{5}{8}$ in. How much did she grow in all that year?

D After a grocer had sold $10\frac{1}{2}$ lb. of fresh green beans, he had $26\frac{1}{2}$ lb. left. How many pounds of fresh green beans did he have to begin with?

E Sally bought a $3\frac{1}{4}$ lb. chicken and $\frac{1}{2}$ lb. of bacon. How many pounds of meat did she buy?

F Nancy's flower garden is in the shape of a triangle. Its sides measure $18\frac{1}{4}$ ft., $18\frac{1}{4}$ ft., and $19\frac{2}{3}$ ft. What is the perimeter of the garden?

G A story in a newspaper said that between 9 o'clock and 10 o'clock the temperature went down $2\frac{7}{10}°$. At 10 o'clock the temperature was $62\frac{3}{10}°$. What had the temperature been at 9 o'clock?

H It is $26\frac{4}{10}$ mi. from Hometown to Springton and $14\frac{6}{10}$ mi. from Springton to Sugar Hill. Mr. Price drove from Hometown to Sugar Hill by way of Springton. He drove how many miles?

I Bob helped his father plant the garden. He worked $\frac{3}{4}$ hr. on Monday, 1 hr. on Tuesday, and $1\frac{1}{2}$ hr. on Wednesday. How many hours did he work in all on these three days?

J Jane's mother made a dress for Jane to wear in a school play. She used 2 yd. of red cloth, $1\frac{3}{8}$ yd. of blue cloth, and $\frac{5}{8}$ yd. of white cloth. She used how many yards of cloth in all?

K Mrs. Green used $1\frac{1}{3}$ cups of sugar to make a cake and $\frac{7}{8}$ cup of sugar to make frosting. She used how many cups of sugar in all for the cake and the frosting?

L Ruth wrapped a present. She used $1\frac{3}{4}$ yd. of blue ribbon and $1\frac{2}{3}$ yd. of white ribbon to trim the package. She used how many yards in all?

183

1 Discuss this problem with the class. Remind them that they have often found the original number when the "number gone" and the "number left" are known. Point out that fractions and mixed numbers are used in problems just as whole numbers are used. Discuss why *n* is found by addition.

2 Assign Problems B to P (M to P on the next page) as written work. Remind pupils to write the proper equation for each, show the computation, and rewrite the equation with the answer in place of *n*.

M One morning when Jim had a cold, his temperature was $99\frac{4}{10}°$. At 4 o'clock in the afternoon his mother took his temperature and found that it had gone up $\frac{8}{10}°$. What was Jim's temperature at 4 o'clock?

N When Jim had a cold, he lost $1\frac{1}{2}$ lb. Then he weighed $62\frac{1}{4}$ lb. How much had Jim weighed before he had the cold?

O Mrs. Price used $4\frac{1}{2}$ c. of flour for bread. Then she found that she had $2\frac{1}{3}$ c. of flour left. How much flour did she have before she made the bread?

P Jim's baseball team practiced $\frac{1}{2}$ hr. on Thursday, $1\frac{1}{2}$ hr. on Friday, and $1\frac{3}{4}$ hr. on Saturday. How many hours did the team practice in all on these three days? 1

Thinking straight

A $\frac{2}{3}+\frac{3}{4}+\frac{1}{3}=n$ 2

Look at these fractions. The sum of which two fractions is 1? Remember to add $\frac{3}{4}$ to 1.

B $\frac{1}{4}+\frac{1}{2}+\frac{3}{4}+\frac{1}{2}=n$ 2

What is the sum of $\frac{1}{4}$ and $\frac{3}{4}$?

What is the sum of $\frac{1}{2}$ and $\frac{1}{2}$?

Why do you not need to use 4 as a common denominator?

C $\frac{1}{4}+\frac{2}{3}+\frac{1}{4}=n$ 3

The sum of $\frac{1}{4}$ and $\frac{1}{4}$ is $\frac{2}{4}$, or $\frac{1}{2}$.

Now add $\frac{2}{3}$ to $\frac{1}{2}$.

Is it easier to add this way than to change $\frac{1}{4}$, $\frac{2}{3}$, and $\frac{1}{4}$ to twelfths?

D $2\frac{1}{5}+7\frac{5}{6}+3\frac{4}{5}=n$ 4

Add $\frac{1}{5}$ and $\frac{4}{5}$. The sum is 1. What is the sum of the whole numbers? Remember to add the 1 and the $\frac{5}{6}$.

E $4\frac{5}{12}+\frac{9}{10}+3\frac{7}{12}+\frac{1}{10}=n$ 4

What is the sum of the twelfths? What is the sum of the tenths? How do you know that the answer is 9?

In each example below, try to add without changing the fraction numerals.

A $\frac{1}{4}+\frac{1}{3}+\frac{1}{4}+1\frac{2}{3}=n$ 5

B $\frac{5}{16}+\frac{2}{5}+1\frac{1}{16}+\frac{3}{5}=n$

C $\frac{3}{8}+\frac{2}{3}+\frac{5}{8}+1=n$

D $\frac{5}{12}+\frac{3}{4}+\frac{1}{12}=n$

E $\frac{3}{8}+\frac{4}{5}+\frac{1}{8}=n$

F $1\frac{1}{2}+5\frac{1}{8}+4\frac{1}{2}+\frac{5}{8}=n$

184

184

1 When the pupils have completed this problem set, supply answers so they can verify their work. Discuss the problems that have caused the most difficulty.

2 Discuss in Examples A and B the possibility of adding fractions that make a whole number in order to make the addition easier.

3 Discuss adding two fractions whose numerals have the same denominator before choosing a common denominator for adding all the fractions.

4 Show the class how these same short cuts work in adding mixed numbers.

5 Assign Exercises A to F as written work. When they have finished, have the pupils explain how they added fractions to shorten their work.

185 Moving forward

Expanded Notes are not considered necessary for this lesson.

Objectives

The child learns how to change numerals for mixed numbers to other numerals as preparation for learning how to subtract mixed numbers.

Vocabulary

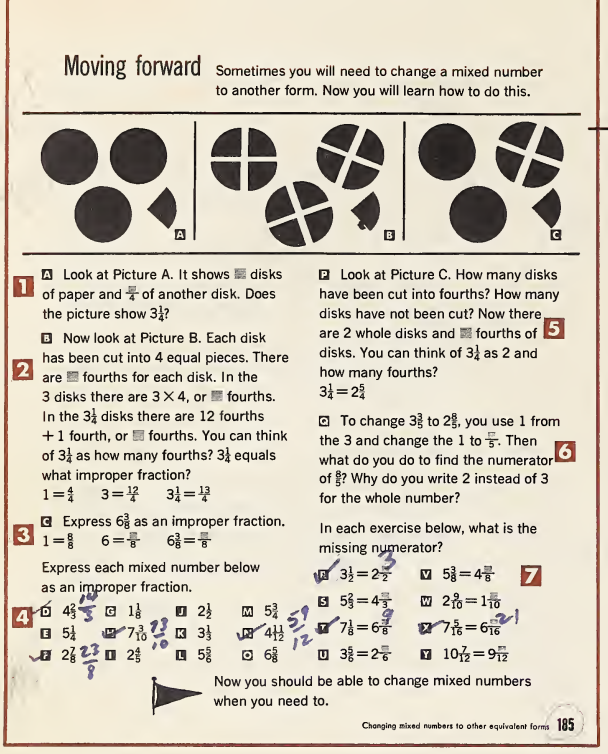
There are no new words.

Comments

In studying subtraction of fractions and mixed numbers, the pupils must know how to change a mixed number to an improper fraction or to another mixed number containing an improper fraction. This lesson shows how to perform this operation. The lesson should be taught as an extension of the principle of regrouping whole numbers. The pupils are already familiar with regrouping in connection with subtraction of whole numbers. As preliminary work you might have the class recall how to regroup a number like 53 into 4 tens and 13 ones so that a number like 9 can be subtracted from it. If necessary, refer the children to Movie B on page 14 of the text to see a picture of this kind of regrouping. Just as a whole number can be regrouped in various ways, so can the mixed number $3\frac{1}{2}$, for example, be regrouped as $\frac{7}{2}$ or as $2\frac{3}{2}$. The ability to regroup in this way is required in subtracting mixed numbers.

Answers

C $\frac{48}{8}$; $\frac{51}{8}$
D $\frac{14}{3}$
E $\frac{21}{4}$
F $\frac{23}{8}$
G $\frac{9}{8}$
H $\frac{73}{10}$
I $\frac{14}{5}$
J $\frac{5}{2}$
K $\frac{10}{3}$
L $\frac{35}{6}$
M $\frac{23}{4}$
N $\frac{59}{12}$
O $\frac{53}{8}$
P —
Q —
R 3
S 5
T 9
U 11
V 11
W 19
X 21
Y 19



Moving forward Sometimes you will need to change a mixed number to another form. Now you will learn how to do this.

A B C

1 A Look at Picture A. It shows ▩ disks of paper and $\frac{\square}{4}$ of another disk. Does the picture show $3\frac{1}{4}$?

2 B Now look at Picture B. Each disk has been cut into 4 equal pieces. There are ▩ fourths for each disk. In the 3 disks there are 3×4, or ▩ fourths. In the $3\frac{1}{4}$ disks there are 12 fourths + 1 fourth, or ▩ fourths. You can think of $3\frac{1}{4}$ as how many fourths? $3\frac{1}{4}$ equals what improper fraction?

$1 = \frac{4}{4}$ $3 = \frac{12}{4}$ $3\frac{1}{4} = \frac{13}{4}$

3 C Express $6\frac{3}{8}$ as an improper fraction.

$1 = \frac{8}{8}$ $6 = \frac{\square}{8}$ $6\frac{3}{8} = \frac{\square}{8}$

Express each mixed number below as an improper fraction.

4

D $4\frac{2}{3}$	G $1\frac{1}{8}$	J $2\frac{1}{2}$	M $5\frac{3}{4}$	P $7\frac{3}{10}$	S $4\frac{11}{12}$
E $5\frac{1}{4}$	H $7\frac{3}{10}$	K $3\frac{1}{3}$	N $4\frac{11}{12}$	Q $2\frac{7}{8}$	T $2\frac{4}{5}$
L $5\frac{5}{6}$	O $6\frac{5}{8}$				

P Look at Picture C. How many disks have been cut into fourths? How many disks have not been cut? Now there are 2 whole disks and ▩ fourths of disks. You can think of $3\frac{1}{4}$ as 2 and how many fourths?

$3\frac{1}{4} = 2\frac{\square}{4}$

5

Q To change $3\frac{3}{8}$ to $2\frac{\square}{8}$, you use 1 from the 3 and change the 1 to $\frac{\square}{8}$. Then what do you do to find the numerator of $\frac{\square}{8}$? Why do you write 2 instead of 3 for the whole number?

6

In each exercise below, what is the missing numerator?

7

R $3\frac{1}{2} = 2\frac{\square}{2}$	V $5\frac{3}{8} = 4\frac{\square}{8}$	
S $5\frac{2}{3} = 4\frac{\square}{3}$	W $2\frac{9}{10} = 1\frac{\square}{10}$	
T $7\frac{1}{8} = 6\frac{\square}{8}$	X $7\frac{5}{16} = 6\frac{\square}{16}$	
U $3\frac{5}{6} = 2\frac{\square}{6}$	Y $10\frac{7}{12} = 9\frac{\square}{12}$	

Now you should be able to change mixed numbers when you need to.

Changing mixed numbers to other equivalent forms 185

185

1 The pupils should see that $3\frac{1}{4}$ disks are shown in Picture A. Be sure that they recognize the gray piece as $\frac{1}{4}$ of a disk.

2 The pupils should realize that the same disks are shown in Picture B and that this picture shows a way of regrouping $3\frac{1}{4}$.

3 Discuss this exercise with the class. You may want to draw pictures on the board or use paper squares or circles to illustrate the exercise.

4 Let the pupils take turns expressing these mixed numbers by improper fraction numerals.

5 Point out that these are the same disks as in the preceding pictures. Help the pupils see that $2\frac{5}{4}$ is another way of regrouping $3\frac{1}{4}$.

6 Be sure that the pupils understand that they use only 1 of the 3 ones in $3\frac{3}{5}$ and express it as $\frac{5}{5}$, then add the $\frac{5}{5}$ to the proper fraction already there.

7 Assign these exercises as written work.

186-189 Learning how

Exponded Notes for this lesson are on pages 328-329.

Objectives

The child learns how to subtract proper fractions.

Vocabulary

There are no new words.

Comments

This lesson applies ideas the pupils already know to new situations. They already understand the meaning of the subtraction process. They have learned how to find common denominators and how to reduce fraction numerals. In the lesson on page 185 they learned that mixed numbers can be expressed in equivalent forms, an idea basic to an understanding of borrowing in subtraction.

If you wish, you may demonstrate the subtraction of fractions by using fractional parts of circles made of oaktag, flannel, or other materials. Plan any such demonstrations to bring out that when one fraction is subtracted from another, the remainder is expressed in simplest form by using fraction numerals that have the same denominator.

It may be wise not to insist that the pupils always find the lowest common denominator. Point out, however, that usually the lowest common denominator is the easiest to use.

The four-step teaching method is used to present the material in this lesson. Note that, as usual, pupils are required to write an equation to show the problem situation ($\frac{2}{3} - \frac{1}{2} = n$). The computational form (vertical arrangement) is shown as an alternative procedure in the TRY step on page 188. There is no advantage in the column arrangement when fractions are to be subtracted, but the vertical form is introduced so that the children will become familiar with it.

The "Keeping skillful" exercises (page 189) provide practice in the addition of fractions and mixed numbers and in the addition and subtraction of whole numbers.

Answers

Page 189:

(block 1)

A $\frac{1}{4}$	H $\frac{1}{4}$	O $\frac{3}{8}$ or $\frac{6}{16}$
B $\frac{1}{2}$ or $\frac{4}{8}$	I $\frac{2}{15}$	P $\frac{11}{20}$
C $\frac{5}{12}$	J $\frac{1}{3}$	Q $\frac{2}{5}$
D $\frac{3}{8}$	K $\frac{5}{12}$	R $\frac{3}{20}$
E $\frac{3}{20}$	L $\frac{3}{8}$	S $\frac{1}{30}$
F $\frac{1}{2}$	M $\frac{9}{16}$	T $\frac{11}{30}$
G $\frac{11}{24}$	N $\frac{13}{24}$	

(block 2)

A $\frac{5}{16}$	H $\frac{5}{16}$
B $\frac{5}{12}$	I $\frac{1}{24}$
C $\frac{7}{24}$	J $\frac{7}{20}$
D $\frac{13}{20}$	K $\frac{3}{16}$
E $\frac{5}{12}$	L $\frac{2}{3}$
F $\frac{1}{12}$	
G $\frac{3}{10}$	

Keeping skillful:

(block 1)

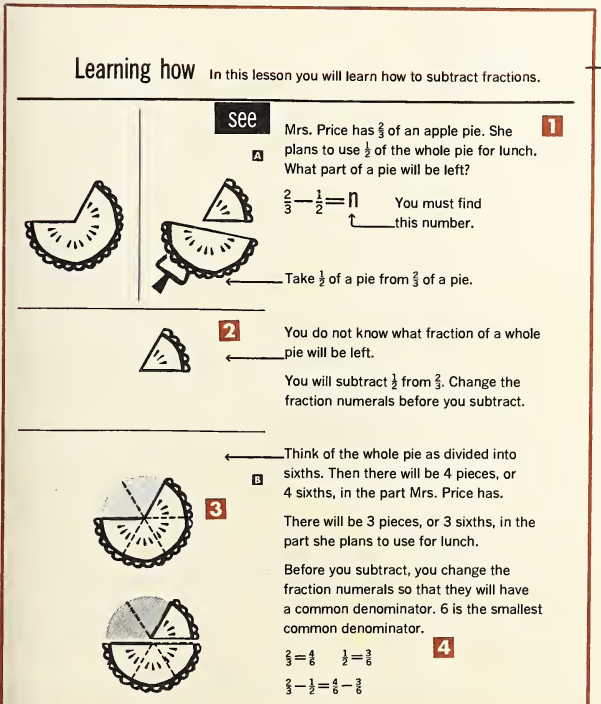
A $2\frac{7}{15}$
B $8\frac{1}{24}$
C $1\frac{7}{12}$
D $15\frac{3}{5}$
E $1\frac{7}{10}$
F $11\frac{1}{12}$
G $5\frac{5}{16}$

(block 2)

A $11\frac{1}{4}$	H $14\frac{5}{6}$ or $14\frac{10}{12}$
B $16\frac{1}{2}$ or $16\frac{3}{6}$	I $1\frac{1}{30}$
C $3\frac{19}{20}$	J $7\frac{1}{10}$
D $1\frac{7}{12}$	K $2\frac{17}{24}$
E $4\frac{3}{16}$	L $9\frac{5}{12}$
F $5\frac{19}{24}$	M $1\frac{2}{15}$
G $1\frac{7}{20}$	N $15\frac{11}{12}$

(block 3)

A 169
B 2068
C 232
D 2379
E 8288
F 3857
G 2551


Learning how
In this lesson you will learn how to subtract fractions.
see
1
A
Mrs. Price has $\frac{2}{3}$ of an apple pie. She plans to use $\frac{1}{2}$ of the whole pie for lunch. What part of a pie will be left?
$\frac{2}{3}-\frac{1}{2}=n$
You must find this number.
Take $\frac{1}{2}$ of a pie from $\frac{2}{3}$ of a pie.
2
You do not know what fraction of a whole pie will be left.
You will subtract $\frac{1}{2}$ from $\frac{2}{3}$. Change the fraction numerals before you subtract.
B
Think of the whole pie as divided into sixths. Then there will be 4 pieces, or 4 sixths, in the part Mrs. Price has.
3
There will be 3 pieces, or 3 sixths, in the part she plans to use for lunch.
Before you subtract, you change the fraction numerals so that they will have a common denominator. 6 is the smallest common denominator.
4
$\frac{2}{3}=\frac{4}{6}$ $\frac{1}{2}=\frac{3}{6}$
$\frac{2}{3}-\frac{1}{2}=\frac{4}{6}-\frac{3}{6}$
186 Subtraction of proper fractions

186

1 Tell the pupils to read Problem A and examine Movie A. Have them relate the action in the three scenes of the movie to the problem. Ask a pupil why it is hard to describe the piece that is left. Be sure the children understand that the *n* holds a place for a number that represents the size of this piece.

2 Get the children to suggest that as yet they cannot subtract to get a single fraction to represent a part of a whole pie because the fraction numerals do not have a common denominator.

3 Get them to explain how Picture B shows that the whole pie can be divided (in imagination) so that both $\frac{2}{3}$ of the pie and $\frac{1}{2}$ of it are divided into equal-sized parts. Pupils should see that the pie must be divided into sixths. Ask how many sixths there are in $\frac{2}{3}$ and in $\frac{1}{2}$.

4 Discuss why 6 is selected as the common denominator for the fraction numerals.

1 ← Take 3 sixths of the pie from the 4 sixths. One sixth will be left.

C

Subtract the numerators to find how many sixths are left. One sixth is left. 2

$\frac{4}{6} - \frac{3}{6} = \frac{1}{6}$ ← Write 1 as the numerator of the answer. You are subtracting sixths. So write 6 as the denominator.

Mrs. Price will have $\frac{1}{6}$ of a pie left.

$\frac{2}{3} - \frac{1}{2} = \frac{1}{6}$

think

Mrs. White has $\frac{5}{6}$ of a cake. She plans to put $\frac{1}{4}$ of the whole cake in Tony's lunch box. How much of a whole cake will she have left? 3

A

$\frac{5}{6} - \frac{1}{4} = n$ You must find this number. 4

What must you do with the fraction numerals before you subtract?

Why do you think of the whole cake as divided into twelfths? How many twelfths are there in the part that Mrs. White has? 5 ←

In the part she is going to put in Tony's lunch box there are ■ twelfths.

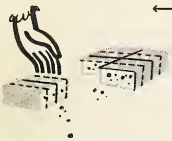
B

Why can you use 12 as the common denominator when you subtract? 6

$\frac{5}{6} = \frac{10}{■}$ $\frac{1}{4} = \frac{3}{■}$ $\frac{5}{6} - \frac{1}{4} = \frac{■}{12} - \frac{■}{12}$

Now turn the page.

187



C ← Take ■ twelfths of the cake away from the ■ twelfths. ■ twelfths will be left.

$\frac{10}{12} - \frac{3}{12} = \frac{■}{12}$ What do you do with the numerators? 1

What do you write as the numerator of the answer? Why do you write 12 as the denominator?

Mrs. White will have $\frac{■}{12}$ of a whole cake left.

$\frac{5}{6} - \frac{1}{4} = \frac{■}{12}$

try

A $\frac{7}{8} - \frac{1}{3} = n$ 2

Use 24 for the denominator when you subtract. Why?

$\frac{21}{24} - \frac{8}{24} = n$

$\frac{21}{24} - \frac{8}{24} = \frac{13}{24}$

You may write your work this way if you want to.

$\frac{7}{8} = \frac{21}{24}$

$\frac{1}{3} = \frac{8}{24}$

$\frac{13}{24}$

B $\frac{11}{15} - \frac{2}{15} = n$ 3

Why do you not change the fraction numerals?

$\frac{11}{15} - \frac{2}{15} = \frac{9}{15}$ or $\frac{3}{5}$

How do you know that $\frac{3}{5}$ is the simplest form for $\frac{9}{15}$?

Your work may be written this way.

$\frac{11}{15}$

$\frac{2}{15}$

$\frac{9}{15}$ or $\frac{3}{5}$

C $\frac{5}{6} - \frac{3}{4} = n$

What common denominator do you use?

$\frac{10}{12} - \frac{9}{12} = n$

$\frac{10}{12} - \frac{9}{12} = \frac{1}{12}$

You may write your work in this way.

$\frac{5}{6} = \frac{10}{12}$

$\frac{3}{4} = \frac{9}{12}$

$\frac{1}{12}$

187

1 Have someone read the text at the top of the page and then explain how Picture C shows that $\frac{3}{6}$ is being removed, leaving $\frac{1}{6}$.

2 Point out that the numerators are subtracted, and have someone explain why. Let the children do this mentally and then see how the answer is written in the equation.

3 Have the new problem read and get the pupils to relate the three scenes of Movie A to the problem.

4 The pupils should explain the equation, relating it to the problem and to the movie.

5 Have someone explain why the whole cake is thought of as divided into twelfths. Let a pupil tell how many twelfths there are in $\frac{5}{6}$ and in $\frac{1}{4}$.

6 Have the pupils discuss why 12 is used as a common denominator. Let someone express $\frac{5}{6}$ and $\frac{1}{4}$ in twelfths.

188

1 The pupils should supply the missing numbers and answer the questions. Let them explain how Picture C shows that the answer may be expressed in twelfths.

2 When you discuss Example A with the pupils, be sure they understand why a common denominator is selected. Ask why 24 is a good choice [because it is the smallest number that can be divided by both 8 and 3]. Let them perform the computation, using the equation form. Then call attention to the computational form and ask the class to subtract $\frac{1}{3}$ from $\frac{7}{8}$, using the computational form.

3 Assign Examples B to D (D is on the next page) as written work. Tell the pupils that they may write their work in the computational form if they wish. Tell them to compare their work for each example with that in the book when they have finished. You may wish to discuss Example B.

D $\frac{11}{12}-\frac{1}{6}=n$

What common denominator do you use?
Why do you not change $\frac{1}{6}$?

$\frac{11}{12}-\frac{2}{12}=n$

$\frac{11}{12}-\frac{2}{12}=\frac{9}{12}$ or $\frac{3}{4}$

Your work may be written this way.

$\frac{11}{12}=\frac{11}{12}$

$\frac{\frac{1}{6}=\frac{2}{12}}{\frac{9}{12}}$ or $\frac{3}{4}$

1

do

2

A Jim bought $\frac{3}{4}$ lb. of peanuts. He took $\frac{1}{2}$ lb. to the park to feed the squirrels. He had what fraction of a pound of peanuts left?
$\frac{3}{4}-\frac{1}{2}=n$

B Susan had $\frac{7}{8}$ yd. of blue cloth. She used $\frac{3}{8}$ yd. to make a dress for her doll. How much blue cloth did Susan have left?
$\frac{7}{8}-\frac{3}{8}=n$

Show your answers in simplest form.

3

C $\frac{11}{12}-\frac{1}{2}=n$
D $\frac{5}{8}-\frac{1}{4}=n$
E $\frac{9}{10}-\frac{3}{4}=n$
F $\frac{5}{8}-\frac{1}{3}=n$
G $\frac{5}{6}-\frac{3}{8}=n$
H $\frac{1}{2}-\frac{1}{4}=n$
I $\frac{5}{6}-\frac{2}{3}=n$
J $\frac{7}{12}-\frac{1}{4}=n$
K $\frac{3}{4}-\frac{1}{3}=n$
L $\frac{7}{8}-\frac{1}{2}=n$
M $\frac{15}{16}-\frac{3}{8}=n$
N $\frac{5}{3}-\frac{1}{8}=n$
O $\frac{7}{15}-\frac{1}{15}=n$
P $\frac{3}{4}-\frac{1}{5}=n$
Q $\frac{7}{10}-\frac{9}{10}=n$
R $\frac{9}{10}-\frac{3}{4}=n$
S $\frac{5}{6}-\frac{3}{8}=n$
T $\frac{5}{3}-\frac{3}{10}=n$
A $\frac{9}{16}-\frac{1}{4}=n$
B $\frac{7}{12}-\frac{1}{6}=n$
C $\frac{7}{8}-\frac{1}{3}=n$
D $\frac{9}{10}-\frac{1}{4}=n$
E $\frac{6}{5}-\frac{1}{4}=n$
F $\frac{3}{4}-\frac{2}{3}=n$
G $\frac{7}{10}-\frac{2}{5}=n$
H $\frac{11}{16}-\frac{3}{8}=n$
I $\frac{7}{8}-\frac{5}{6}=n$
J $\frac{3}{5}-\frac{1}{4}=n$
K $\frac{9}{16}-\frac{3}{8}=n$
L $\frac{11}{12}-\frac{1}{4}=n$

Now you should be able to subtract one fraction from another.

Keeping skillful **4**

A $\frac{3}{5}+\frac{3}{5}+1\frac{1}{5}=n$
B $2\frac{6}{8}+5\frac{1}{8}=n$
C $\frac{1}{4}+\frac{1}{2}+\frac{5}{6}=n$
D $8\frac{5}{8}+7\frac{2}{8}=n$
E $\frac{9}{10}+\frac{5}{5}+\frac{3}{5}=n$
F $4\frac{1}{3}+6\frac{2}{3}=n$
G $3\frac{3}{4}+1\frac{9}{16}=n$

A $n-4\frac{1}{2}=6\frac{3}{4}$
B $11\frac{5}{8}+4\frac{3}{8}=n$
C $n-1\frac{1}{2}=2\frac{3}{4}$
D $\frac{11}{12}+\frac{1}{6}+\frac{1}{2}=n$
E $n-1\frac{5}{16}=3\frac{1}{4}$
F $n-\frac{7}{8}=4\frac{11}{12}$
G $\frac{4}{5}+\frac{1}{4}+\frac{3}{10}=n$

H $8\frac{11}{12}+5\frac{11}{12}=n$
I $n-\frac{1}{3}=\frac{7}{10}$
J $4\frac{1}{2}+2\frac{3}{5}=n$
K $\frac{7}{8}+1\frac{5}{8}=n$
L $n-6\frac{2}{9}=2\frac{3}{4}$
M $n-\frac{4}{5}=\frac{1}{3}$
N $7\frac{3}{4}+8\frac{5}{8}=n$

A $1006-837=n$
B $n+795=2863$
C $54+82+96=n$
D $7816-n=5437$
E $n-2287=6001$
F $1354+n=5211$
G $n+3089=5640$

189

189

1 Use the procedure suggested in Note 3 of the notes for page 188.
2 Assign Exercises A to T as independent written work. Allow the pupils to use either way to show their computation.
3 Use these exercises for pupils who need additional practice.
4 Assign the "Keeping skillful" exercises as separate work. Supply the children with answers so they can verify their work.

190-195 Learning how

Expanded Notes for this lesson are on pages 329-331.

Objectives

The child learns to subtract mixed numbers.

Vocabulary

New words page 192 lard; page 193 watermelon

Comments

This lesson on subtraction of mixed numbers makes use of concepts and skills the pupils have already learned in various preceding lessons. If the children understood the lesson on page 185 on changing mixed numbers to improper fractions, and if they have mastered finding common denominators, they should have little trouble with this lesson.

The most important thing for the children to understand in this lesson is how to proceed when they find that the fraction in the number to be subtracted is larger than the fraction in the number from which it is subtracted.

This process of rewriting mixed numbers in different forms can be compared to the regrouping that the pupils have to do in subtraction of whole numbers. Help the children see that now they think of 1 (taken from the whole number) as an improper fraction with the same denominator as that of the fraction part of the mixed number, just as they took 1 ten, for example, from the tens' place and changed it to 10 ones.

Answers

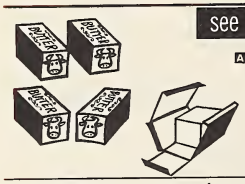
Page 195:

(block 1)		(block 2)	
A $1\frac{3}{4}$	H $4\frac{13}{16}$	A 6	H $7\frac{11}{16}$
B $2\frac{1}{2}$	I $6\frac{3}{5}$	B $2\frac{1}{2}$	I $19\frac{4}{5}$
C $1\frac{5}{6}$	J $5\frac{3}{8}$	C $\frac{1}{24}$	J $16\frac{3}{8}$
D $\frac{13}{24}$	K $4\frac{7}{12}$	D $2\frac{5}{8}$	K $12\frac{5}{12}$
E $\frac{13}{15}$	L $6\frac{7}{15}$	E $1\frac{7}{16}$	L $8\frac{4}{5}$
F $3\frac{3}{4}$	M $3\frac{19}{30}$	F $5\frac{13}{20}$	
G $4\frac{1}{2}$	N $12\frac{7}{10}$	G $1\frac{2}{3}$	

Learning how

Now you will learn how to subtract, using mixed numbers.

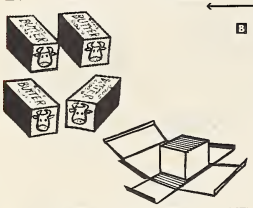
See



A

Mrs. Diamond has $4\frac{2}{3}$ lb. of butter. She plans to use $2\frac{3}{4}$ lb. for baking. How many pounds of butter will she have left? **1**

$4\frac{2}{3} - 2\frac{3}{4} = n$ ← You must find this number.

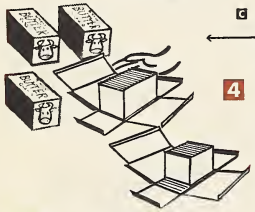


B

Before you can find out whether or not you can take $\frac{3}{4}$ lb. from the $\frac{8}{9}$ lb., you must think of $\frac{8}{9}$ lb. as $\frac{8}{12}$ lb. You must also think of $\frac{3}{4}$ lb. as $\frac{9}{12}$ lb. You cannot take $\frac{9}{12}$ lb. from $\frac{8}{12}$ lb. **2**

$4\frac{2}{3} = 4\frac{8}{12}$ ← Before you subtract, change the fraction numerals so that they will have a common denominator. You cannot subtract $\frac{9}{12}$ from $\frac{8}{12}$. **3**

$2\frac{3}{4} = 2\frac{9}{12}$

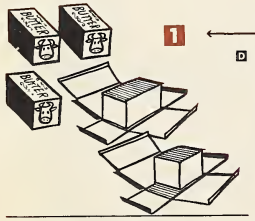


C

Before you can take away $\frac{9}{12}$ lb., you must have more twelfths. Open another pound of butter and put it with the $\frac{8}{12}$ lb. Think of the 1 pound as 12 twelfths. **4**

$4\frac{2}{3} = 4\frac{8}{12}$ ← Before you can subtract $\frac{9}{12}$, you must have more twelfths. Take 1 of the 4 ones and think of it as $\frac{12}{12}$. Put $\frac{12}{12}$ with $\frac{8}{12}$.

$2\frac{3}{4} = 2\frac{9}{12}$



D

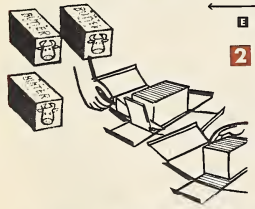
1

Now you have 3 whole pounds and $\frac{20}{12}$ lb.

$3\frac{20}{12}$ ← Now there are 3 ones and $\frac{20}{12}$.

$4\frac{2}{3} = 4\frac{8}{12}$

$2\frac{3}{4} = 2\frac{9}{12}$



E

Take away $\frac{9}{12}$ lb. The amount left is $\frac{11}{12}$ lb.

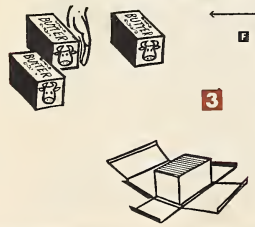
2

$3\frac{20}{12}$ ← Subtract $\frac{9}{12}$ from $\frac{20}{12}$. There are $\frac{11}{12}$ left.

$4\frac{2}{3} = 4\frac{8}{12}$

$2\frac{3}{4} = 2\frac{9}{12}$

$\frac{11}{12}$ ← Write $\frac{11}{12}$ here.



F

Next take away 2 whole pounds. 1 pound will be left.

3

$3\frac{20}{12}$ ← Subtract 2 from 3. The answer is 1.

$4\frac{2}{3} = 4\frac{8}{12}$

$2\frac{3}{4} = 2\frac{9}{12}$

$1\frac{11}{12}$ ← Write the 1 here.

Mrs. Diamond will have $1\frac{11}{12}$ lb. of butter left.

$4\frac{2}{3} - 2\frac{3}{4} = 1\frac{11}{12}$

190

1. Call on a pupil to read Problem A aloud and ask him to verify the amount of butter shown in Picture A. Then discuss with the class why the equation correctly describes the problem situation.
2. Guide the pupils to see that no subtraction of fractions can take place yet because the denominators are not the same.
3. Have someone explain why 12 is the correct common denominator, and have the children rewrite the fraction numerals with this denominator. Have the pupils note that the $\frac{2}{3}$ of a pound of butter in Picture B has been marked to show $\frac{8}{12}$.
4. Call on a pupil to read the text and tell how Picture C relates to it. Point out that nothing has been done with the computation as yet, but that the text tells what must be done next with the numerals.

191

1. Explain that the changes that occurred in Picture C are now made here. Also explain where the $\frac{20}{12}$ came from and tell how the picture shows the mixed number, $3\frac{20}{12}$.
2. Ask the class to verify the computation shown in the text and relate the computation to the action shown in the picture.
3. Have a pupil point out the whole pound and the $\frac{11}{12}$ pound of butter that are left in Picture F.

see

A

Mrs. Brown had 4 lb. of lard. She used $1\frac{7}{8}$ lb. to cook doughnuts. How many pounds of lard did she have left?

1

$4 - 1\frac{7}{8} = n$ ← You must find this number.

2

You cannot take away $\frac{7}{8}$ of a pound until a whole pound has been opened. Think of the opened pound as made up of eighths.

B

Now there are 3 whole pounds and 8 eighths of a pound.

$3\frac{8}{8}$ ← Before you can subtract $\frac{7}{8}$, you must take 1 of the 4 ones and think of it as $\frac{8}{8}$. Now there are 3 ones and $\frac{8}{8}$.

$\not{4}$

$1\frac{7}{8}$

3

C

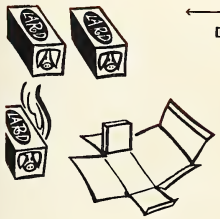
Now take $\frac{7}{8}$ of a pound from the $\frac{8}{8}$ of a pound. $\frac{1}{8}$ of a pound will be left.

$3\frac{8}{8}$ ← Subtract $\frac{7}{8}$ from $\frac{8}{8}$. $\frac{1}{8}$ is left.

$\not{4}$

$1\frac{7}{8}$

$\frac{1}{8}$ ← Write the $\frac{1}{8}$ here.



D

Take 1 pound from 3 pounds. 2 pounds will be left.

$3\frac{8}{8}$ ← Subtract 1 from 3. The answer is 2.

$\not{4}$ 1

$1\frac{7}{8}$

$2\frac{1}{8}$ ← Write the 2 here.

Mrs. Brown had $2\frac{1}{8}$ pounds of lard left.

$4 - 1\frac{7}{8} = 2\frac{1}{8}$

think

A

Mr. Cross had $5\frac{1}{4}$ watermelons when he opened his store. Mr. Baker bought $4\frac{3}{4}$ watermelons for a picnic. How many watermelons did Mr. Cross have left?

2

$5\frac{1}{4} - 4\frac{3}{4} = n$ ← You must find this number.

B

First take away the $\frac{3}{4}$ watermelon. How do you know right away that another watermelon will have to be cut? Into how many equal parts should it be cut? Why?

3

$5\frac{1}{4}$ ← Do you have to change the fraction numerals?

$4\frac{3}{4}$ ← How do you know that you need more fourths before you can subtract?

Now turn the page.

192

1. Ask the pupils to read the new problem and call on someone to explain each symbol in the equation.
2. Discuss with the pupils that they cannot take $\frac{7}{8}$ of a pound away without changing one of the whole pounds into eighths, as shown in Picture B. Have someone explain why and how 4 was changed to $3\frac{8}{8}$. Ask questions to make the children realize why it is not necessary to rewrite the example as they did on page 191.
3. In Picture C, have the pupils note that $\frac{7}{8}$ of the pound of lard is being removed and relate this to the computation.

193

1. Have the pupils verify the computation and match the numerals in the answer with the objects in Picture D.
2. For the problem in the THINK step, have a pupil explain the equation and tell why it is correct. Point out that the fractions are already expressed with a common denominator.
3. Have pupils take turns answering the questions in the text. Have the pupils look at the computational form and answer the related questions.

C 1 What do you do with the cut-up watermelon? Now there are how many whole watermelons? Now there are how many fourths?

$4\frac{3}{4}$ ← How do you get $4\frac{3}{4}$?

$5\frac{1}{4}$

$4\frac{3}{4}$

D 2 Can you take the $\frac{3}{4}$ of a watermelon from the $\frac{1}{4}$? How many fourths will be left?

$4\frac{5}{4}$ ← Subtract $\frac{3}{4}$ from $\frac{5}{4}$. $\frac{2}{4}$ will be left.

$5\frac{1}{4}$

$4\frac{3}{4}$

$\frac{2}{4}$ ← Why do you write $\frac{2}{4}$ here? Can you also express $\frac{2}{4}$ as $\frac{1}{2}$?

E 3 How many whole watermelons do you take away? How many will be left?

$4\frac{5}{4}$ ← Subtract ▩ from ▩. What is left?

$5\frac{1}{4}$

$4\frac{3}{4}$

$\frac{2}{4}$ ← Why do you not write a numeral here?

Mr. Cross had $\frac{2}{4}$, or $\frac{1}{2}$, of a watermelon left.

$5\frac{1}{4}-4\frac{3}{4}=\frac{\square}{\square}$ or $\frac{\square}{\square}$

try

A Mrs. Woods bought 5 grapefruit. She used $3\frac{1}{2}$ of them for breakfast. How many grapefruit did she have left?

$5-3\frac{1}{2}=n$ 1

$4\frac{2}{2}$ ← Why do you write 5 as $4\frac{2}{2}$?

5

$3\frac{1}{2}$

$1\frac{1}{2}$

B Jane had $6\frac{3}{4}$ yd. of cloth. She used $2\frac{3}{4}$ yd. How much cloth did she have left?

$6\frac{3}{4}-2\frac{3}{4}=n$

$6\frac{3}{4}$

$2\frac{3}{4}$

4 ← Why do you not write a fraction numeral here?

C $8\frac{7}{8}-5\frac{1}{3}=n$

$8\frac{7}{8}=8\frac{21}{24}$ ← Why do you not have to use 1 of the 8 ones?

$5\frac{1}{3}=5\frac{8}{24}$

$3\frac{13}{24}$

D $11\frac{9}{10}-7=n$

$11\frac{9}{10}$ 2

7

$4\frac{9}{10}$

E $9\frac{1}{6}-8\frac{5}{6}=n$

$9\frac{1}{6}=8\frac{7}{6}$

$8\frac{5}{6}=8\frac{5}{6}$

$\frac{3}{6}$ or $\frac{1}{2}$

do

A Mrs. Fisher bought a 5-lb. box of sugar. She used $3\frac{1}{4}$ lb. for canning. How much sugar was left? 3

$5-3\frac{1}{4}=n$

B Tony's mother collected $7\frac{1}{2}$ doz. eggs. She sold 5 doz. of them. How many dozen eggs did she have left?

$7\frac{1}{2}-5=n$

Show your answers in simplest form.

G $4\frac{1}{12}-3\frac{1}{12}=n$	I $8\frac{3}{8}-2=n$	A $9\frac{1}{2}-3\frac{1}{2}=n$	C $18\frac{1}{3}-16\frac{2}{3}=n$	4
O $1\frac{1}{6}-\frac{5}{6}=n$	J $11\frac{1}{8}-5\frac{3}{8}=n$	B $5\frac{7}{10}-3\frac{1}{5}=n$	H $26\frac{11}{16}-19=n$	
E $6\frac{2}{3}-5\frac{4}{3}=n$	K $12\frac{1}{3}-7\frac{2}{3}=n$	C $1\frac{1}{6}-1\frac{3}{8}=n$	I $40\frac{9}{10}-21\frac{1}{10}=n$	
F $4\frac{1}{12}-1\frac{5}{6}=n$	L $16\frac{4}{5}-10\frac{1}{3}=n$	D $7\frac{1}{2}-4\frac{7}{8}=n$	J $34-17\frac{5}{8}=n$	
G $7\frac{3}{4}-3\frac{1}{4}=n$	M $23\frac{1}{3}-19\frac{7}{10}=n$	E $8\frac{3}{4}-7\frac{5}{16}=n$	K $27\frac{1}{6}-14\frac{2}{3}=n$	
H $5\frac{3}{8}-\frac{9}{16}=n$	N $28-15\frac{3}{10}=n$	F $6\frac{2}{3}-\frac{3}{4}=n$	L $18\frac{1}{2}-9\frac{7}{10}=n$	

Now you should be able to subtract when you have mixed numbers.

194

1 Ask a pupil to refer to the picture to answer the questions in the text.

2 After the questions in the text have been answered, relate what is being done in Picture D to the computation.

3 Have the class verify the computation. Ask what fraction numerals can be written in place of the screens in the final equation.

195

1 Ask the children to do the problems and examples in the TRY step on their own. Tell them to put the work in computational form and find the answers, then compare their work with that in the book. If they have no trouble with the first two or three examples, tell them to go on to the exercises in the DO step. Help pupils who need assistance.

2 In discussing the examples in the TRY step, call attention to Example D. Point out that since there is no fraction to be subtracted from $\frac{9}{10}$ it can be brought down immediately in the answer, and only the whole numbers need to be subtracted.

3 Let the children go on to Exercises A to N when they finish their work on the TRY step. Afterwards, discuss all exercises, and find out what the difficulties are.

4 Assign this set to the pupils who need more work in subtracting mixed numbers.

196-197 Using arithmetic

Expanded Notes for this lesson are on page 331.

Objectives

The child solves problems that require subtraction of fractions and mixed numbers.

Vocabulary

New words page 197 west*, bus*

Comments

The children have previously studied the four problem types presented in this lesson in connection with their work with whole numbers. They now solve these same types of problems, but the numerals represent fractions and mixed numbers.

Problems A, B, C, and D on page 196 represent the four problem types, all of which are solved by subtraction. The equation for each is given in order to help the pupils recognize them as familiar situations they have encountered before. You may find it wise to discuss Problems A to D with the class to be sure they understand the equations.

Answers

A $6\frac{1}{2}-[3\frac{1}{4}]=3\frac{1}{4}$

B $[2\frac{3}{4}]+1\frac{1}{4}=4$

C $100-31\frac{3}{4}=[68\frac{1}{4}]$

D $25\frac{7}{10}+[14\frac{3}{10}]=40$

E $1\frac{1}{2}+[1\frac{3}{4}]=3\frac{1}{4}$ or $3\frac{1}{4}-1\frac{1}{2}=[1\frac{3}{4}]$

F $45\frac{1}{2}-[35]=10\frac{1}{2}$ or $45\frac{1}{2}-10\frac{1}{2}=[35]$

G $[1\frac{1}{2}]+1\frac{3}{4}=3\frac{1}{4}$; $[1\frac{2}{4}]$

H $5\frac{1}{2}-4\frac{1}{3}=[1\frac{1}{6}]$

I $36\frac{7}{8}-13\frac{3}{4}=[23\frac{1}{8}]$

J $26-[22\frac{3}{4}]=3\frac{1}{4}$

K $1\frac{2}{3}+[\frac{5}{6}]=2\frac{1}{2}$

L $6\frac{1}{2}-[4\frac{1}{2}]=2$ or $6\frac{1}{2}-2=[4\frac{1}{2}]$

M $36-15\frac{1}{2}=[20\frac{1}{2}]$ or $15\frac{1}{2}+[20\frac{1}{2}]=36$

N $[\frac{3}{4}]+\frac{3}{4}=1\frac{1}{2}$

O $2\frac{1}{2}+[2\frac{1}{4}]=4\frac{3}{4}$

P $\frac{1}{4}+[1\frac{13}{20}]=1\frac{9}{10}$ or $1\frac{9}{10}-\frac{1}{4}=[1\frac{13}{20}]$

Using arithmetic

A Mr. Field had $6\frac{1}{2}$ bu. of potatoes. He planted some of them and then had $3\frac{1}{4}$ bu. left. How many bushels of potatoes had he planted? 1

Number of bushels he had at first.
Bushels he planted.
Bushels left.
$6\frac{1}{2}-n=3\frac{1}{4}$ You must find this number.

Subtract $3\frac{1}{4}$ from $6\frac{1}{2}$.

B On Friday Jane practiced $1\frac{1}{4}$ hr. for the school play. Then she said, "I have practiced 4 hours so far this week." How many hours had she practiced before Friday? 2

Number of hours she practiced before Friday.
Hours she practiced on Friday.
$n+1\frac{1}{4}=4$
Hours in all.
You must find this number.

Subtract $1\frac{1}{4}$ from 4.

C A farmer had 100 bu. of peaches. Then he sold $31\frac{3}{4}$ bu. How many bushels of peaches did he have left? 3

$100-31\frac{3}{4}=n$

D Mr. Page drove $25\frac{7}{10}$ mi. from his home to Springton. He then drove to Sugar Hill. When he reached there, he had driven 40 mi. How far did he drive going from Springton to Sugar Hill?

First distance he drove. 4
Next distance he drove.
Total distance.
$25\frac{7}{10}+n=40$ You must find this number.

Subtract $25\frac{7}{10}$ from 40.

E Ann and Mary bought hot dogs for a picnic. Mary bought $1\frac{1}{2}$ lb., and Ann bought the rest. Altogether they had $3\frac{1}{4}$ lb. of hot dogs. How many pounds of hot dogs did Ann buy? 5

F Jim grew $45\frac{1}{2}$ bu. of corn. He saved $10\frac{1}{2}$ bu. for seed and sold the rest. How many bushels did he sell?

G One morning, Tony worked for Mr. Price. In the afternoon he worked $1\frac{3}{4}$ hr. for Mr. Page. He said, "I have worked $3\frac{3}{4}$ hours today." How many hours had he worked for Mr. Price?

H When Jack made a dog house, he used $4\frac{1}{3}$ ft. from a board that was $5\frac{1}{2}$ ft. long. How long was the unused part of the board?

196

1 Have the pupils read Problem A. They should see that the equation accurately describes the problem situation. Have the pupils compute.

2 Follow the same procedure for this problem; discuss the equation and why subtraction is used in the computation.

3 Pupils should easily see that this is a subtractive situation. Have them subtract to find the answer.

4 Follow the same procedure as for Problems A and B.

5 Assign Problems E to P as written work. (Problems I to P are on page 197.) Be sure pupils write an equation for each problem and show their computation.

I A store had $36\frac{7}{8}$ yd. of printed cloth. $13\frac{3}{4}$ yd. were sold in 2 hr. Then how many yards of the cloth were left to be sold?

J Susan's mother bought 26 lb. of tomatoes. After she had canned some, she had $3\frac{1}{4}$ lb. of tomatoes left. How many pounds of tomatoes had she canned?

K Mrs. Hope needed $2\frac{1}{2}$ cups of powdered sugar to make frosting. She found she had only $1\frac{2}{3}$ cups. How much more powdered sugar did she need?

L Mrs. Hope made $6\frac{1}{2}$ doz. doughnuts. She kept 2 doz. for her family and gave the rest to be sold at the school fair. How many dozen did she give for the school fair?

M A very large truck and its load together weighed 36 T. The truck weighed $15\frac{1}{2}$ T. How heavy was the load?

N Jim raked leaves before dinner one day. Then he worked for $\frac{3}{4}$ of an hour after dinner to finish. He had worked $1\frac{1}{2}$ hr. in all raking leaves. How long had he raked leaves before dinner?

O Pat made a model boat in $4\frac{3}{4}$ hr. He worked $2\frac{1}{2}$ hr. on Friday, and did the rest of the work on Saturday. How long did he work on Saturday?

P The library is $1\frac{9}{10}$ mi. west of Bob's house. To get there, Bob walked $\frac{1}{4}$ mi. west to the bus and then rode the rest of the way. How far did Bob ride on the bus?

1

Checking up

Test 1	Test 2	Test 3	Test 4
A $\frac{3}{8}+\frac{5}{8}=n$	A $2\frac{1}{6}+3\frac{5}{6}=n$	A $\frac{7}{12}-\frac{1}{12}=n$	A $11\frac{3}{5}-2\frac{4}{5}=n$
B $\frac{1}{2}+\frac{3}{4}=n$	B $1\frac{1}{4}+\frac{4}{5}+2\frac{1}{2}=n$	B $\frac{4}{5}-\frac{2}{3}=n$	B $13\frac{7}{10}-6=n$
C $\frac{1}{3}+\frac{3}{5}+\frac{4}{5}=n$	C $3\frac{3}{4}+6\frac{11}{12}=n$	C $\frac{5}{6}-\frac{3}{8}=n$	C $5\frac{1}{4}-4\frac{2}{3}=n$
D $\frac{5}{6}+\frac{1}{4}+\frac{3}{4}=n$	D $6\frac{9}{10}+7\frac{1}{4}=n$	D $\frac{3}{4}-\frac{1}{8}=n$	D $9\frac{3}{8}-2\frac{2}{7}=n$
E $\frac{7}{12}+\frac{1}{2}+\frac{5}{8}=n$	E $9\frac{3}{16}+11\frac{1}{4}=n$	E $\frac{7}{8}-\frac{2}{3}=n$	E $8-5\frac{1}{2}=n$
F $\frac{9}{10}+\frac{7}{10}=n$	F $\frac{3}{5}+\frac{1}{2}+6\frac{7}{10}=n$	F $\frac{7}{10}-\frac{1}{5}=n$	F $6\frac{5}{8}-5\frac{5}{8}=n$
G $\frac{6}{5}+\frac{3}{4}+\frac{3}{5}=n$	G $17\frac{1}{3}+21\frac{3}{8}=n$	G $\frac{13}{16}-\frac{1}{16}=n$	G $2\frac{2}{3}-1\frac{8}{9}=n$
H $\frac{5}{8}+\frac{11}{16}=n$	H $24+16\frac{4}{5}=n$	H $\frac{9}{10}-\frac{1}{2}=n$	H $7\frac{5}{12}-\frac{3}{8}=n$
I $\frac{2}{3}+\frac{3}{4}=n$	I $10\frac{3}{8}+6\frac{3}{4}=n$	I $\frac{3}{5}-\frac{1}{4}=n$	I $3\frac{1}{10}-2\frac{9}{10}=n$

End-of-block tests on addition and subtraction of fractions 197

197

1 Supply answers for all the problems. Then discuss any that caused the pupils trouble. If necessary, let the pupils use objects to dramatize the situations and solutions.

197 Checking up

Exponded Notes are not considered necessary for this lesson.

Objectives

The child tests his knowledge of addition and subtraction of fractions and mixed numbers.

Vocabulary

There are no new words.

Comments

These tests should be used for diagnosis of difficulties, and sufficient time should be allowed so that most children finish the tests. Test 1 covers addition of proper fractions; Test 2, addition of mixed numbers; Test 3, subtraction of proper fractions; and Test 4, subtraction of mixed numbers.

If the children have considerable difficulty, reteaching is, of course, necessary. In reteaching for Test 1, use pages 171-176. In reteaching for Test 2, use pages 177-182. For Test 3, use pages 186-189. For Test 4, use page 185 and pages 190-195.

An activity for able pupils that might be used in connection with this lesson is described on page 259 in the Expanded Notes for page 18 and in Activity 9, page 360.

Answers

Test 1	Test 2	Test 3	Test 4
A 1	A 6	A $\frac{1}{2}$ or $\frac{6}{12}$	A $8\frac{4}{5}$
B $1\frac{1}{4}$	B $4\frac{11}{20}$	B $\frac{2}{15}$	B $7\frac{7}{10}$
C $1\frac{11}{15}$	C $10\frac{2}{3}$ or $10\frac{8}{12}$	C $\frac{11}{24}$	C $\frac{7}{12}$
D $1\frac{17}{24}$	D $14\frac{3}{20}$	D $\frac{7}{12}$	D $6\frac{13}{24}$
E $1\frac{11}{12}$	E $20\frac{7}{16}$	E $\frac{5}{24}$	E $2\frac{5}{12}$
F $1\frac{3}{5}$ or $1\frac{6}{10}$	F $7\frac{4}{5}$ or $7\frac{8}{10}$	F $\frac{1}{2}$ or $\frac{5}{10}$	F 1
G $2\frac{3}{20}$	G $38\frac{23}{24}$	G $\frac{3}{4}$ or $\frac{12}{16}$	G $\frac{13}{15}$
H $1\frac{5}{16}$	H $40\frac{4}{5}$	H $\frac{2}{5}$ or $\frac{4}{10}$	H $6\frac{3}{4}$ or $6\frac{9}{12}$
I $1\frac{5}{12}$	I $17\frac{1}{8}$	I $\frac{7}{20}$	I $\frac{1}{5}$ or $\frac{2}{10}$

I A store had $36\frac{7}{8}$ yd. of printed cloth. $13\frac{3}{4}$ yd. were sold in 2 hr. Then how many yards of the cloth were left to be sold?

J Susan's mother bought 26 lb. of tomatoes. After she had canned some, she had $3\frac{1}{4}$ lb. of tomatoes left. How many pounds of tomatoes had she canned?

K Mrs. Hope needed $2\frac{1}{2}$ cups of powdered sugar to make frosting. She found she had only $1\frac{2}{3}$ cups. How much more powdered sugar did she need?

L Mrs. Hope made $6\frac{1}{2}$ doz. doughnuts. She kept 2 doz. for her family and gave the rest to be sold at the school fair. How many dozen did she give for the school fair?

M A very large truck and its load together weighed 36 T. The truck weighed $15\frac{1}{2}$ T. How heavy was the load?

N Jim raked leaves before dinner one day. Then he worked for $\frac{3}{4}$ of an hour after dinner to finish. He had worked $1\frac{1}{2}$ hr. in all raking leaves. How long had he raked leaves before dinner?

O Pat made a model boat in $4\frac{3}{4}$ hr. He worked $2\frac{1}{2}$ hr. on Friday, and did the rest of the work on Saturday. How long did he work on Saturday?

P The library is $1\frac{9}{10}$ mi. west of Bob's house. To get there, Bob walked $\frac{1}{4}$ mi. west to the bus and then rode the rest of the way. How far did Bob ride on the bus?

Checking up 1

Test 1	Test 2	Test 3	Test 4
A $\frac{3}{8}+\frac{5}{8}=n$	A $2\frac{1}{6}+3\frac{5}{6}=n$	A $\frac{7}{12}-\frac{1}{12}=n$	A $11\frac{3}{5}-2\frac{4}{5}=n$
B $\frac{1}{2}+\frac{3}{4}=n$	B $1\frac{1}{4}+\frac{4}{5}+2\frac{1}{2}=n$	B $\frac{4}{5}-\frac{2}{3}=n$	B $13\frac{7}{10}-6=n$
C $\frac{1}{3}+\frac{3}{5}+\frac{4}{6}=n$	C $3\frac{3}{4}+6\frac{11}{12}=n$	C $\frac{5}{6}-\frac{3}{8}=n$	C $5\frac{1}{4}-4\frac{2}{3}=n$
D $\frac{5}{6}+\frac{1}{8}+\frac{3}{4}=n$	D $6\frac{9}{10}+7\frac{1}{4}=n$	D $\frac{3}{4}-\frac{1}{6}=n$	D $9\frac{3}{8}-2\frac{5}{6}=n$
E $\frac{7}{12}+\frac{1}{2}+\frac{5}{8}=n$	E $9\frac{9}{16}+11\frac{1}{4}=n$	E $\frac{7}{8}-\frac{2}{3}=n$	E $8-5\frac{7}{12}=n$
F $\frac{9}{10}+\frac{7}{10}=n$	F $\frac{3}{5}+\frac{1}{2}+6\frac{7}{10}=n$	F $\frac{7}{10}-\frac{1}{5}=n$	F $6\frac{5}{8}-5\frac{5}{8}=n$
G $\frac{4}{5}+\frac{3}{4}+\frac{3}{5}=n$	G $17\frac{1}{3}+21\frac{5}{6}=n$	G $13\frac{13}{16}-\frac{1}{16}=n$	G $2\frac{2}{3}-1\frac{5}{6}=n$
H $\frac{5}{8}+\frac{11}{16}=n$	H $24+16\frac{4}{5}=n$	H $\frac{9}{10}-\frac{1}{2}=n$	H $7\frac{5}{12}-\frac{2}{3}=n$
I $\frac{2}{3}+\frac{3}{4}=n$	I $10\frac{3}{8}+6\frac{2}{4}=n$	I $\frac{3}{5}-\frac{1}{4}=n$	I $3\frac{1}{10}-2\frac{9}{10}=n$

End-of-block tests on addition and subtraction of fractions 197

197

1 Have the pupils work on these tests independently. Tell them to write their answers in simplest form.

2 After the pupils have corrected their work, do any reteaching that is necessary.

198-199 Side trip

Expanded Notes for this lesson are on pages 332-333.

Objectives

The child learns about time zones, standard time, and daylight-saving time. He also learns about the 24-hour method of expressing time.

Vocabulary

New words page 198 world*, zones, east*, standard, Pacific, telephone*, New York*; page 199 extra*

Comments

You might like to correlate this lesson with what pupils have been learning in their science classes about the rotation of the earth and how it causes day and night. Suggestions for discussion will be found in the Expanded Notes for this lesson.

Once the children realize that it cannot be noon at the same moment everywhere, they are prepared to understand how the time changes as one travels across the United States. The three-hour difference between San Francisco time and New York time then ceases to puzzle the children, and they understand the reason for moving the hands of a timepiece forward when traveling from west to east, and backward when traveling from east to west.

It is important to make sure that the pupils understand the meaning and proper use of A.M. and P.M. Bring out the fact that a "day" has 24 hours and includes a night, that the old day ends at midnight and a new day begins.

Children should know that the 24-hour method of expressing time is used by military services, as well as by some air lines and "ham" radio operators.

In discussing "daylight-saving" time, help the children discover that this system benefits some people and inconveniences others. Bring out the fact that setting the clock ahead does not actually add more daylight to the amount of daylight in 24 hours, but that to the city dweller it seems to do so, since setting the clock ahead and getting up an hour earlier

is a means of getting an extra hour of daylight in the evening.

Answers

A Five

B —

C 9 A.M.

D Mountain; Central

E Turn your clock forward 1 hour for each time zone; turn it back 1 hour for each time zone

F —

G Great confusion would result. Instead of having four time zones in the United States, we would have hundreds. It would be very difficult to make appointments, to conduct business efficiently, to have train or plane schedules, etc.

H To avoid confusion

I Setting the clock forward an hour adds an extra hour of daylight to the end of the day because then it is an hour later when it gets dark. This hour of daylight comes from starting the working day earlier according to standard time.

J 8:20 A.M.; 8:20 P.M.

K 1300

L 0100

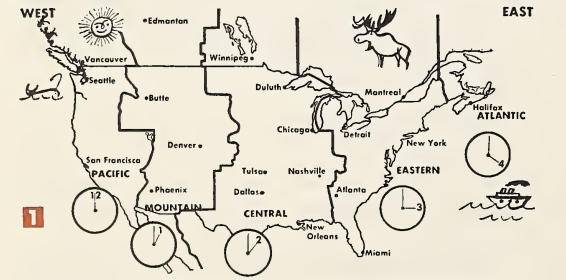
M 1235

N 1530

O 2400; 0000

P 1200; 2400

Side trip



1

A The world is divided into 24 time zones. How many are shown on the map? In which zone do you live?

B Name the zones east of you. Name the zones west of you.

C Look at the clocks on the map above. They show what the standard time is in each of the other zones when it is noon in the Pacific Time Zone. Imagine that you live in California and that you want to telephone a friend in New York. If you know that your friend wants you to call him at noon by his clock, when should you telephone by Pacific Standard Time?

2

D In what time zone will it be noon when it is two o'clock in the afternoon in the Eastern Time Zone? In what zone will it be one o'clock?

E If you travel from west to east, crossing several time zones, how should you change your clock? How should you change your clock when you travel from east to west?

F What time is it now in your classroom? What time is it now in each zone on the map?

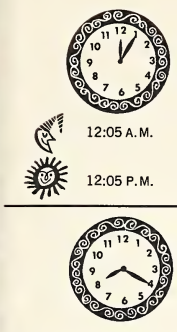
G What would happen if everyone would set his clock at 12 noon when the sun is exactly overhead? 3

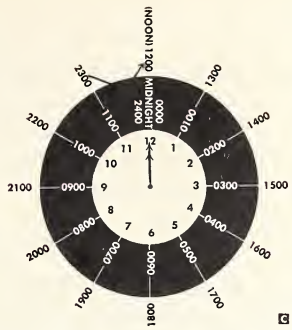
198

1 Have the pupils study the map. Point out that the easternmost time zone shown is outside of the United States. Let the pupils answer the questions in Exercises A and B.

2 Let the pupils discover what the difference in time is between one zone and the next. Let someone tell what the time difference is between San Francisco and New York. Then let the pupils answer the questions in Exercises C to F. (Pairs of able pupils might take turns dramatizing phone calls while the rest of the class checks the accuracy of the statements made. For example, "Hello. Mary? This is Jean, in Denver. It's 6 P.M. here. What time is it in Atlanta?")

3 Discuss the complications that would arise if there were hundreds of different times in use in the United States. For example, how would we know at what time we could get programs on radio and TV, meet trains, etc.?





H What towns near you have the same time as you do? Why is it a good thing to set all clocks in a zone so that they tell the same time? **1**

I In some places, "daylight-saving" time is used in summer. At 6 P.M. standard time, a clock that has been set for daylight-saving time shows 7 o'clock. How does this add an extra hour of daylight to the end of the day? Where does this hour of daylight come from? **2**

J Look at Clock B above. How do you write the time it shows if it is morning? How do you write the time if it is night? Clock A will help you. **3**

K Picture C shows another way to write time. The P.M. hours are shown with green numerals. How is 1 P.M. expressed? **4**

L A.M. hours are shown with white numerals. How is 1 A.M. expressed?

M 12:35 A.M. is written as 0035. How is 12:35 P.M. written?

N If it is 3:30 P.M., how do you write the time in this new way?

O Picture C shows two ways to write the time at midnight. What are they?

P How would you write the next hour after 1100? How would you write the next hour after 2300?

199

199

1 Have pupils name some towns in their time zone.

2 Help the pupils see that if it gets dark at 8 P.M. standard time, and we set the clock forward 1 hour, it will then get dark at 9 P.M. This hour of daylight is not added to the number of daylight hours in a day, but simply put in a different place. Help the pupils see that this hour comes from the hour we might have spent in bed in the morning. By getting up an hour earlier (standard time), we have more light at the end of the day. We do not lose any sleep, because we keep the same daily schedule.

3 Let someone explain the meaning and use of A.M. and P.M. before Exercise J is discussed.

4 After pupils have answered the questions in Exercises K to P and studied Picture C, discuss the advantages of the 24-hour method.

200 Thinking straight

Expanded Notes for this lesson are on page 333.

Objectives

The child learns to use short cuts that help him to multiply mentally. He also practices mental calculation in addition and subtraction.

Vocabulary

There are no new words.

Comments

This lesson not only introduces the children to mental multiplication, but also makes explicit use of the mathematical principle known as the distributive law. Suppose, for example, 38 is to be multiplied by 6. Then 38 may be thought of as 30 plus 8, and you may multiply 30 by 6 and multiply 8 by 6. The sum of these two products will equal the product 6×38. While this principle is pointed out and used in this lesson, it is neither necessary nor desirable to use the mathematical name (distributive law). It is enough to show how a number may be expressed as a sum or a difference of two numbers, how these may then be multiplied separately and the products added to find the final answer. You may wish to demonstrate the application of the distributive law on the board in this way:

$6 \times 38 = n$ $\quad$ $38 = 30 + 8$

$$\begin{array}{rl} 180 & (6 \times 30) \\ 48 & (6 \times 8) \\ \hline 228 & \end{array} \quad \text{or} \quad \begin{array}{rl} 48 & (6 \times 8) \\ 180 & (6 \times 30) \\ \hline 228 & \end{array}$$

You may point out that although in mental calculation it is frequently easier to multiply the tens before the ones, the answer obtained is the same no matter which order is used in multiplying and then adding the partial products.

A different approach will sometimes be easier. Consider 6×38 again. 38 can be thought of as $40 - 2$. This time we can multiply as follows: $6 \times 40 = 240$; $6 \times 2 = 12$. Since the multiplicand was increased by 2, then 6×2 must be subtracted; $240 - 12 = 228$. When there are 6 or more ones in the multiplicand, many pupils will prefer the method just outlined.

Encourage the child to use any method he finds convenient; the opportunity this gives him to develop his own ingenuity is one that should not be missed. There is usually more than one way to compute mentally, and as long as the solution is correct, the way one goes about it is not important.

The last block of exercises on the page provides a brief review of mental calculation in addition and subtraction, originally taught on page 156. Some children may need to look again at the work on page 156 before proceeding with these exercises.

Be careful not to set up unreasonable standards of achievement, and do not require any child to work beyond his capacity when he calculates "in his head." Let him think of mental calculation as an ability that will be convenient in many everyday situations.

Answers

(block 1)

A 180 is a partial answer, obtained by multiplying part of the number (30) by 6. When the other part of the number (8) is multiplied by 6, this product (48) is another partial answer. To find the final product, the two partial products must be added.

B Andy increased the number to be multiplied (38) by 2; that is, his product was 6×2, or 12, more than the product of 38 multiplied by 6.

C 7; 7; 420+28; 448

D 8; 8; 8; 632

E 2; $8\times50=400$; $8\times2=16$; $400+16=416$

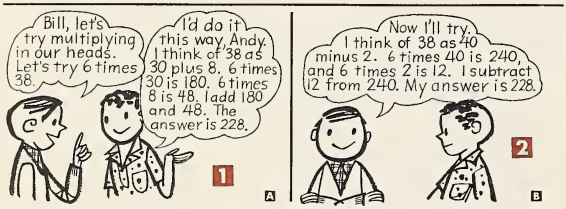
F 50; $4\times50=200$; $4\times3=12$; $200-12=188$

G 7; $4\times40=160$; $4\times7=28$; $160+28=188$

H Responses will vary.

(block 2)				(block 3)			
A	144	F	342	A	117	F	24
B	133	G	138	B	27	G	530
C	228	H	168	C	38	H	286
D	320	I	312	D	105	I	14
E	144	J	243	E	118	J	535

Thinking straight


Bill, let's try multiplying in our heads. Let's try 6 times 38.
I'd do it this way, Andy. I think of 38 as 30 plus 8. 6 times 30 is 180. 6 times 8 is 48. I add 180 and 48. The answer is 228.
1
A
Now I'll try. I think of 38 as 40 minus 2. 6 times 40 is 240, and 6 times 2 is 12. I subtract 12 from 240. My answer is 228.
2
B

A In Picture A, why did Bill add 48 to 180?

B In Picture B, why did Andy subtract 12 from 240?

3 C To multiply 64 by 7, think of 64 as 60+4. Then multiply 60 by ☐ and multiply 4 by ☐. Now what do you add? What is the answer?

D To multiply 79 by 8, think of 79 as 80−1. Then multiply 80 by ☐ and multiply 1 by ☐. Now subtract ☐ from 640. What is the answer?

E Multiply 52 by 8 in your head. Use the way shown in Picture A. Think of 52 as 50+☐.

F Multiply 47 by 4 in your head. Use the way shown in Picture B. Think of 47 as ☐−3.

G Use the way shown in Picture A to multiply 47 by 4. This time think of 47 as 40+☐.

H When you multiply 47 by 4, is the work easier if you think of 47 as 40+7 or as 50−3?

Multiply in your head. Use the way that is easier for you. 4

A	$6\times24=n$	F	$9\times38=n$
B	$7\times19=n$	G	$3\times46=n$
C	$3\times76=n$	H	$7\times24=n$
D	$5\times64=n$	I	$6\times52=n$
E	$8\times18=n$	J	$9\times27=n$

Do these examples in your head.

A	$54+63=n$	F	$149-125=n$
B	$46-19=n$	G	$218+312=n$
C	$81-43=n$	H	$624-338=n$
D	$28+77=n$	I	$101-87=n$
E	$95+23=n$	J	$446+89=n$

5

200

1 Have two boys act out Picture A. Ask another child to answer Exercise A. The rest of the class may participate by doing the computation mentally, if they can, or by using penciled notes.

2 Dramatize Picture B and discuss Exercise B in the same way.

3 Discuss Exercises C to H, calling on as many children as you can. For Exercise H, encourage the children to justify their preference.

4 Let pairs of children choose exercises from this block to act out. Let the class decide if the methods chosen are good.

5 Before the children do these exercises, you may wish to review methods of mental calculation in addition and subtraction (see page 156 and the teaching notes for that page).

201-202 Checking up

Expanded Notes are not considered necessary for this lesson.

Objectives

The child tests his ability to solve problems and to compute with whole numbers and fractions.

Vocabulary

New words page 201 automobile*

Comments

An achievement test on problem solving is presented on page 201. Allow enough time so that everyone has an opportunity to complete the test. The ten tests which follow on page 202 are not to be treated as speed tests; use only as many of them as the group can reasonably be expected to complete in one period.

An activity for the able pupils that might be used with this lesson is described in the Expanded Notes for page 18, page 259, and in Activity 9, page 360.

Reteaching chart

The pages listed below tell you where to find reteaching materials if you need them. The page references are to Book 5 unless otherwise indicated.

Test 1

Problem	Pages	Problem	Pages
A	43-45	G	20
B	96-100, 19	H	20
C	101-104	I	21-22
D	19-22	J	40-42, 19
E	96-100, 19	K	25, 86, 138-139
F	21	L	24

Test	Pages
2	Book 4: 24-27, 36-41, 45 Book 5: 20 (for Exercises C, D, E, G, H, and I)
3	Book 4: 24-27
4	Book 4: 36-41, 45, 54 Book 5: 43-45 (for Exercises A, B, and G), 40-42 (for Exercises D, H, and I)
5	61-64, 77, 164, 138-139 (for Exercises C and E)
6	78-84, 91, 24 (for Exercises C, D, F, H, and I)
7	171-176, 186-189, 20 (for Exercises C, D, F, G and H)
8	177-182, 190-195, 20 (for Exercises C, D, F, G, and H)
9	171-176, 186-189, 40-42 (for Exercises B and E), 43-45 (for Exercises A, G, and H)
10	177-182, 190-195, 40-42 (for Exercises E and G), 43-45 (for Exercises C, D, F, and I)
11	140-142

Answers

Test 1

A $144-[36]=108$
B $\$10.00-(\$7.18)=[\$2.82]$
C $(16)\div[4]=4$
D $37-23=[14]$ or $23+[14]=37$
E $(\$.75)+(\$.36)+(\$.90)=[\$2.01]$
F $\$8.75-\$6.95=[\$1.80]$
G $\$19.32+[\$7.18]=\$26.50$
H $[\$3.75]+\$1.25=\$5.00$
I $\$67.95-\$52.00=[\$15.95]$
J $[203]-87=116$ or $87+116=[203]$
K $\$5.46\div\$.07=[78]$ or $[78]\times\$.07=\$5.46$
L $200\div[25]=8$

Test 2	Test 3	Test 4	Test 5
A 11,718	A 2055	A 4416	A 146,083
B 12,015	B 19,659	B 1259	B 134,234
C 2688	C 10,790	C 4707	C 66
D 1618	D 21,170	D 5214	D 17,725
E 1084	E 2176	E 4245	E 29
F 6302	F 14,800	F 16,762	F 211,864
G 6057	G 100,011	G 47,802	G 446,775
H 801	H 101,070	H 130,107	H 355,352
I 2608		I 159,727	I 53,628
			J 84,112

Test 6	Test 7	Test 8	Test 9
A 123 and 60 rm.	A 1	A $10\frac{13}{16}$	A $\frac{1}{4}$ or $\frac{3}{12}$
B 28 and 13 rm.	B $1\frac{7}{24}$	B $10\frac{5}{12}$	B $\frac{7}{8}$
C 27	C $\frac{4}{15}$	C $3\frac{3}{8}$	C $\frac{7}{12}$
D 148	D $\frac{3}{10}$	D $3\frac{1}{15}$	D $\frac{13}{30}$
E 81 and 3 rm.	E $1\frac{11}{24}$	E $11\frac{1}{6}$	E $\frac{13}{20}$
F 581	F $\frac{5}{16}$	F $2\frac{4}{5}$ or $2\frac{8}{10}$	F $\frac{1}{24}$
G 194 and 29 rm.	G $\frac{5}{16}$	G $2\frac{1}{8}$	G $\frac{5}{12}$
H 503	H $\frac{1}{10}$	H $2\frac{17}{24}$	H $\frac{1}{5}$ or $\frac{2}{10}$
I 86	I $1\frac{7}{12}$	I $16\frac{1}{16}$	I $\frac{7}{24}$
J 587 and 39 rm.			

Test 10

A $5\frac{5}{8}$
B $6\frac{11}{12}$
C $3\frac{1}{30}$
D $14\frac{1}{2}$ or $14\frac{3}{6}$
E $12\frac{15}{16}$
F $7\frac{5}{6}$ or $7\frac{10}{12}$
G $28\frac{1}{2}$ or $28\frac{5}{10}$
H $5\frac{1}{2}$ or $5\frac{8}{16}$
I $5\frac{1}{24}$

Test 11

A 12/30
B 24/3
C 4/5
D 4/1
E 25/45
F 9/30
G 50/12
H 8/2

Checking up 1

Test 1

A Mrs. Wood had 144 flower bulbs. Then she gave some of her bulbs to a neighbor. She had 108 bulbs left to plant. How many bulbs did Mrs. Wood give to her neighbor?

B Mr. Diamond bought a gallon of paint for $4.29 and a paint brush for $2.89. He gave the clerk at the paint store a 10-dollar bill. How much money should the clerk have given him in change?

C Tony mailed some packages at the post office. The packages weighed 3 lb., 5 lb., 2 lb., and 6 lb. What was the average weight of the packages?

D Miss Parks asked each of the 37 children in the fifth grade to bring a news story to school. 23 children brought news stories to school in the morning. How many of the children still had to bring their news stories?

E Sue went to the post office to buy some stamps. She bought twenty-five 3¢ stamps, eighteen 2¢ stamps, and fifteen 6¢ stamps. How much money did she spend for all the stamps?

F Kathy bought a blue dress for $8.75 and a red dress for $6.95. The blue dress cost how much more than the red dress?.

G The steam engine that Don wants to buy costs $26.50. He has $19.32. How much more money does he need before he can buy the steam engine?

H Mary had some money in her bank. Then she earned $1.25 and put it in her bank. Now she has $5 altogether. How much money was in her bank before she earned the $1.25?

I Jim's bicycle cost $52, and Don's bicycle cost $67.95. Jim's bicycle cost how much less than Don's?

J After Bob and his father had gone 87 mi. on an automobile trip from their home to Newmarket, they saw a sign that said, "116 mi. to Newmarket." How far was it from their home to Newmarket?

K Kathy sold ice cream at the school fair. She sold it at 7¢ a dish. At the end of the afternoon, she had sold $5.46 worth. How many dishes of ice cream had she sold?

L The fifth-grade boys and girls hoped to sell 200 tickets for a play. Eight of the girls said they would make the tickets. If each of the girls made the same number, how many tickets did each girl make? 2

Test 2 1

A $9176+2542=n$
B $7670+4345=n$
C $5006+n=7694$
D $1684+n=3302$
E $n+5874=6958$
F $2654+3648=n$
G $795+n=6852$
H $n+8947=9748$
I $n+1655=4263$

Test 3

Find the sum.

A 365, 812, 581, 297
B 6634, 8701, 4324
C 2448, 6763, 1579
D 9072, 7248, 4850
E 919, 632, 455, 170
F 2001, 7244, 5555
G 48369, 51642
H 93287, 7783

Test 4

A $5634-n=1218$
B $4006-n=2747$
C $9097-4390=n$
D $n-3804=1410$
E $5944-1699=n$
F $86470-69708=n$
G $78944-n=31142$
H $n-42559=87548$
I $n-68406=91321$

Test 5

A $287\times509=n$
B $82\times1637=n$
C $n\times53=3498$
D $25\times709=n$
E $71\times n=2059$
F $373\times568=n$
G $925\times483=n$
H $4132\times86=n$
I $164\times327=n$
J $28\times3004=n$

Test 6

A $7809\div63=n$
B $545\div19=n$
C $1485\div n=55$
D $9176\div n=62$
E $6483\div80=n$
F $52871\div n=91$
G $14191\div73=n$
H $13581\div n=27$
I $3354\div n=39$
J $27041\div46=n$

Test 7

A $\frac{5}{8}+\frac{1}{8}=n$
B $\frac{7}{8}+\frac{5}{12}=n$
C $n+\frac{2}{5}=\frac{4}{5}$
D $\frac{3}{5}+n=\frac{9}{10}$
E $\frac{3}{8}+\frac{5}{6}=n$
F $n+\frac{1}{8}=\frac{7}{16}$
G $\frac{7}{16}+n=\frac{3}{4}$
H $n+\frac{7}{10}=\frac{4}{5}$
I $\frac{5}{6}+\frac{3}{4}=n$

Test 8

A $3\frac{3}{8}+7\frac{3}{16}=n$
B $9\frac{3}{4}+\frac{2}{3}=n$
C $n+12\frac{3}{8}=16\frac{1}{8}$
D $5\frac{2}{3}+n=8\frac{2}{3}$
E $4\frac{2}{3}+6\frac{1}{2}=n$
F $n+9\frac{7}{10}=12\frac{1}{2}$
G $2\frac{5}{8}+n=4\frac{3}{4}$
H $11\frac{5}{8}+n=14\frac{1}{3}$
I $7\frac{5}{16}+8\frac{3}{4}=n$

Test 9

A $\frac{7}{12}-n=\frac{1}{3}$
B $n-\frac{3}{8}=\frac{1}{2}$
C $\frac{3}{4}-\frac{1}{6}=n$
D $\frac{5}{6}-\frac{2}{5}=n$
E $n-\frac{1}{4}=\frac{2}{5}$
F $\frac{7}{8}-\frac{5}{6}=n$
G $1\frac{1}{2}-n=\frac{1}{2}$
H $\frac{9}{10}-n=\frac{7}{10}$
I $\frac{3}{8}-\frac{1}{3}=n$

Test 10

A $9\frac{3}{4}-4\frac{1}{8}=n$
B $13\frac{1}{2}-6\frac{7}{12}=n$
C $5\frac{7}{10}-n=2\frac{2}{5}$
D $17\frac{5}{8}-n=3\frac{1}{4}$
E $n-8\frac{7}{16}=4\frac{1}{2}$
F $9\frac{3}{4}-n=1\frac{11}{12}$
G $n-21\frac{2}{3}=7\frac{1}{10}$
H $11\frac{1}{16}-5\frac{9}{16}=n$
I $15\frac{7}{8}-n=10\frac{3}{8}$

Test 11

A $\frac{2}{5}=\frac{\square}{30}$
B $\frac{8}{1}=\frac{24}{\square}$
C $\frac{32}{40}=\frac{4}{\square}$
D $\frac{12}{3}=\frac{\square}{1}$
E $\frac{5}{9}=\frac{25}{\square}$
F $\frac{3}{10}=\frac{\square}{30}$
G $\frac{25}{6}=\frac{50}{\square}$
H $\frac{64}{16}=\frac{\square}{2}$ 2

201

1 Give this test without a time limit. Direct the pupils to write an equation for each problem. Remind them that in a multiple-step problem they should use the screen to hold a place for preliminary answers in the equation, and an *n* for the final answer.

2 Supply the children with correct answers, and let them verify their own work. Determine which problems have given the class the most trouble. Then refer to the Reteaching Chart on page 185 and do whatever reteaching is necessary.

202

1 Have the children work independently on these tests. Assign only as many as you think the pupils can reasonably complete in the allotted time. Use as many class periods as necessary to complete all the tests.

2 After the pupils have corrected their work, refer to the Reteaching Chart for any needed reteaching. In some cases the error will lie in the pupil's computation; in others, the pupil will have misinterpreted the equation. Plan your reteaching according to which type of error is involved.

203 Keeping skillful

Expanded Notes are not considered necessary for this lesson.

Objectives

The child extends his knowledge of division to include dividing by numbers whose numerals have three figures.

Vocabulary

There are no new words.

Comments

The principles that were used for one-figure and two-figure divisors are used when the divisor has three figures. It is a good idea to let children show one another the different ways to work a division problem. Do not expect all children to be very efficient at estimating partial quotients, particularly when they are working with larger numbers. A method of estimating quotients is taught on pages 205-206.

Answers

(block 1)

B 7 and 22 rm. D 410 F 104
C 187 and 350 rm. E 615 and 2 rm. G 34 and 141 rm.
H 82 K 205 M 90
I 49 and 5 rm. L 97 N 802 and 42 rm.
J 16 and 205 rm.

(block 2)

A 170 G 15 M 57 and 153 rm.
B 907 H 82 N 750 and 24 rm.
C 782 I 73 and 72 rm. O 408
D 91 J 83 P 816
E 36 K 272 Q 370 and 135 rm.
F 85 L 682 R 629

(block 3)

A $\frac{13}{18}$ E $1\frac{3}{16}$ I $1\frac{4}{15}$ M $\frac{4}{9}$ or $\frac{8}{18}$
B $\frac{2}{9}$ F $8\frac{7}{12}$ J $3\frac{11}{24}$ N $3\frac{1}{18}$
C $3\frac{4}{5}$ or $3\frac{8}{10}$ G $2\frac{7}{8}$ K $2\frac{1}{16}$ O $1\frac{1}{3}$ or $1\frac{4}{12}$
D $9\frac{1}{2}$ or $9\frac{8}{16}$ H 23 L $7\frac{5}{9}$ P $9\frac{1}{12}$

(block 4)

A 648 hr. F 46 wk. K 7 yd. P 492 things
B 140 pk. G 18 c. L 56 qt. Q 1440 min.
C 92 tbs. H 34,000 lb. M 120 yd. R 33 bu.
D 1152 in. I 3 min. N 636 mo.
E 7 gal. J 11 qt. O 36 pt.

Keeping skillful

A $147232 \div 287 = n$

1

287 | 147232 | 300
86100
61132 | 200
57400
3732 | 10
2870
862 | 2
574
288 | 1
287
1 | 513

287 | 147232 | 500
143500
3732 | 10
2870
862 | 3
861
1 | 513

2

B $862 \div 120 = n$
C $92354 \div 492 = n$
D $160310 \div 391 = n$
E $547352 \div 890 = n$
F $28600 \div 275 = n$
G $9899 \div 287 = n$
H $41574 \div 507 = n$
I $30238 \div 617 = n$
J $14829 \div 914 = n$
K $32595 \div 159 = n$
L $75369 \div 777 = n$
M $54810 \div 609 = n$
N $364952 \div 455 = n$

3

A $1360 \div n = 8$
B $8163 \div 9 = n$
C $5474 \div n = 7$
D $546 \div n = 6$
E $684 \div 19 = n$
F $1955 \div n = 23$
G $1275 \div 85 = n$
H $6478 \div n = 79$
I $5401 \div 73 = n$
J $4233 \div 51 = n$
K $25568 \div 94 = n$
L $19778 \div n = 29$
M $9900 \div 171 = n$
N $93774 \div 125 = n$
O $83640 \div n = 205$
P $613632 \div n = 752$
Q $67845 \div 183 = n$
R $512635 \div n = 815$

3

A $\frac{2}{3} + \frac{1}{2} = n$
B $\frac{8}{9} - \frac{2}{3} = n$
C $n + 1\frac{1}{2} = 5\frac{3}{10}$
D $12\frac{15}{16} + n = 22\frac{7}{16}$
E $\frac{7}{16} + \frac{3}{4} = n$
F $n - 6\frac{3}{4} = 1\frac{5}{8}$
G $6\frac{3}{4} - n = 3\frac{7}{8}$
H $14\frac{7}{12} + 8\frac{5}{12} = n$
I $11\frac{2}{5} + n = 12\frac{2}{3}$
J $8\frac{5}{8} - n = 5\frac{3}{8}$
K $10\frac{1}{4} + n = 12\frac{5}{16}$
L $8 - n = \frac{4}{9}$
M $n + \frac{1}{6} = \frac{11}{18}$
N $\frac{13}{18} + 2\frac{1}{3} = n$
O $n - \frac{3}{4} = \frac{7}{12}$
P $2\frac{1}{4} + 6\frac{5}{6} = n$

4

A 27 da. = ▪ hr.
B 35 bu. = ▪ pk.
C ▪ tbs. = 46 fl. oz.
D ▪ in. = 96 ft.
E 56 pt. = ▪ gal.
F 322 da. = ▪ wk.
G 144 fl. oz. = ▪ c.
H ▪ lb. = 17 T.
I 180 sec. = ▪ min.
J 22 pt. = ▪ qt.
K 252 in. = ▪ yd.
L ▪ qt. = 14 gal.
M 360 ft. = ▪ yd.
N ▪ mo. = 53 yr.
O 72 c. = ▪ pt.
P ▪ things = 41 doz.
Q 24 hr. = ▪ min.
R 132 pk. = ▪ bu.

203

203

1 Discuss these examples, bringing out the fact that the same procedure is used with three-figure divisors as with two-figure divisors. Point out that the second example involves less work because a larger partial quotient was chosen at the beginning.

2 Assign this block of exercises and let the pupils work independently. Then provide answers so everyone can verify his own work.

3 Have the pupils work on these exercises independently.

4 Allow the pupils to refer to the table of equivalent measures on page 93 as they need to in working these exercises.

204 Thinking straight

Expanded Notes are not considered necessary for this lesson.

Objectives

The child learns how to round off numbers to suit various situations. He also uses the principle that he can divide a divisor and a dividend by the same multiple of ten without affecting the quotient.

Vocabulary

New words page 204 zero

Comments

The main purpose of this lesson is to teach some techniques that the child will use when he learns a new procedure for estimating partial quotients in division. One important idea that he should get from this lesson is that numbers can be rounded either up or down and to any multiple of ten (hundreds, thousands, etc.) as may be desired.

The children have already studied some techniques for rounding off numbers. (See page 23.) In the earlier lesson the emphasis was on rounding off numbers in the millions to the nearest thousand, hundred thousand, or million. This lesson deals with rounding off numbers in the hundreds and thousands.

Some time should be spent in discussing the value of rounding off numbers; for example, whether numbers should be rounded up or down, and what degree of approximation is desired; that is, whether a number rounded to the nearest hundred, thousand, etc., is close enough to the original number to be useful. Point out, too, that sometimes no rounding off should be done because exact quantities should be used.

The lesson also applies the principle that if both the divisor and dividend are multiplied or divided by the same multiple of ten, the quotient will not be affected. This idea was introduced in the lesson on pages 136-137. In this lesson the children see what happens when both dividend and divisor end in one or two zeros. They are shown that the best short-cut way to divide is by either 10 or 100, depending upon whether both numerals end in one or two zeros. The children should see that the result of dividing by either 10 or 100 may be shown by crossing out one or two zeros.

Answers

(block 1)

	higher hundred	lower hundred	nearer hundred
A	600	500	500
B	900	800	900
C	500	400	500
D	400	300	400
E	200	100	100
F	900	800	800
G	3200	3100	3200
H	300	200	200
I	1000	900	1000
J	2100	2000	2100
K	1000	900	900
L	1600	1500	1600

(block 2)

	higher thousand	lower thousand	nearer thousand
M	7000	6000	7000
N	6000	5000	6000
O	9000	8000	8000
P	10,000	9000	10,000
Q	24,000	23,000	24,000
R	12,000	11,000	11,000
S	73,000	72,000	72,000
T	41,000	40,000	41,000
U	30,000	29,000	30,000
V	58,000	57,000	57,000
W	83,000	82,000	83,000
X	75,000	74,000	75,000

(block 3)

A Yes; because the dividend, 162, and the divisor, 27, have been divided by the same number, 3

B Yes; because both 180 and 30 have been divided by 10

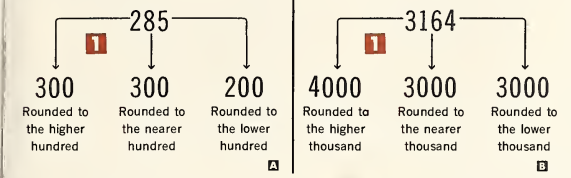
C 6

D 8

E 30; because both 1500 and 50 must be divided by the same number (50 cannot be divided by 100, only 10)

F 90	I 8	L 4	O 44
G 3	J 60	M 12	P 9
H 32	K 90	N 4	Q 220

Thinking straight



2 Round each of the following numbers to the higher hundred, the lower hundred, and the nearer hundred. For Example A you would have 600, 500, and 500.

A	503	E	144	I	970
B	872	F	823	J	2061
C	455	G	3197	K	935
D	391	H	208	L	1580

2 Round each of the following numbers to the higher thousand, the lower thousand, and the nearer thousand.

M	6542	Q	23769	U	29880
N	5579	R	11368	V	57301
O	8116	S	72384	W	82500
P	9774	T	40698	X	74615

A $162 \div 3 = 54$, and $27 \div 3 = 9$. Will $54 \div 9$ give the same answer as $162 \div 27$? Why? 3

B $180 \div 10 = 18$, and $30 \div 10 = 3$. Will $18 \div 3$ give the same answer as $180 \div 30$? Why? 4

C The short-cut way to divide 180 by 30 is first to divide each number by 10. $180 \div 30 = 18\not{0} \div 3\not{0}$. The answer is ■. 5

D The short-cut way to divide 3200 by 400 is first to divide each number by 100. $3200 \div 400 = 32\not{0}\not{0} \div 4\not{0}\not{0}$. The answer is ■. 5

E $1500 \div 50 = 150\not{0} \div 5\not{0}$, or ■. Why do you cross out one zero in 1500? 6

Use the short cut and then divide. 7

F	$2700 \div 30 = n$	L	$1600 \div 400 = n$
G	$210 \div 70 = n$	M	$10800 \div 900 = n$
H	$1600 \div 50 = n$	N	$3600 \div 900 = n$
I	$640 \div 80 = n$	O	$22000 \div 500 = n$
J	$4800 \div 80 = n$	P	$1800 \div 200 = n$
K	$5400 \div 60 = n$	Q	$44000 \div 200 = n$

204 Rounding numbers: preparation for estimating quotients

204

1 Have the pupils study these two charts to see examples of rounding to the higher, nearer, and lower hundred and thousand. In Chart B, they should see that the nearer thousand to 3164 is 3000 because 164 is not over half the way from 3000 to 4000.

2 Let the pupils take turns answering these exercises.

3 In discussing this exercise, remind the pupils of their earlier work on page 136.

4 Relate this exercise to Exercise A. Point out that both numerals end in zero.

5 Have the children verify the results by long division.

6 Point out that only one zero can be crossed out because 50 can be divided only by 10.

7 Assign these exercises as written work.

205-206 Thinking straight

Expanded Notes are not considered necessary for this lesson.

Objectives

The child learns a method for estimating partial quotients.

Vocabulary

New words page 206 choose*

Comments

A useful way of estimating partial quotients in division is presented in this lesson. In this method the child never gets a partial quotient that is too large. He may occasionally estimate a quotient that is smaller than the maximum he could use, but this smaller partial quotient is not discarded—instead, it can be used in the computation.

The method, presented by means of a cartoon and examples in the textbook, involves these steps:

(1) For divisors less than 100, round up the divisor to the next 10 or 100; for divisors less than 1000, round them up to the next 1000. Round down the dividend to a lower multiple of 100, 1000, etc., so that the number represented by the first one or two figures is exactly divisible by the one represented by the first figure of the rounded-up divisor. For example, suppose you want to divide 2963 by 84. Round 84 up to 90. Round 2963 down to 2700, because you want a number divisible by 9.

(2) Divide both trial divisor and trial dividend by 10 or 100. Thus, in the example above, think of $2700 \div 90$ as $270 \div 9$.

(3) Now a good partial quotient for the original division problem is easily found. For this example, 30 is a good partial quotient to start with.

Some of the children will be able to use this method without pencil and paper; others will need to use scratch paper to write the rounded numbers and find the partial quotient. In some division problems, no rounding off is necessary, since a good partial quo-

tient will be immediately apparent. Or in some cases, either the dividend or the divisor will be in usable form already, and only one of the numbers will have to be rounded off.

The work on the preceding page (204) involving rounding numbers and dividing by 10 and 100 should have given the children the necessary background for understanding the work in this lesson.

Very slow learners may find any definite method for estimating quotients difficult. Instead, they may be inclined to use a trial-and-error method. Experience has shown that when they use a trial-and-error method, they will generally get a trial quotient that is smaller than the maximum partial quotient that can be used. The division takes longer than if a maximum partial quotient were found at the beginning, but this does not mean that their work will be incorrect or wasted.

Answers

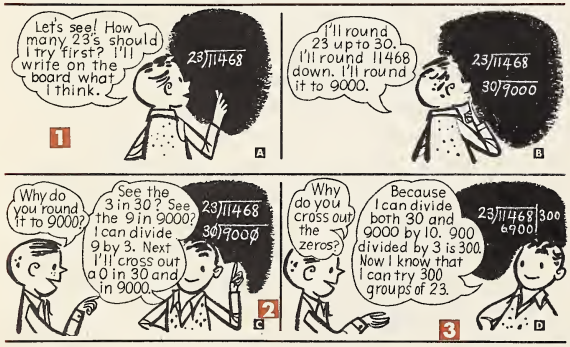
(block 1)

A 74 and 441 rm.
B 56 and 2 rm.
C 183 and 191 rm.

(block 2)

A 396 and 27 rm.
B 68 and 22 rm.
C 174 and 348 rm.
D 113
E 193 and 251 rm.
F 724 and 23 rm.
G 14
H 134 and 22 rm.
I 5 and 306 rm.
J 542
K 410 and 27 rm.
L 135 and 277 rm.
M 505
N 492 and 46 rm.
O 130 and 364 rm.
P 541 and 54 rm.
Q 322 and 27 rm.
R 85 and 78 rm.
S 86
T 141 and 478 rm.
U 95 and 176 rm.
V 748
W 376

Thinking straight


Let's see! How many 23's should I try first? I'll write on the board what I think.
23)11468
A
I'll round 23 up to 30. I'll round 11468 down. I'll round it to 9000.
23)11468
30)9000
B
1
Why do you round it to 9000?
See the 3 in 30? See the 9 in 9000; I can divide 9 by 3. Next I'll cross out a 0 in 30 and in 9000.
23)11468
30)900ø
C
2
Why do you cross out the zeros?
Because I can divide both 30 and 9000 by 10. 900 divided by 3 is 300. Now I know that I can try 300 groups of 23.
23)11468 300
6900
D
3

A $64895 \div 871 = n$

Round the divisor up to 900.

$871\overline{)64895}$ ← Round this number down to 63,000 because the 63 in 63,000 can be divided by the 9 in 900.

4

$9ø\overline{)63øøø}$ ← Why can you cross out 2 zeros in each numeral? Now divide 630 by 9. You can either do this work in your head or write it on paper.

$871\overline{)64895}\ 70$ ← Now you know that you can try 70 groups of 871. $70 \times 871 = 60970$

60970

Now turn the page.

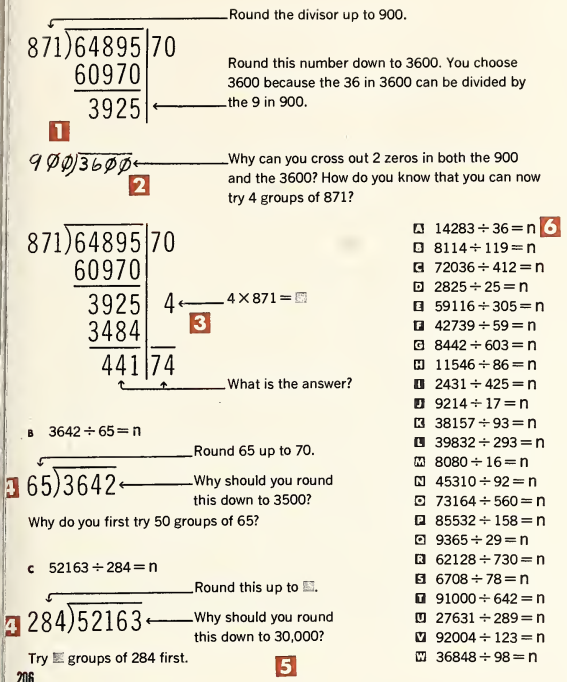
205

1 Let two children read the dialogue. Be sure the class understands that first the divisor is rounded to the next higher decade. Then point out that the number to be divided (do not say *dividend*) is rounded down to a number easily divided by the divisor.

2 Ask why the zeros are crossed out. Relate this to the work done on page 204.

3 Have the children try multiplying 23 by 300 mentally to see if they get 6900. You may discuss what to do next.

4 Have the pupils notice that the divisor in the new example is rounded up to the next hundred and that the dividend is rounded down to 63,000 because 63 in 63,000 is divisible by 9 in 900. Point out that both numbers are divided by 100. Let the children compute on scratch paper, if they need to, and also multiply 871 by 70 to check the product given.


Round the divisor up to 900.
871)64895|70
60970
3925
Round this number down to 3600. You choose 3600 because the 36 in 3600 can be divided by the 9 in 900.
1
900)3600
Why can you cross out 2 zeros in both the 900 and the 3600? How do you know that you can now try 4 groups of 871?
2
871)64895|70
60970
3925 | 4
3484
441 | 74
4 × 871 = ▢
3
What is the answer?
B 3642 ÷ 65 = n
Round 65 up to 70.
4 65)3642
Why should you round this down to 3500?
Why do you first try 50 groups of 65?
C 52163 ÷ 284 = n
Round this up to ▢.
4 284)52163
Why should you round this down to 30,000?
Try ▢ groups of 284 first.
5
A 14283 ÷ 36 = n 6
D 8114 ÷ 119 = n
G 72036 ÷ 412 = n
O 2825 ÷ 25 = n
E 59116 ÷ 305 = n
F 42739 ÷ 59 = n
C 8442 ÷ 603 = n
N 11546 ÷ 86 = n
I 2431 ÷ 425 = n
J 9214 ÷ 17 = n
K 38157 ÷ 93 = n
L 39832 ÷ 293 = n
M 8080 ÷ 16 = n
N 45310 ÷ 92 = n
O 73164 ÷ 560 = n
P 85532 ÷ 158 = n
C 9365 ÷ 29 = n
R 62128 ÷ 730 = n
S 6708 ÷ 78 = n
T 91000 ÷ 642 = n
U 27631 ÷ 289 = n
V 92004 ÷ 123 = n
W 36848 ÷ 98 = n
206

206

1 In discussing the rounding procedures, point out that 871 is rounded up to the same number it was before, and that 3925 is rounded down to 3600 because the 36 is divisible by 9.

2 Ask why the result of dividing the divisor and dividend by 100 can be shown by crossing out zeros.

3 Be sure the pupils understand why 4 is used and that it means there are at least 4 groups of 871 in 3925. Let them verify the multiplication and subtraction. Have someone give the final answer.

4 Let a pupil go to the chalkboard and find the answer to this example. He should discuss each step with the class.

5 Let another pupil finish the example in the cartoon on page 205.

6 Assign these exercises as written work.

207-208 Using arithmetic

Expanded Notes are not considered necessary for this lesson.

Objectives

The child learns to solve problems that require him to find how much more or less one fraction or mixed number is than another. He also learns the meaning of the word *difference* as applied to subtraction.

Vocabulary

New words page 207 difference*; page 208 farther*

Comments

In this lesson, for the first time, the pupils encounter comparison problems that involve fractions and mixed numbers. They are also introduced to the word *difference* and learn that finding the difference is the same as finding how much greater or how much smaller one quantity is than another.

Answers

A $3\frac{1}{2}-2\frac{7}{8}=[\frac{5}{8}]$

B $\frac{5}{8}$ yd.

C $57\frac{1}{2}-28\frac{1}{4}=[29\frac{1}{4}]$

D $29\frac{1}{4}$ lb.

E $52\frac{3}{4}-30\frac{1}{4}=[22\frac{1}{4}]$; $[22\frac{2}{4}]$

F $6\frac{1}{2}-4=[2\frac{1}{2}]$

G $2\frac{1}{4}-1\frac{2}{3}=[\frac{7}{12}]$

H $50\frac{1}{2}-48\frac{1}{4}=[2\frac{1}{4}]$

I $31\frac{1}{2}-26\frac{1}{2}=[5]$

J $3-2\frac{1}{4}=[\frac{3}{4}]$

K $40-29\frac{1}{2}=[10\frac{1}{2}]$

L $3\frac{1}{4}-\frac{3}{4}=[2\frac{1}{2}]$; $[2\frac{2}{4}]$

M $5\frac{1}{4}-4\frac{1}{2}=[\frac{3}{4}]$

N $37\frac{1}{2}-34\frac{3}{4}=[2\frac{3}{4}]$

O $\frac{3}{4}-\frac{1}{2}=[\frac{1}{4}]$

P $11-9\frac{3}{4}=[1\frac{1}{4}]$

Q $5\frac{3}{8}-2\frac{1}{2}=[2\frac{7}{8}]$

R $1\frac{9}{10}-1\frac{3}{10}=[\frac{3}{5}]$; $[\frac{6}{10}]$

S $2\frac{1}{4}-1\frac{7}{8}=[\frac{3}{8}]$

Keeping skillful:

(block 1)		(block 2)	(block 3)
A $12\frac{3}{8}$	H $\frac{3}{5}$	A $\frac{13}{16}$	A 389,478
B 568	I 3804	B $19\frac{11}{24}$	B 304
C $9\frac{11}{12}$	J $2\frac{7}{24}$	C $3\frac{5}{24}$	C 774
D $10\frac{1}{10}$	K 34	D 41	D 82,081
E 188	L 1787	E $18\frac{1}{2}$ or $18\frac{6}{12}$	E 61,446
F 1503	M 491	F $6\frac{1}{5}$ or $6\frac{2}{10}$	F 65
G $2\frac{5}{8}$	N $7\frac{3}{8}$ or $7\frac{6}{16}$	G $2\frac{1}{2}$ or $2\frac{6}{12}$	G 134,676
			H 820
			I 99,958

Using arithmetic

A Jane has $3\frac{1}{2}$ yd. of red cloth and $2\frac{7}{8}$ yd. of blue cloth. Jane's piece of red cloth is how much longer than her piece of blue cloth?

1

Number of yards of red cloth.

Number of yards of red cloth equal to the number of yards of blue cloth. Imagine these as gone.

$3\frac{1}{2} - 2\frac{7}{8} = n$

You must find this number.

Subtract $2\frac{7}{8}$ from $3\frac{1}{2}$.
Jane's piece of red cloth is ▩ yd. longer than her piece of blue cloth.

B The answer for Problem A is the difference in the lengths of the two pieces of cloth. The difference in the lengths is ▩ yd.

2

C Jack weighs $57\frac{1}{2}$ lb., and his little brother weighs $28\frac{1}{4}$ lb. Jack's brother weighs how much less than Jack?

Subtract $28\frac{1}{4}$ from $57\frac{1}{2}$.
Jack's brother weighs ▩ lb. less than Jack.

3

D The answer for Problem C is the difference in the weights of the two boys. The difference in their weights is ▩ lb.

4

E Bob is $52\frac{3}{4}$ in. tall, and his little brother is $30\frac{1}{4}$ in. tall. What is the difference in their heights?

5

THINK To find the difference in their heights, you find how much taller or how much shorter one boy is than the other.

F Mrs. Brown's family uses $6\frac{1}{2}$ gal. of milk each week. Mrs. Page's family uses 4 gal. of milk each week. In a week, Mrs. Page's family uses how much less milk than Mrs. Brown's family?

6

G Mrs. Diamond used $2\frac{1}{4}$ c. of flour for bread and $1\frac{2}{3}$ c. of flour for cookies. She used how much more flour for bread than she did for cookies?

H Jim is $50\frac{1}{2}$ in. tall, and his sister is $48\frac{1}{4}$ in. tall. Jim's sister is how much shorter than Jim?

I Jim's father measured two trees that he had cut down. One was $31\frac{1}{2}$ ft. high, and the other was $26\frac{1}{2}$ ft. high. Find the difference in the heights of the two trees.

J Susan goes to school 3 hr. in the morning and $2\frac{1}{4}$ hr. in the afternoon. She goes to school how much longer in the morning than in the afternoon?

K A grocer sold $29\frac{1}{2}$ lb. of butter on Wednesday and 40 lb. of butter on Thursday. He sold how much less butter on Wednesday than he did on Thursday?

1

L Sally's father worked in the yard for $\frac{3}{4}$ hr. on Friday and for $3\frac{1}{4}$ hr. on Saturday. He worked how many fewer hours on Friday than on Saturday?

M Mrs. Park used $4\frac{1}{2}$ lb. of sugar for canning apples and $5\frac{1}{4}$ lb. for canning peaches. She used how much more sugar for the peaches than she did for the apples?

N Sally measured two pieces of cloth. One was $34\frac{3}{8}$ in. long, and the other was $37\frac{1}{2}$ in. long. Find the difference in the lengths of the two pieces of cloth.

O Paul practiced his cornet for $\frac{1}{2}$ hr. on Friday and for $\frac{3}{4}$ hr. on Saturday. He practiced how much longer on Saturday than he did on Friday?

P Jim's room is 11 ft. wide, and Tony's room is $9\frac{3}{4}$ ft. wide. Find the difference in the widths of the two rooms.

Q Mrs. Banks bought a chicken that weighed $5\frac{3}{8}$ lb. and a whitefish that weighed $2\frac{1}{2}$ lb. The chicken weighed how much more than the fish?

R The school is $1\frac{9}{10}$ mi. from Bill's house, and the library is $1\frac{9}{10}$ mi. from his house. The library is how much farther away from Bill's house than the school is?

S Tom weighed the two largest stones in his collection. One weighed $2\frac{1}{4}$ lb., and the other weighed $1\frac{7}{8}$ lb. What was the difference in the weights of the two stones?

2

Keeping skillful

3

Find the difference.

A	$18\frac{7}{8}, 6\frac{1}{2}$	**H**	$9\frac{1}{5}, 8\frac{2}{5}$
B	987, 419	**I**	6301, 2497
C	$11\frac{3}{4}, 1\frac{5}{8}$	**J**	$6\frac{3}{8}, 4\frac{1}{12}$
D	25, $14\frac{9}{10}$	**K**	$72\frac{2}{6}, 38\frac{5}{6}$
E	1000, 812	**L**	2700, 913
F	9142, 7639	**M**	8468, 7977
G	$33\frac{1}{2}, 30\frac{7}{8}$	**N**	$21\frac{2}{16}, 14\frac{3}{16}$

4

Find the sum.

A	$\frac{1}{8}, \frac{1}{4}, \frac{7}{16}$
B	$6\frac{3}{8}, 3\frac{1}{8}, 9\frac{1}{2}$
C	$\frac{11}{12}, \frac{7}{8}, \frac{2}{3}, \frac{3}{4}$
D	$29\frac{1}{2}, 11\frac{1}{2}$
E	$7\frac{1}{4}, 9\frac{7}{12}, 1\frac{2}{3}$
F	$1\frac{9}{10}, 3\frac{2}{5}, \frac{4}{5}, \frac{1}{10}$
G	$\frac{3}{4}, \frac{5}{6}, \frac{7}{12}, \frac{1}{3}$

5

A	$834 \times 467 = n$
B	$n \times 27 = 8208$
C	$56 \times n = 43344$
D	$79 \times 1039 = n$
E	$98 \times 627 = n$
F	$49 \times n = 3185$
G	$86 \times 1566 = n$
H	$n \times 54 = 44280$
I	$106 \times 943 = n$

207

1. Make sure the pupils see that this is a comparative situation, and that they can find how much longer one piece is than the other by subtraction. Then discuss the equation. Ask a pupil to perform the subtraction at the board.
2. Discuss the word *difference*. Make sure the pupils understand that here it means finding how much longer one piece of cloth is than the other.
3. Remind the pupils that finding how much less involves the same type of computation as finding how much more. Have the equation and computation put on the board.
4. Again discuss the use of the word *difference*.
5. Point out that difference can be thought of as meaning either how much taller or how much shorter.
6. Assign Problems F to S (K to S are on the next page) as independent written work.

208

1. Have the children continue to solve the problems. As usual, they should write an equation for each problem and show the computation.
2. When the children have finished, provide answers and discuss those problems that caused difficulty.
3. Before assigning this block of work, tell the children that "finding the difference" can apply to any two numbers and that it means finding how much more or less one number is than another.
4. Tell the pupils to use the computational form when they do these exercises.
5. Have pupils multiply or divide as necessary.

209 Keeping skillful

Expanded Notes are not considered necessary for this lesson.

Objectives

The child practices using ratios to express rates and comparisons.

Vocabulary

New words page 209 decorations

Comments

The purpose of this lesson is to give the children experience that will help them reinforce their understanding of the use of ratios.

When ratios are used in the text of this teaching guide, they are written as 12/14, 4/5, etc., to distinguish them from fraction numerals. Do not, however, write them in this way for the children.

In discussing the answers to Problems A to O, be sure the pupils are aware that each rate or comparison situation can be expressed by many different ratios. Thus either of two numerals can be considered as the first term, and reductions can be made both upward and downward. For example, for Problem A, the ratio can be 18/24, which can be reduced to 3/4, 9/12, 36/48, 54/72, etc. Or the children can think of comparing 24 with 18, in which case the ratio becomes 24/18. This can be reduced to 4/3, 12/9, 48/36, 72/54, etc. Since there are so many possible answers for these problems, no answers are given in the answer key.

Answers

(block 2)

Only the numerals that replace the screen are given.

A 2	E 18	I 9	M 30	Q 48	U 36
B 20	F 10	J 5	N 40	R 6	V 3
C 3	G 30	K 22	O 17	S 8	W 4
D 6	H 4	L 3	P 36	T 18	X 30

Keeping skillful

For each exercise below, write a ratio to express a rate or a comparison. When you have done this, write another ratio to express the same rate or comparison. Your work for Exercise A might look like this.

A $\frac{18}{24}=\frac{3}{4}$ or A $\frac{24}{18}=\frac{4}{3}$

1 A Nancy made 18 decorations. Carol made 24 decorations.

B Bill walked 4 mi. in 2 hr.

2 C Mary made 14 decorations in 40 minutes.

D Mrs. Price used 2 c. of sugar for 4 c. of apple juice.

E Mr. Price drove 260 mi. in 6 hr.

F Bob had 10 white mice. Tony had 5 white mice.

G Jim had 27 model cowboys. Bill had 36 model cowboys.

H Tony has read 54 pages of a book about stars. Don has read 100 pages of the same book.

I Mrs. West paid 39¢ for 1 doz. oranges.

J Mrs. West used 2 t. of baking powder for 4 c. of flour.

K Jane bought 4 oz. of candy for 21¢.

L Bob bought 25 sheets of notebook paper. Don bought 50 sheets of the same kind of paper.

M Jane bought 1 doz. flowers for 90¢.

N Tom earned $1.20. Jack earned 75¢.

O Ann sold 42 birthday cards. Kathy sold 20 birthday cards.

Find the missing numeral in each example below.

3

A $\frac{18}{6}=\frac{6}{\square}$ I $\frac{3}{4}=\frac{\square}{12}$ Q $\frac{7}{8}=\frac{42}{\square}$

B $\frac{2}{3}=\frac{\square}{30}$ J $\frac{24}{60}=\frac{2}{\square}$ R $\frac{18}{21}=\frac{\square}{7}$

C $\frac{15}{25}=\frac{\square}{5}$ K $\frac{11}{6}=\frac{\square}{12}$ S $\frac{48}{32}=\frac{12}{\square}$

D $\frac{4}{3}=\frac{8}{\square}$ L $\frac{27}{18}=\frac{\square}{2}$ T $\frac{3}{5}=\frac{\square}{30}$

E $\frac{9}{10}=\frac{\square}{20}$ M $\frac{7}{15}=\frac{14}{\square}$ U $\frac{5}{9}=\frac{20}{\square}$

F $\frac{14}{5}=\frac{28}{\square}$ N $\frac{10}{9}=\frac{\square}{36}$ V $\frac{44}{12}=\frac{11}{\square}$

G $\frac{5}{6}=\frac{\square}{36}$ O $\frac{51}{24}=\frac{\square}{8}$ W $\frac{16}{36}=\frac{\square}{9}$

H $\frac{35}{28}=\frac{5}{\square}$ P $\frac{20}{72}=\frac{10}{\square}$ X $\frac{24}{10}=\frac{72}{\square}$

209

1 Discuss Example A. Have the pupils observe that either numeral can be used as the first term of the ratio. Ask the pupils to give other ratios for Example A.

2 After the pupils have finished writing the answers to this set, discuss each problem. Point out that reductions could be made both upward and downward. Have the pupils tell whether each problem describes a rate situation or a comparison situation.

3 Assign this set of exercises as written work.

210-212 Moving forward

Exponded Notes are not considered necessary for this lesson.

Objectives

The child learns how to use a remainder in division to express the answer as a mixed number.

Vocabulary

There are no new words.

Comments

With traditional methods of teaching division, as soon as the children have had an introduction to fractions, they are taught to use the remainder with the divisor to form a fraction. This fraction together with the whole number is used to express the answer as a mixed number. The whole procedure is carried out more or less mechanically without any real attention to what the answer means. This lesson gives the children some idea of the significance of writing a fraction numeral in the answer when there is a remainder in a division problem.

The work on these pages shows that the meaning of the fraction depends upon whether the problem involves partitive or quotitive division. In partitive division, the fraction represents a part of one of the objects that are being put into equal groups. Thus, in the example on page 210, the fraction numeral $\frac{3}{4}$ represents $\frac{3}{4}$ of an apple.

In quotitive division, on the other hand, the fraction represents a part of one of the *groups* into which the objects are divided. Thus, in the example on page 211, the $\frac{1}{2}$ represents $\frac{1}{2}$ of a group (dozen). In the quotitive division case, therefore, the fraction, although it represents a part of one group, can also represent a number of whole objects—in this case, 6 eggs.

A more detailed explanation of the significance and use of the fraction in the answer will be given in Grade 6. Expressing the answer, when appropriate, as a mixed number has been introduced at this point because it is traditional in the fifth grade.

If the pupils have trouble understanding the example on page 210, it might be helpful to dramatize the situation with objects. In dividing the 3 remaining apples among the boys, you should make clear that each time one of the apples is cut into fourths, each boy will get one of the equal pieces until all 3 apples are used up.

The children must understand that they should never show both a remainder and a fraction in the answer. In the text, the numeral that expresses the remainder is crossed out when a fraction numeral is used to express the answer as a mixed number.

Answers

Page 211:

C $6\frac{1}{2}$	F $9\frac{2}{3}$	I $68\frac{1}{2}$	L $473\frac{2}{3}$
D $9\frac{1}{4}$	G $5\frac{1}{2}$	J $81\frac{1}{2}$	M $351\frac{5}{7}$
E $8\frac{2}{9}$	H $54\frac{3}{5}$	K $77\frac{3}{8}$	N $89\frac{1}{3}$

Page 212:

A $2\frac{3}{4}$	E $5\frac{2}{3}$	I $17\frac{1}{2}$	M $6\frac{3}{4}$
B $7\frac{1}{4}$	F $7\frac{3}{5}$	J $24\frac{1}{2}$	N $39\frac{2}{3}$
C $12\frac{1}{2}$	G $17\frac{3}{4}$	K $2\frac{17}{35}$	
D $11\frac{1}{2}$	H $9\frac{1}{2}$	L $4\frac{3}{4}$	

Keeping skillful:

(block 1)

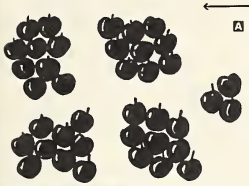
A $6\frac{1}{6}$ ft.	F $2\frac{2}{3}$ doz.	K $16\frac{1}{3}$ yd.	P $2\frac{1}{12}$ yd.
B $11\frac{1}{2}$ qt.	G $3\frac{1}{4}$ yr.	L 5 lb.	Q $2\frac{1}{6}$ da.
C $3\frac{4}{7}$ wk.	H $11\frac{3}{4}$ bu.	M $1\frac{1}{2}$ yr.	R $6\frac{1}{3}$ yd.
D 23 pt.	I $2\frac{1}{8}$ gal.	N $2\frac{5}{8}$ gal.	S $6\frac{3}{4}$ gal.
E $3\frac{1}{4}$ lb.	J $3\frac{1}{2}$ c.	O 4 min.	T $1\frac{1}{2}$ lb.

(block 2)

A $12\frac{3}{8}$	F $1\frac{3}{4}$ or $1\frac{9}{12}$	
B 2	G 3	
C $22\frac{1}{6}$	H $9\frac{5}{8}$	
D $5\frac{17}{24}$	I $15\frac{2}{3}$ or $15\frac{8}{12}$	
E $9\frac{5}{6}$ or $9\frac{10}{12}$	J $17\frac{1}{4}$ or $17\frac{3}{12}$	

Moving forward

Often there is a remainder in a division example. Now you will learn how to write a fraction numeral in the answer when there is a remainder.



A

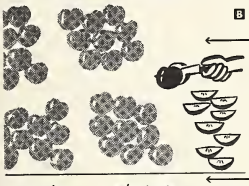
Four boys decided to divide 39 apples equally among them. How many apples would each boy get? 1

$39 \div n = 4$ Divide 39 by 4.

$4\overline{)39}\ \ 9$ ← Each boy would get 9 whole apples.

$\underline{36}$ 2

3 ← 3 apples would be left over.

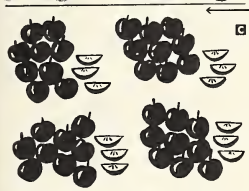


B

To divide the 3 left-over apples, the boys would have to cut each apple into 4 equal pieces, or fourths. 3

Each boy would get 3 of the equal pieces.

Each boy would get $\frac{3}{4}$ of an apple.



C

4

$4\overline{)39}\ \ 9\frac{3}{4}$

$\underline{36}$

6 3

Instead of leaving 3 as a remainder, write a fraction numeral in the answer. The numerator will be the remainder, 3. The denominator will be the divisor, 4. 5

Each boy would get $9\frac{3}{4}$ apples.

A Patsy and Sue made 27 cupcakes and shared them equally. How many cupcakes did each girl get? 1

$27 \div n = 2$ Divide 27 by 2.

$27 \div 2 =$ ▨ and ▨ remainder

You may say that each girl got $13\frac{1}{2}$ cupcakes. How do you get the $\frac{1}{2}$?

B Kathy weighs 74 lb., Sue weighs 68 lb., Carol weighs 75 lb., and Ruth weighs 69 lb. What is the average weight of the 4 girls? 2

▨ $\div n = 4$ Divide 286 by 4.

Why do you use 4 in the equation?

The average weight of the girls is $71\frac{\square}{4}$ lb., or $71\frac{\square}{2}$ lb.

Use each remainder to write a fraction numeral in simplest form in the answer. 3

C $52 \div n = 8$	F $58 \div n = 6$	I $137 \div n = 2$	L $1421 \div n = 3$
D $37 \div n = 4$	G $22 \div n = 4$	J $326 \div n = 4$	M $2462 \div n = 7$
E $74 \div n = 9$	H $273 \div n = 5$	K $619 \div n = 8$	N $804 \div n = 9$

A

Ruth gathered 30 eggs. She gathered how many dozen eggs? 4

$30 \div 12 = n$

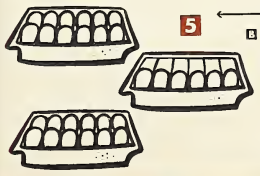
$12\overline{)30}\ \ 2$ ← There are 2 dozen eggs.

$\underline{24}$

6 ← 6 eggs are left over.

The 6 left-over eggs are part of a dozen.

They are $\frac{6}{12}$, or $\frac{1}{2}$, of a dozen.



B

5

$12\overline{)30}\ \ 2\frac{6}{12}$

$\underline{24}$

ϕ 6

Use the remainder to write a fraction numeral in the answer. Write ▨ for the numerator. Write ▨ for the denominator.

Ruth gathered $2\frac{6}{12}$ doz., or $2\frac{1}{2}$ doz., eggs.

210

1 Ask the pupils to read the problem. They should see that this is a sharing type of problem.

2 Discuss the computation and relate it to the picture.

3 Point out that the 3 apples must be shared equally by the 4 boys. Ask for suggestions as to how the 4 boys can share the 3 apples. Then have someone read the text and explain what is happening in the picture.

4 Have someone explain how the new answer, $9\frac{3}{4}$, is shown in the picture and have him compare it with the first answer, 9 and 3 remainder, and Picture A.

5 Have the pupils read the text opposite the quotient. Be sure they understand what to use for the numerator and for the denominator.

6 Have pupils note that the remainder 3 is crossed out when $\frac{3}{4}$ is written in the answer.

211

1 Read the problem together and discuss the equation. Then have a pupil divide 27 by 2 at the board. Ask how the 1 remaining cupcake can be divided so each of the 2 girls gets an equal part. Remind pupils that the $\frac{1}{2}$ should be written with the answer and the remainder 1 should be crossed out.

2 Adapt Note 1 to this problem.

3 Assign these exercises as written work.

4 Have the class read the new problem and note that they are to find the number of dozens, not the size of the groups. Discuss the equation and division.

5 Note that the 6 leftover eggs fill only part of a carton holding 12 eggs. The fraction represented is $\frac{6}{12}$. Discuss where $\frac{6}{12}$ is written in the computation. Point out that the remainder 6 is crossed out.

A Mrs. Long said, "We have used 11 quarts of milk this week." This was **1** equal to how many gallons?
$11 \div 4 = n$
$11 \div 4 =$ ▩ and ▩ remainder

Why do you divide by 4? What is the remainder? You may write $\frac{3}{4}$ in the answer. Does the $\frac{3}{4}$ mean quarts or gallons? 11 qt. $= 2\frac{3}{4}$ gal.

B Jane measured a piece of crepe paper and found that it was 87 in. long. She said, "I have $7\frac{1}{4}$ ft. of crepe paper." Was Jane correct? **2**
$87 \div 12 = n$
$87 \div 12 =$ ▩ and ▩ remainder

Why do you divide 87 by 12? The remainder is ▩ in. 3 in. are $\frac{3}{12}$, or $\frac{1}{4}$, of a foot. 87 in. $= 7\frac{3}{4}$ ft.

Use each remainder to write a fraction numeral in simplest form in the answer.

3

C $25 \div 2 = n$	**F** $38 \div 5 = n$	**I** $105 \div 6 = n$	**L** $76 \div 16 = n$
D $46 \div 4 = n$	**G** $71 \div 4 = n$	**J** $294 \div 12 = n$	**M** $54 \div 8 = n$
E $17 \div 3 = n$	**H** $19 \div 2 = n$	**K** $87 \div 35 = n$	**N** $119 \div 3 = n$

Now you should be able to write a fraction numeral in the answer to a division example when there is a remainder.

Keeping skillful

4

If there is a remainder, remember to write a fraction numeral in simplest form in the answer.

4

A 74 in. = ▩ ft.
B 23 pt. = ▩ qt.
C 25 da. = ▩ wk.
D 46 c. = ▩ pt.
E 52 oz. = ▩ lb.
F 32 things = ▩ doz.
G 39 mo. = ▩ yr.
H 47 pk. = ▩ bu.
I 17 pt. = ▩ gal.
J 28 fl. oz. = ▩ c.
K 49 ft. = ▩ yd.
L 80 oz. = ▩ lb.
M 18 mo. = ▩ yr.
N 21 pt. = ▩ gal.
O 240 sec. = ▩ min.
P 75 in. = ▩ yd.
Q 52 hr. = ▩ da.
R 19 ft. = ▩ yd.
S 27 qt. = ▩ gal.
T 24 oz. = ▩ lb.

Find the sum.

A $6\frac{1}{2}, 1\frac{1}{4}, 3\frac{1}{8}, 2\frac{1}{2}$
B $\frac{2}{3}, \frac{3}{5}, \frac{2}{5}, \frac{1}{3}$
C $10\frac{8}{6}, 11\frac{1}{3}$
D $\frac{5}{8}, 3\frac{1}{4}, 1\frac{5}{6}$
E $4\frac{11}{12}, 3\frac{2}{3}, 1\frac{1}{4}$
F $\frac{3}{4}, \frac{1}{3}, \frac{2}{3}$
G $1\frac{7}{10}, \frac{3}{5}, \frac{3}{10}, \frac{2}{5}$
H $2\frac{1}{2}, \frac{1}{4}, 6\frac{7}{8}$
I $5\frac{1}{6}, 2\frac{11}{12}, 7\frac{7}{12}$
J $6\frac{3}{4}, 1\frac{5}{6}, 8\frac{2}{3}$

212

212

1 Have the pupils read the problem and discuss the equation. Be sure they understand that the 3 remaining quarts represent $\frac{3}{4}$ of a gallon.

2 Ask the pupils to read the problem and discuss the equation. Then have them answer the questions in the text. Have someone compute at the board; make sure that he puts the numerals in the correct places. At this point you might have the pupils generalize that the remainder always tells what the numerator of the fraction numeral should be, and the divisor tells what the denominator should be.

3 Assign Exercises C to N as written work. Let pupils each put a different exercise on the board, and let the rest of the class discuss their work

4 Assign these exercises as written work.

213-214 Exploring problems

Expanded Notes are not considered necessary for this lesson.

Objectives

The child learns to make judgments in handling remainders in division problems.

Vocabulary

New words page 213 sensible; page 214 gasoline

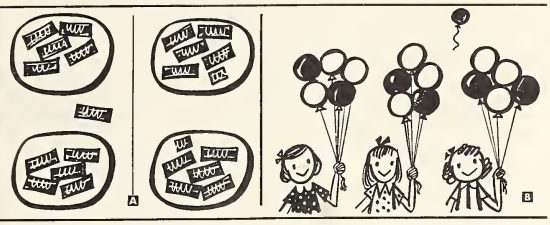
Comments

A previous lesson on pages 105-106 directed attention to the development of judgment about the use of remainders in division. Refinement of this judgment is possible now that the child knows how to express the answer as a mixed number. See Charting the Course, page 335.

Answers

A $5\frac{1}{2}$; 1; It was divided into 2 equal pieces. Yes; yes
B 5; It is better to have a remainder because it is not possible to have a fraction of a balloon.
C 4 packages
D 4 children in each of 5 cars and 5 in 1 car
E $(20) \div 12 = n$; $1\frac{2}{3}$ doz.
F $94¢ \div n = 3$; $32¢$; The usual practice is to increase the amount to the next higher cent.
G $49 \div n = 4$; $12\frac{1}{4}$ in.; because it is possible to measure and cut fractions of an inch
H $14 \div n = 6$; $2\frac{1}{3}$; yes; because a sandwich can be cut into thirds
I $(24) \div 16 = n$; $1\frac{1}{2}$ lb.
J $\$1.27 \div n = 8$; $15\frac{7}{8}¢$, about $16¢$
K $(165) \div n = 6$; $27\frac{1}{2}$ min., or about 27 min.; No, an average often does not need to be exact.
L $15 \div n = 4$; $3\frac{3}{4}$ doz.
M $75 \div 16 = n$; 4, 11 rm.; 5 boxes, since the 11 remaining stones will partly fill another box
N $\$2.00 \div \$.32 = n$; $6\frac{1}{4}$ gal.; because it is possible to measure and buy a fraction of a gallon
O $69¢ \div n = 10$; $6\frac{9}{10}¢$, about $7¢$
P $14 \div 6 = n$; $2\frac{1}{3}$; 3, because to get the 2 extra buttons she needs, she must buy another card

Exploring problems



A Bill and David shared 11 candy bars equally. How many candy bars did each boy get?
$11 \div n = 2$

What is the remainder when you divide? Look at Picture A. The candy bars have been put into two equal groups. What happened to the 1 left-over bar? Is it possible to have $\frac{1}{2}$ candy bar? Can you say each boy got $5\frac{1}{2}$ bars?

B Three girls had 16 balloons to share equally. How many balloons should each girl have taken?
$16 \div n = 3$

Look at Picture B. Each girl could have ▩ balloons.
Is it more sensible to use a fraction in the answer or to have a remainder? Why?

C Mrs. Block wants to buy 30 buns. The buns are sold in packages of 8. How many packages should she buy?
$30 \div 8 = n$

THINK Which answer is most sensible, 3 packages, $3\frac{3}{4}$ packages, or 4 packages?

D Twenty-five children are going to the zoo in 6 cars. If they try to divide up into equal groups, how many children should ride in each car?
$25 \div n = 6$

THINK Is it sensible to have a fraction in the answer to Problem D? What should be done with the 1 child who is not in any of the equal groups?

E Ann bought 2 packages of cookies. There were 10 cookies in a package. She bought how many dozen cookies?

Developing judgment about the remainder in division: when to express the answer using a fraction; when to round off the answer to a whole number **213**

F Ellen bought 1 can of peaches at the rate of 3 cans for 94¢. Do you think she had to pay 31¢, $31\frac{1}{3}$¢, or 32¢ for the 1 can? Why do you think so?

THINK A fraction of a cent is usually counted as a whole cent.

G Jane has a ribbon 49 in. long. She wants to cut it into 4 equal pieces. How long should she make each piece?

THINK Why is it sensible to use a fraction in the answer?

H Six girls took 14 sandwiches on a picnic and shared them equally. How many sandwiches did each girl get?

THINK In Problem H, is it sensible to have a fraction in the answer? Why?

I Kathy bought 3 cans of salted nuts. Each can held 8 oz. of nuts. How many pounds of nuts did she buy?

THINK How many ounces are in 1 lb.?

J Don saved $1.27 in 8 weeks. He saved an average of how much per week?

THINK Is $15\frac{7}{8}$¢ about 15¢ or about 16¢? Which answer is more sensible for Problem J, about 15¢ or about 16¢?

K Bill practiced his cornet 30 min. on Monday, 20 min. on Tuesday, 35 min. on Wednesday, 15 min. on Thursday, 40 min. on Friday, and 25 min. on Saturday. He practiced an average of how many minutes per day for the 6 days?

THINK Is the remainder important in Problem K? Why or why not?

L Mrs. Turner bought 15 doz. eggs. She shared them equally with 3 other women. How many dozen eggs did each of the 4 women get?

M Paul is putting his rock collection in boxes. He has 75 rocks, and he wants to put 16 rocks in each box. How many boxes will he need?

N Mr. Turner spent $2 for gasoline for his car. The gasoline cost 32¢ per gallon. How many gallons of gasoline did he buy?

THINK Why should you have a fraction in the answer to Problem N?

O Mrs. Turner bought a 10-lb. bag of potatoes for 69¢. The potatoes cost about how much per pound?

P Mrs. Long needs 14 buttons to trim Patsy's dress. The buttons are sold on cards of 6. How many cards of buttons should she buy?

Sometimes you use the remainder in a division problem to express the answer with a fraction. Sometimes you round off the answer to a whole number.

213

1. Read Problem A and discuss Picture A. Get the children to decide that since a candy bar can be divided into equal parts, $5\frac{1}{2}$ bars is a sensible answer. Then have the balance of Problem A read and the questions answered.
2. Proceed in a similar way with Problem B and Picture B. Now the pupils should decide that it is not sensible to use a fraction in the answer, since a balloon is destroyed if cut into parts.
3. The pupils should see that this answer must be rounded up to 4 because Mrs. Black needs 30 buns.
4. Here again emphasize the common-sense approach. The remainder (1 child) cannot be disregarded, and the answer should not be expressed as a mixed number.
5. The pupils should see that a mixed-number answer is acceptable here.

214

1. The children should work these problems independently. Remind them to pay special attention to the suggestions labeled "Think."
2. You may wish to discuss the problem solutions with the class. Be sure the pupils can justify the various ways they have expressed their answers.

215-220 Exploring problems

Expanded Notes for this lesson are on pages 337-340.

Objectives

The child learns to apply his knowledge of rate to problem solving.

Vocabulary

New words page 219 music*; page 220 custard, hall*, received*

Comments

All problems about buying items at a stated price, traveling at so many miles per hour, etc., are based on the concept of rate. Traditionally, at this grade level, the child has been taught to handle only the special case in which the rate is given for one item (10¢ for 1 candy bar, 50 miles per hour, etc.) and the problem is to find the total for several of these items. In solving problems of this kind, the children have been taught to use multiplication (in the examples above, $6\times10¢$ for 6 candy bars, 3×50 miles for 3 hours). For the more general case in which the rate may be given in such forms as 10¢ for 3 bars or 150 miles in 4 hours, the child has had no definite procedure to follow. Such general problems have usually been left for the sixth grade, at which time the child was taught to attack them as multiple-step problems, but was given very little help in solving them.

This lesson gives the child a general mathematical method which can be used to solve rate problems of both the general and the specialized types described above. In all problems of this nature, two ratios (both expressing the same rate) are involved. Of the four terms in the two ratios, three terms are known and one term is unknown. To solve the problem, it is necessary to make two equal ratios and then compute to find the value of the unknown term. Suppose you want to buy 9 cans of soup, and the soup is priced at 3 cans for 43¢. How much money will you need for 9 cans? Two expressions of rate are evident in this problem: 3 cans per 43¢ and 9 cans per *n*¢. Three terms are known: 3, 43, 9. The unknown term *(n)* represents the cost of 9 cans. The rate at which soup is sold can be expressed by the ratio $3/43$. It can also be expressed by the ratio $9/n$. The equation $3/43=9/n$ then gives a convenient way to find the numeral to replace *n*. Since the children have already had experience in writing ratios to express rates and have learned how to find a numeral to complete one of a pair of equal ratios (see pages 140-142 of the pupils' book), all that remains for them to learn here is how to make the proper equation.

Stress the idea that it is not important which numeral becomes the first term in the first ratio. But once the child has selected the numeral that is to be the first term of that ratio, he must be consistent when he selects the first term in the equal ratio. The first terms of the two ratios must represent the same thing —dollars, miles, items, etc. In the above example, if he chooses to write the price for 3 cans of soup as the first term of the first ratio, he must write *n* to hold a place for the price of 9 cans of soup as the first term of the equal ratio. Thus, instead of the equation above, his equation would look like this: $43/3=n/9$. Point out to the pupils that all of the illustrated examples in the text could have been written in another order; the equation for the problem on pages 215-216 could have been $3/10=6/n$; the equation for the problem on pages 216-217 could have been $8/60=2/n$, etc.

Answers

Pages 219-220:

A $65/2=[325]/10$

B $27/18=3/[2]$

C $25/10=75/[30]$

D $140/8=[35]/2$; \$.35

E $4/30=24/[180]$ or $30/4=[180]/24$

F $12/300=4/[100]$ or $300/12=[100]/4$; \$1.00

G $5/56=[25]/280$ or $56/5=280/[25]$

H $6/4=[3]/2$ or $4/6=2/[3]$

I $9/2=[27]/6$ or $2/9=6/[27]$

J $30/100=1260/[4200]$ or $100/30=[4200]/1260$

K $78/12=26/[4]$ or $12/78=[4]/26$

L $3/23=12/[92]$ or $23/3=[92]/12$; \$.92

M $2/25=20/[250]$ or $25/2=[250]/20$; \$2.50

N $146/4=73/[2]$ or $4/146=[2]/73$

O $100/50=[20]/10$ or $50/100=10/[20]$

P $2/36=(6)/[108]$ or $36/2=[108]/(6)$; 1 hr. 48 min.

Exploring problems

see

A

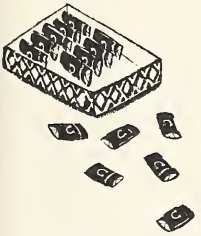
Candy bars are priced at 10¢ for 3 bars. Kathy wants to buy 6 bars. How much will she have to pay for 6 bars? **1**

$\frac{10}{3}$ **2** — This ratio shows that the bars are sold at the rate of 10¢ per 3 bars.

B

Kathy wants to buy 6 bars. You do not know how much she will have to pay for them.

3 Kathy will pay for 6 bars at the same rate as for 3 bars, but now the rate will be expressed by two different numerals.



4 $\frac{n}{6}$ — n: Number of cents Kathy will pay for 6 bars; 6: Number of bars Kathy wants to buy

5 $\frac{10}{3}=\frac{n}{6}$ — You must find the numeral that replaces n. The ratio n per 6 expresses the same rate as 10 per 3.

Now turn the page.

compute

C

It takes 2 groups of 3 bars to make the group of 6 bars that Kathy wants to buy.

$\frac{10}{3}=\frac{n}{6}$ **1** — You can multiply 3 by 2 to get 6.

D

It takes 10¢ to pay for each group of 3 bars. It will take 2 groups of 10¢ to pay for 2 groups of 3 bars. It will take 20¢.

2

$\frac{10}{3}=\frac{n}{6}$ **3** — 6 can be found by multiplying 3 by 2. So the numeral that replaces n can be found by multiplying 10 by 2. $n=20$

Kathy will have to pay 20¢ for the 6 bars.

see

A

Mrs. Cook bought 8 bars of soap for 60¢. At this rate, how much would 2 bars cost?

4 $\frac{60}{8}$ — This ratio shows that Mrs. Cook bought soap at the rate of 60¢ for 8 bars.

215

1 Help the pupils see that a rate is stated in this problem, and that the problem is to find another way to express this same rate, using 6 as part of the new ratio.

2 Discuss the written arrangement of the rate as a ratio. Make sure the pupils understand what the numerals in the ratio represent.

3 Point out that they already know one of the numerals for the new ratio [6], but that they must find the other numeral, which will represent the cost of 6 bars of candy.

4 Discuss this ratio. Point out that the unknown price is written above the line, as was the price in the original ratio, and that the numeral for the number of bars is placed below, as it was in the original ratio.

5 Point out that the numeral that replaces n must make the equation correct; that is, the second ratio must equal the first.

216

1 Remind the pupils that they have already studied how to change a ratio to an equivalent ratio by multiplying or dividing both terms by the same number. Relate the text to Picture C. Point out that since they can multiply 3 by 2 to get 6, they can also multiply 10 by 2.

2 Have the pupils note in Picture D that for each group of 3 bars there is a group of 10¢. They can see that 2 groups of 10¢ make 20¢.

3 After someone has read the text to the right of the equation, write the completed equation, $10/3=20/6$, on the board. Be sure the pupils understand that the equation is correct because both 10 and 3 can be multiplied by the same number, 2, to get the 20 and 6. Discuss the fact that 20 means 20¢. Have the pupils read 20/6 as a rate [20 per 6].

4 Have pupils relate the picture to the problem and to the ratio that shows the rate involved.

B

You do not know how much 2 bars would cost.

1

$\frac{n}{2}$ ← Number of cents 2 bars will cost; Number of bars

The rate for 2 bars is the same as the rate for 8 bars, but the rate has now been expressed by different numerals.

2

$\frac{60}{8}=\frac{n}{2}$ ← You must find the numeral that replaces n.

compute

C

You can think of the 8 bars as divided into groups of 2 bars. There are 4 groups.

3

$\frac{60}{8}=\frac{n}{2}$

You can divide 8 by 4 to get 2.

D

It takes 60¢ to pay for 4 groups of 2 bars. So divide 60¢ into 4 equal groups to find how much each group of 2 bars cost.

4

$\frac{60}{8}=\frac{n}{2}$

2 can be found by dividing 8 by 4. So the numeral that replaces n can be found by dividing 60 by 4. $n=15$

5

2 bars would cost 15¢.

see

A

Mrs. Parks paid 57¢ for some grapefruit. They were priced at 19¢ for 2. How many grapefruit did she buy?

1

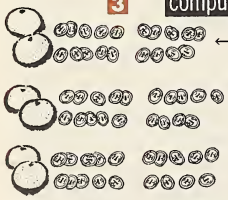
What do 19 and 2 stand for?

2

What do 57 and n stand for?

$\frac{19}{2}=\frac{57}{n}$ ← You must find the numeral that replaces n.

3 compute



B

How many groups of 19¢ make the 57¢ that Mrs. Parks spent for grapefruit? How many groups of 2 grapefruit did she buy?

$\frac{19}{2}=\frac{57}{n}$

You can multiply 19 by ■ to get 57. So you can multiply 2 by ■ to get the numeral that replaces n.

4

Mrs. Parks bought ■ grapefruit.

see

A

Mrs. Parks also bought 6 cans of baby food for 76¢. The baby food was selling at the rate of how many cans for 38¢?

5

What does each numeral in this ratio stand for?

What do 38 and n in this ratio stand for?

$\frac{76}{6}=\frac{38}{n}$ ← You must find the numeral that replaces n.

217

1 Remind the pupils that in the ratio 60/8 the cost was placed above the line. Therefore, the cost [n] should be placed above the line here, and the 2 that stands for the number of bars should be written below.

2 Discuss the fact that the two ratios are equal and express the same rate.

3 Point out that the pupils know the cost of 8 bars and want to find the cost of 2 bars, so it is helpful to think of 8 bars as 4 groups of 2 bars. Pupils must see that since 8 can be divided by 4 to get 2, 60 should also be divided by 4.

4 Point out in Picture D that since the 8 bars were divided into 4 groups of 2, the 60¢ should be divided into 4 equal groups.

5 Have someone read the text beside the equation. Then write the completed equation, 60/8=15/2, on the board. Ask: "How do we know that 15 is the number we want? Why divide both 60 and 8 by 4, and not by 2?"

218

1 Have the class read the problem and study Picture A.

2 Discuss the equation and have the pupils take turns answering the questions about the equation. Then ask: "Why is 57 written above the line in the second ratio? Why is n written below?"

3 The pupils should see in Picture B that for every group of 19¢ there are 2 grapefruit. They can see that there are 3 groups of 19¢ and 3 groups of 2 grapefruit.

4 Have a pupil read the text beside the equation, filling in the answers as he reads. Then put the completed equation on the board. Ask: "How do we know that 6 is the number we want? Why do we multiply both 19 and 2 by 3, instead of by some other number?"

5 Have pupils read the problem and answer the questions about the equation. Then ask: "Why is 38 written above the line? Why is n written below?"

compute

B

Think of the 76¢ as divided into groups of 38¢. There are ■ groups of 38¢. Then think of 6 cans as ■ groups of ■ cans. 1

$$\frac{76}{6}=\frac{38}{n}$$ 2

You can divide 76 by ■ to get 38. So you can divide 6 by ■ to get the numeral that replaces n.

The baby food was selling at the rate of ■ cans for 38¢.

A Mr. Cook traveled 65 mi. in 2 hr. in his car. At this rate, how far could he travel in 10 hr. in his car? 3

$$\frac{65}{2}=\frac{n}{10}$$

65 and the numeral you will get for n tell how many miles.

2 means 2 hr.

10 means 10 hr.

B There were 27 pupils in the music class. The class had 18 music books. This was a rate of how many books for every 3 pupils?

$$\frac{27}{18}=\frac{3}{n}$$

What do 27 and 3 stand for?

What do 18 and n stand for?

C Paul paid 75¢ for some balloons. They were priced at 10 for 25¢. How many balloons did Paul buy?

$$\frac{25}{10}=\frac{75}{n}$$

What do 25 and 75 stand for?

What do 10 and n stand for?

D Mrs. White bought 8 cans of corn for $1.40. At the same rate, how much would 2 cans of this corn have cost? 4

$$\frac{140}{8}=\frac{n}{2}$$

What do 140 and n stand for?

What do 8 and 2 stand for?

E Sally is making 24 invitations to a party. She thinks that she can make 4 invitations in about 30 min. It will take her about how many minutes to make the 24 invitations? 1

$\frac{4}{30}=\frac{24}{n}$ or $\frac{30}{4}=\frac{n}{24}$

F David saved $3 in 12 weeks. He saved at the same rate during all this time. How much money did he save in 4 weeks? 2

$\frac{12}{300}=\frac{4}{n}$ or $\frac{300}{12}=\frac{n}{4}$

Why can $3 be thought of as 300?

G John has a new book that has 280 pages. He has read 56 pages in 5 days. At this rate, how many days will it take him to read the whole book?

$\frac{5}{56}=\frac{n}{280}$ or $\frac{56}{5}=\frac{280}{n}$

H When Mrs. White made a custard, she used 6 eggs for 4 cups of milk. How many eggs should she have used if she had used only 2 cups of milk?

$\frac{6}{4}=\frac{n}{2}$ or $\frac{4}{6}=\frac{2}{n}$

I On a bicycle trip, Bill and his friends rode 9 miles in 2 hours. At this rate, how many miles could they ride in 6 hours? 3

J The fifth graders collected paper and sold it for 30¢ per 100 pounds. They received $12.60 for all the paper that they sold. How many pounds of paper did they sell?

K Mrs. Brown paid 78¢ for 1 dozen cakes of soap. She bought the soap at the rate of how many cakes for 26¢? 4

L At another store Mrs. Brown saw the same kind of soap priced at 3 cakes for 23¢. How much would 12 cakes of soap cost at this store?

M Ruth bought 20 noisemakers for a party. They were priced at 2 for 25¢. How much did she pay for all of them?

N On a trip, Mr. Black drove 146 mi. in 4 hr. Then he still had 73 mi. to go. He traveled at the same rate for the next 73 mi. How long did it take him to travel the 73 mi.?

O Kathy used 100 ft. of crepe paper to decorate 50 ft. of the school hall. She still had 10 ft. of the hall to decorate. How many feet of crepe paper did she need for the 10 ft.?

P Paul and his friends went on an 8-mi. hike. They walked the first 2 mi. in 36 min. If they kept walking at this rate, how long did it take them to go the rest of the way? 5

Now you should know how to express rates by using ratios and how to use these ratios to solve problems. 6

219

1 Pupils should be able to fill in the numerals in place of the screens in the text by referring to the picture.

2 Have someone read the text at the right of the equation and fill in the numerals that replace the screens. Then ask: "How do we know that 3 is the number we want? Why do we choose to divide both 76 and 6 by 2, instead of by some other number?"

3 For the remaining problems on this page, discuss the equations for each. Be sure the pupils can explain what each numeral, including n, stands for in each equation. Be sure they can tell you why they can multiply or divide by a certain number to get the numeral that replaces n. Then have them find the answers and state them as rates [325 mi. per 10 hr.].

4 In Problem D, be sure the pupils understand that the $1.40 is expressed in the equation as 140 cents. The answer will represent cents.

220

1 In each of the first four problems, discuss why both of the given equations are correct. Then have the class find the answers.

2 In this problem, be sure the pupils understand that they can think of the $3.00 as 300 cents, so that the numeral they put in the first ratio is 300. Remind them that their answer will represent cents.

3 Assign Problems I to O as written work. Remind the pupils to think of money as cents in the ratio equations, then change the answers to dollars if necessary.

4 The pupils should think of 1 dozen cakes of soap as 12 cakes of soap.

5 Assign this problem to the able pupils.

6 Write the completed equations on the board and let the pupils verify their work. For each problem, discuss how the answer was found. Give individual help as necessary.

221-226 Exploring problems

Expanded Notes for this lesson are on pages 340-342.

Objectives

The child learns to solve problems involving comparisons by using ratios for making equations.

Vocabulary

New words page 225 while*, machine*; page 226 Steve, arrived*, band*

Comments

The children have just learned how to use ratios to make ratio equations for rate situations (see page 198 of these Lesson Briefs). They should have no trouble working problems involving comparisons. Both types of problems are solved in the same way. The result of a comparison is expressed by a ratio, and an equivalent ratio is to be made, one term of which is known and the other is to be found. As you did for rate problems, stress the fact that it is not important which numeral is the first term of the first ratio, but that the first term in the second ratio must represent the same kind of things. For example, if the first term of one ratio refers to Bob's money, the first term of the equal ratio must also refer to Bob's money. The pupils should understand that equations for the problems in the text could have been written in different ways; thus, the equation for the problem on pages 221-222 could have been $15/25=n/125$.

Answers

Pages 225-226:

A $5/6=30/[36]$ C $50/120=10/[24]$

B $2/3=[6]/9$ D $10/12=[5]/6$

E $4/6=56/[84]$ or $6/4=[84]/56$

F $16/12=[4]/3$ or $12/16=3/[4]$

G $30/35=[180]/210$ or $35/30=210/[180]$

H $500/350=[50]/35$ or $350/500=35/[50]$

I $18/12=90/[60]$ or $12/18=[60]/90$

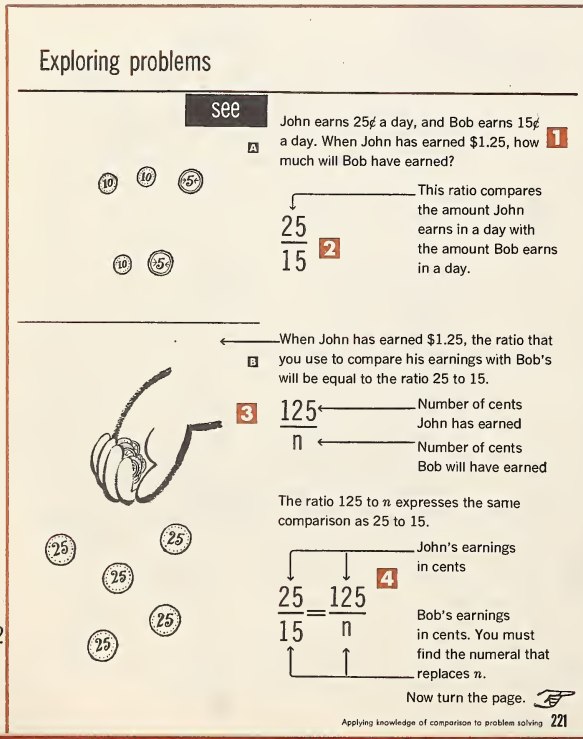
J $60/80=15/[20]$ or $80/60=[20]/15$; 20¢

K $3/4=[9]/12$ or $4/3=12/[9]$

L $160/120=40/[30]$ or $120/160=[30]/40$

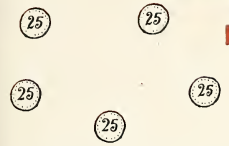
M $150/80=15/[8]$ or $80/150=[8]/15$

N $39/26=[3]/2$ or $26/39=2/[3]$


Exploring problems
see
A
John earns 25¢ a day, and Bob earns 15¢ a day. When John has earned $1.25, how much will Bob have earned?
1
This ratio compares the amount John earns in a day with the amount Bob earns in a day.
25/15
2
B
When John has earned $1.25, the ratio that you use to compare his earnings with Bob's will be equal to the ratio 25 to 15.
3
125/n
Number of cents John has earned
Number of cents Bob will have earned
The ratio 125 to n expresses the same comparison as 25 to 15.
John's earnings in cents
25/15 = 125/n
4
Bob's earnings in cents. You must find the numeral that replaces n.
Now turn the page.
Applying knowledge of comparison to problem solving
221
202

221

1. Ask the pupils to read the problem and indicate which group of coins represents John's earnings and which represents Bob's. Help them to state the comparison made in this problem and to see that they are to find another way to express it.
2. Discuss the ratio that expresses the comparison. Point out that the numeral for John's money is above the line and that the numeral for Bob's is below the line.
3. Be sure that the pupils understand that we are now going to express the comparison with different numerals, one of which we know. Ask: "The numeral for whose money is above the line? *n* holds a place for what numeral?" Relate the ratio to Picture B.
4. Help the pupils see that these two ratios are equal because any comparison between John's money and Bob's money is always to be the same.



compute

C

1

John has to earn 5 groups of 25¢ before he has earned a total of $1.25. Think of $1.25 as 125¢.

$$\frac{25}{15}=\frac{125}{n}$$ 2

You can multiply 25 by 5 to get 125.

D

Every time John earns 25¢, Bob earns 15¢. So when John has earned 5 groups of 25¢, or $1.25, Bob will have earned 5 groups of 15¢. He will have earned 75¢.

3

$$\frac{25}{15}=\frac{125}{n}$$ 4

You can multiply 25 by 5 to get 125. So you can multiply 15 by 5 to get the numeral that replaces n.

$n = 75$

Bob will have earned 75¢ when John has earned $1.25.

see

A

5

One day Tony picked 12 qt. of berries. In the same time, Bill picked 8 qt. How many quarts did Tony pick for each 2 qt. that Bill picked?

$$\frac{12}{8}$$

This ratio compares the number of quarts Tony picked with the number of quarts that Bill picked.

222

B

You do not know how many quarts Tony picked for each group of 2 qt. Bill picked.

But you can make a ratio that compares the unknown number of quarts that Tony picked with the 2 qt. that Bill picked.

1

$\frac{n}{2}$ ← Number of quarts Tony picked

← Number of quarts Bill picked

The ratio n to 2 expresses the same comparison as the ratio 12 to 8.

$$\frac{12}{8}=\frac{n}{2}$$ 2

Number of quarts Tony picked

Number of quarts Bill picked

compute

C

3

Think of Bill's 8 qt. as divided into groups of 2 qt. There are 4 groups.

$$\frac{12}{8}=\frac{n}{2}$$ 5

2 can be found by dividing 8 by 4.

D

4

Think of Tony's 12 quarts as divided into 4 equal groups of 3 quarts.

$$\frac{12}{8}=\frac{n}{2}$$

2 can be found by dividing 8 by 4. So the numeral that replaces n can be found by dividing 12 by 4. $n = 3$

Tony picked 3 qt. for each 2 qt. Bill picked.

223

222

1 Have the pupils relate the text to the picture.

2 Draw attention to the equation. Remind the pupils that to change a ratio to an equivalent one, they must multiply or divide the numbers represented in both terms by the same number.

3 Have the pupils note in Picture D that for each 25¢ of John's there is a matching 15¢ of Bob's. Since there are 5 groups of 25¢, there are 5 groups of 15¢, or a total of 75¢.

4 Point out that the completed equation $25/15 = 125/75$ is correct because when both 25 and 15 are multiplied by 5 the result is 125 and 75.

5 Have the pupils read the new problem and relate Picture A to it and to the ratio. Point out that the number of quarts Bill picked is represented by the numeral below the line.

223

1 Have someone explain the new ratio and why it is $n/2$ instead of $2/n$.

2 Explain that the two ratios are equal because the result of the comparison must be the same both times it is made.

3 Get the pupils to see that Bill has 4 groups of 2 quarts.

4 Be sure the pupils understand that Tony's 12 quarts must be thought of as divided into 4 equal groups, just as Bill's are.

5 Ask: "By what number can we divide 8 to get the 2? Which picture shows this? By what number can we also divide 12 to find the number whose numeral replaces n? Which picture shows this?"

see

A

Jim buys 20 stamps a month for his collection, and Don buys 16 stamps a month. How many stamps will Jim have bought when Don has bought 48 stamps? 1

$$\frac{20}{16} = \frac{n}{48}$$

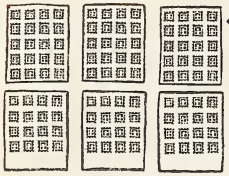
What do the 20 and 16 stand for?

What do the n and 48 stand for? 2

3 compute

B

How many groups of 16 stamps make the 48 stamps that Don has bought? Jim will then have bought ■ groups of 20 stamps.



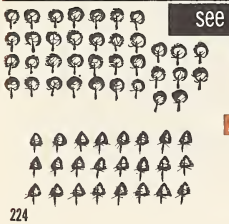
4 48 can be found by multiplying 16 by ■. How do you find the numeral that replaces n?

$$\frac{20}{16} = \frac{n}{48}$$

Jim will have bought ■ stamps when Don has bought 48 stamps.

see

A



Paul planted 36 young trees, and Tom planted 24. How many trees did Tom plant for each group of 18 that Paul planted? 5

$$\frac{36}{24} = \frac{18}{n}$$

What does each numeral stand for?

What do the 18 and n stand for?

224

compute

B

You can think of the 36 trees as divided into groups of 18 trees. There are ■ groups. 1

$$\frac{36}{24} = \frac{18}{n}$$

18 can be found by dividing 36 by ■. How do you find the numeral that replaces n?

Tom planted ■ trees for each 18 trees that Paul planted.

A Mr. Price's car uses 5 gal. of gasoline while Mr. Brown's car uses 6 gal. When Mr. Price's car has used 30 gal. of gasoline, how many gallons will Mr. Brown's car have used?

$$\frac{5}{6} = \frac{30}{n}$$

The 5 and the 30 stand for the number of gallons used by Mr. Price's car. 2

The 6 and the n stand for the number of gallons used by Mr. Brown's car.

B Jack cut up 2 bu. of seed potatoes while Jim cut up 3 bu. How many bushels of potatoes did Jack cut up while Jim cut up 9 bu.? 3

$$\frac{2}{3} = \frac{n}{9}$$

What do the 2 and the n stand for?

What do the 3 and the 9 stand for?

C A large wheel on a machine turns 50 times a minute. A smaller wheel on the same machine turns 120 times a minute. When the larger wheel has turned 10 times, how many times has the smaller wheel turned? 4

$$\frac{50}{120} = \frac{10}{n}$$

What do the 50 and the 10 stand for?

What do the 120 and the n stand for?

D Mrs. Bird has a picture that is 10 in. wide and 12 in. long. She wants to have a small print made that will be only 6 in. long. How wide will this small print be? 5

$$\frac{10}{12} = \frac{n}{6}$$

What do the 10 and the n stand for?

What do the 12 and the 6 stand for?

224

1 Have the pupils read the problem. In Picture A have someone tell which group of stamps is Jim's and which is Don's.

2 Ask a pupil to answer the questions about the equation. Emphasize the idea that 20 and n must both represent the number of Jim's stamps at different times and that both 16 and 48 must represent the number of Don's stamps at those same times.

3 Ask: "How many groups of 16 stamps equal the 48 stamps that Don bought? Why must there also be 3 groups of 20 stamps for Jim?"

4 Ask: "By what number can 16 be multiplied to get 48? By what must 20 be multiplied to find the number that n represents? What would be wrong if we multiplied 20 by some other number?"

5 Ask the pupils to relate the equation to the problem and picture. Discuss why 18 is written above the line and n below the line.

225

1 Let someone use the picture to explain the procedures for finding the numeral that replaces the screen. Ask, "By what number must 24 be divided to find the number that n represents?"

2 Discuss the reasons for writing the second ratio as $30/n$ instead of $n/30$. Ask: "By what do we multiply 5 to get 30? How do we find the number that n represents?"

3 Discuss how this problem differs from Problem A. Ask, "How do we know that we should multiply 2 by 3?"

4 Direct attention to the first terms, 50 and 10. Ask: "What can we do with 50 to get 10? What must we do with 120 to get the number that n represents?"

5 Discuss the ways in which this equation is similar to and different from the equation for Problem C. Ask: "Do we divide or multiply? By what? Why?"

E Patsy saves 4 pennies a day, and Jane saves 6 pennies a day. When Patsy has saved 56 pennies, how many pennies will Jane have saved?

$\frac{4}{6}=\frac{56}{n}$ or $\frac{6}{4}=\frac{n}{56}$

F Bob rode 16 mi. on his bicycle during the same time that Jim rode 12 mi. on his bicycle. How far did Bob ride during the time that Jim rode 3 mi.?

$\frac{16}{12}=\frac{n}{3}$ or $\frac{12}{16}=\frac{3}{n}$

G Mr. Page and Mr. White left at the same time to drive from Garden City to Grand Springs, a distance of 210 mi. Mr. Page drove at 30 mi. per hour, and Mr. White drove at 35 mi. per hour. How far had Mr. Page driven when Mr. White had arrived at Grand Springs?

$\frac{30}{35}=\frac{n}{210}$ or $\frac{35}{30}=\frac{210}{n}$

H In a race, one car traveled 500 mi. while a second car traveled 350 mi. How far had the first car traveled when the second car had traveled 35 mi.?

$\frac{500}{350}=\frac{n}{35}$ or $\frac{350}{500}=\frac{35}{n}$

I The Point Diamond lighthouse flashes 18 times while the Point Jupiter lighthouse flashes 12 times. When the Point Diamond lighthouse has flashed 90 times, how many times has the Point Jupiter lighthouse flashed?

J Mary saved 60¢, and Tony saved 80¢. How much money did Tony save for each 15¢ that Mary saved?

K Steve has a picture that is 3 in. wide and 4 in. long. He plans to have a larger picture made that will be 12 in. long. How wide will this larger picture be?

L A machine made 160 parts a minute. Another machine made 120 parts of the same kind in a minute. When the first machine had made 40 parts, how many parts had the second machine made?

M Kathy skated 150 ft. while Sally skated 80 ft. For each 15 ft. that Kathy skated, how far did Sally skate?

N At the Burns School, 39 boys and 26 girls play in the school band. How many boys play in the band for each two girls that play in the band?

Now you should be able to express comparisons by using ratios. You should also be able to use the ratios to solve problems.

226

226

1 Discuss the two equations for this problem, and be sure that the pupils understand that either way of writing the ratios is correct. Emphasize the fact that the numerals above the line must represent numbers that belong to the same idea—for example, Patsy's pennies. Discuss the computational procedure for finding the number that *n* represents.

2 Assign these problems as written work. Tell pupils they may write the equation whichever way they want to, but they must be consistent. Afterwards, have the pupils discuss why they multiplied or divided in each equation.

227-230 Exploring problems

Expanded Notes for this lesson are on pages 342-344.

Objectives

The child learns to solve problems in which the result of a comparison of two groups of objects is expressed in terms of "times as many."

Vocabulary

New words page 230 twice*

Comments

Problems involving "times as many" have traditionally been taught by a short-cut method involving division. Thus Problem A on page 227 would be solved by dividing 16 by 8, but the reason for this method of solution has never been made clear in traditional arithmetic programs. In *Seeing Through Arithmetic 4,* an introductory lesson sought to show the children why division could be used and to help them see that the problem situation was one of comparison. (See the lesson beginning on page 170, Book 4.) Now that the children have studied how to change a ratio to an equivalent ratio in comparative problem situations, they are much more ready to understand the solution of the "times as many" situation.

Problem A is really the same type as the problem discussed on pages 222-223, except that the expressed comparison 16 to 8, or 16/8, is to be changed to an equivalent ratio in which the second term is 1. The real problem is to find how many shells Sally found for every one that Kathy found. An equation that describes the problem can be written in the form $16/8 = n/1$. The children must be helped to understand that the expression "times as many" is a way of expressing something compared with 1. This is done in this problem by changing the ratio 16 to 8 to an equal ratio in which 1 takes the place of 8.

You will want to continue to emphasize the fact that the children must be consistent in making ratios. If the numeral that represents Sally's shells is above the line in the first ratio of the equation, then the other

numeral that also represents Sally's shells must be above the line in the second ratio in the equation.

A problem situation is also presented in which the children know that one group is a certain number of times as many as another group. They also know the number of things in one group and must find the number of things in the other group. Problem B on page 228 is this type of problem and is illustrated by a picture that shows that "3 times as many" means 3 buns for every 1 bun. This fact is represented by the ratio 3/1. The children also know that Ann has 12 buns. The equation for the problem situation is $3/1 = n/12$. Problem C concerns a situation in which the 20 arrowheads Tony has are known to be 4 times as many as Paul has. The second term of the desired ratio is to be found, and the equation is $4/1 = 20/n$.

Answers

Pages 229-230:

D 5/1=[90]/18; 90¢
E 9/3=[3]/1
F 3/1=15/[5]
G 12/4=[3]/1
H 3/1=[69]/23
I 2/1=40/[20]
J 2/1=80/[40]
K 30/15=[2]/1
L 6/1=[510]/85; $5.10
M 12/2=[6]/1
N 2/1=28/[14]
O 3/1=[36]/12
P 2/1=[18]/9
Q 50/25=[2]/1
R 3/1=[21]/7

Exploring problems

A Sally found 16 shells, and Kathy found 8 shells. Sally found how many times as many shells as Kathy?

1

To answer the question, you must find how many shells Sally found for each shell that Kathy found.

First compare the 16 shells that Sally found with the 8 shells that Kathy found. Look at Picture A. Write a ratio to express this comparison.

2

$$\frac{16}{8} \leftarrow \text{Number of Sally's shells} \quad \leftarrow \text{Number of Kathy's shells}$$

You can also write a ratio to show how many shells Sally found for each one that Kathy found.

3

$$\frac{n}{1} \leftarrow \text{Number of Sally's shells} \quad \leftarrow \text{Number of Kathy's shells}$$

Now you have expressed the same comparison with two equal ratios. Look at the equation below.

$$\frac{16}{8} = \frac{n}{1}$$

4 1 can be found by dividing 8 by 8. So the numeral that replaces n can be found by dividing 16 by 8. $n = 2$

$$\frac{16}{8} = \frac{2}{1}$$

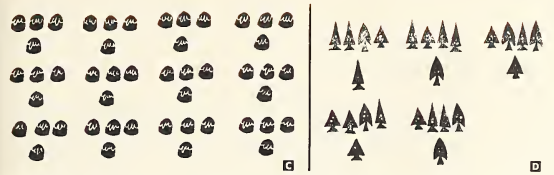
5 Sally found 2 shells for each shell that Kathy found. Look at Picture B.

Sally found 2 times as many shells as Kathy.

In working with the ratios, you found the numeral that replaces n by dividing 16 by 8. Why do you divide by 8?

227

1. Tell the pupils to read the problem and identify the girls' shells in Picture A. Help them see that this is a comparative situation.
2. Discuss how to express the comparison by a ratio.
3. Tell them to make another ratio telling how many of Sally's shells there are for each 1 of Kathy's. Point out that the numeral representing Kathy's shells was written below the line in the first ratio. So the numeral 1, which also represents her shells, must also be written below the line. Be sure they understand that n stands for the number of Sally's shells.
4. Have pupils look at the 8 and the 1 in the equation and see that 1 can be found by dividing 8 by 8. Discuss the fact that 16 must also be divided by 8.
5. Point out that the new ratio, 2 to 1, and Picture B show there are 2 of Sally's shells for every 1 of Kathy's.



B Ann bought 12 buns. Ruth bought 3 times as many buns as Ann. How many buns did Ruth buy?

When you say that Ruth bought 3 times as many buns as Ann, you know that she bought 3 buns for each bun that Ann bought.

Look at Picture C. You can compare Ruth's buns with Ann's buns by using the ratio 3 to 1.

You can also compare Ruth's buns with Ann's buns by using the ratio n to 12.

$$\frac{3}{1}=\frac{n}{12}$$

12 can be found by multiplying 1 by 12. So the numeral that replaces n can be found by multiplying 3 by 12. $n=36$

$$\frac{3}{1}=\frac{36}{12}$$

Ruth bought 36 buns.

In working with the ratios, you found the numeral that replaces n by multiplying 3 by 12. Why do you multiply by 12?

C Tony has 20 arrowheads. He has 4 times as many arrowheads as Paul. How many arrowheads has Paul?

When you say that Tony has 4 times as many arrowheads as Paul, you know that Tony has 4 arrowheads for each one that Paul has. Look at Picture D.

Use the ratio 4 to 1 to compare Tony's arrowheads with Paul's. The ratio 20 to n also compares Tony's arrowheads with Paul's. The two ratios express the same comparison and are equal. Look at the equation below.

$$\frac{4}{1}=\frac{20}{n}$$

20 can be found by multiplying 4 by 5. So the numeral that replaces n can be found by multiplying 1 by 5. $n=5$

$$\frac{4}{1}=\frac{20}{5}$$

Paul has 5 arrowheads.

In working with the ratios, you found the numeral that replaces n by first dividing 20 by 4. Then you multiplied 1 by ■.

D Don has 18¢. Jim has 5 times as much money as Don. How much money does Jim have?

When you say that Jim has 5 times as much money as Don, you know that Jim has 5 pennies for each one of Don's.

$$\frac{5}{1}=\frac{n}{18}$$

Jim's money in cents

Don's money in cents

18 can be found by multiplying 1 by 18. How can you find the numeral that replaces n?

Why can you find how much money Jim has by multiplying 5 by 18?

E Andy made 9 model boats. Tom made 3 model boats. Andy made how many times as many boats as Tom?

One ratio must tell how many boats Andy made for each boat Tom made. What must the other ratio tell? The two ratios are equal.

$$\frac{9}{3}=\frac{n}{1}$$

Number of Andy's model boats

Number of Tom's model boats

1 can be found by dividing 3 by 3. How do you find the numeral that replaces n?

To find how many times as many boats Andy made as Tom, divide ■ by 3. Why do you divide by 3?

F Susan picked 15 daisies. She picked 3 times as many daisies as Nancy. How many did Nancy pick?

Susan picked 3 daisies for each one that Nancy picked.

$$\frac{3}{1}=\frac{15}{n}$$

Number of Susan's daisies

Number of Nancy's daisies

15 can be found by multiplying 3 by ■. How do you find the numeral that replaces n?

To find how many daisies Nancy picked, first divide 15 by ■. Then multiply 1 by ■.

228

1 Ask a pupil to read this problem and find how it differs from Problem A on page 227. [Here they are *told* how many "times as many."]

2 Discuss the phrase "3 times as many," and have pupils look at Picture C to see that for every 1 of Ann's buns there are 3 of Ruth's.

3 Get the children to see that one ratio is 3/1, and that the equal ratio must be written as $n/12$. Discuss the meaning of n and 12. They should see that 12 can be obtained by multiplying 1 by 12. Ask: "Why should we multiply 3 by 12?"

4 Ask pupils to read Problem C. They should realize that "4 times as many" means 4 of Tony's arrowheads for every 1 of Paul's. They can see this comparison demonstrated in Picture D.

5 Have them look at the equation and explain why 20 has to be above the line of the second ratio and why n must be below. Ask: "Why do we multiply 1 by 5?"

229

1 Discuss how the answer 5 was obtained.

2 Have the pupils read Problem D. Have a pupil express "5 times as much" as a ratio. Discuss changing this ratio to an equal ratio. Point out that they know the amount of Don's money. Ask someone why the numeral representing it should be below the line of the second ratio and why n has to be above. Help the pupils discover that if 1 is multiplied by 18 the result is 18. Ask why it is important to discover this.

3 Discuss the remaining problems on this page in terms of the work done on the preceding problems.

G George collected 12 masks, and Don collected 4 masks. George collected how many times as many masks as Don?

1

$$\frac{12}{4}=\frac{n}{1}$$

THINK The ratio 12 to 4 compares George's masks with Don's masks. What does the ratio n to 1 compare? Why can you divide 12 by 4 to get the numeral that replaces n?

H Pat has 23 coins in his collection. Andy has 3 times as many coins as Pat. How many coins does Andy have?

1

$$\frac{3}{1}=\frac{n}{23}$$

THINK What does the ratio 3 to 1 compare? What does the ratio n to 23 compare?

I Ellen has 40 butterflies. She has 2 times as many as Carol has. How many butterflies does Carol have?

1

$$\frac{2}{1}=\frac{40}{n}$$

J June earned 80¢. She earned twice as much as George. How much money did George earn?

2

THINK When you say "twice as much," you mean "2 times as much."

K Ellen used 30 ft. of crepe paper for decorations, and Patsy used 15 ft. Ellen used how many times as much crepe paper as Patsy?

3

L Don saved 85¢. Steve said that he saved 6 times as much as Don. How much money did Steve save?

M Ann bought 12 noisemakers, and June bought 2. Ann bought how many times as many noisemakers as June?

N Paul found 28 rocks. This was 2 times as many rocks as Jack found. How many rocks did Jack find?

O One week Tony rode 12 mi. on his bicycle. That week Jim rode 3 times as many miles as Tony. How far did Jim ride?

P Paul bought 9 postcards. May bought twice as many postcards as Paul did. How many postcards did May buy?

Q Mr. Cook harvested 50 bu. of potatoes. Mr. Miner harvested 25 bu. Mr. Cook harvested how many times as many bushels as Mr. Miner?

R Kathy wrote 7 invitations for a party. June wrote 3 times as many invitations as Kathy did. How many invitations did June write?

Now you should understand how to use ratios to make equations to solve problems like those in this lesson.

230

230

1 For each of the problems illustrated with an equation, have the pupils explain why the numerals, including n, are in the places they are. Then have them solve each equation.

2 Point out that "twice as much" means the same as "2 times as much." Let the pupils give instances of where they can use these expressions interchangeably.

3 Assign the remaining problems as written work. Tell the pupils to write each equation first with the letter n in the proper place, then rewrite the equation with the answer in it. Supply the answers and discuss those problems that caused any trouble for more than a few pupils.

231 Keeping skillful

Expanded Notes are not considered necessary for this lesson.

Objectives

The child tests his skill in interpreting equations, in working with fractions, in reducing measures, and in finding the value of an unknown term in equal ratios.

Vocabulary

There are no new words.

Comments

The exercises in this lesson provide practice on many of the things that the children have previously studied. The additive and subtractive equations in Block 1 provide for review of the processes of addition and subtraction with whole numbers. Block 2 covers multiplication with whole numbers. Block 3 covers division with whole numbers. Block 4 requires the child to interpret multiplicative and divisive equations. Block 5 tests the children's ability to add fractions and mixed numbers. Block 6 covers subtraction of fractions and mixed numbers. The additive and subtractive equations in Block 7 require proper interpretation before the computation with fractions and mixed numbers can be performed. For the work in Block 8 (on reduction of measures), allow the children to use the tables of equivalents on page 93 if they need to. The final block tests the children's ability to find the value of an unknown term in equal ratios.

This page of materials for practice will indicate how proficient the children are in interpreting and solving equations. They may enjoy and profit from the activity of making up problems for the equations. Children who need help should restudy the original lessons as indicated by the exercises they miss.

Answers

(block 1)

A 68,086	E 15,109	I 130,775
B 189	F 12,811	J 69,803
C 8705	G 13,690	K 10,542
D 4840	H 5607	

Answers

(block 2)

A 52,164
B 306,436
C 271,056
D 144,200
E 659,025
F 15,725
G 359,040
H 259,200
I 317,376
J 261,162
K 324,486

(block 3)

A $74\frac{1}{2}$ or $74\frac{18}{36}$
B 346
C $156\frac{1}{4}$ or $156\frac{8}{32}$
D 38
E 119
F 92
G $235\frac{1}{8}$ or $235\frac{9}{72}$
H $686\frac{2}{7}$ or $686\frac{250}{875}$

(block 4)

A 71,198
B 293
C 3,738,420
D $322\frac{11}{12}$
E 193
F 205,416
G 39
H 92
I 423,024

(block 5)

A $\frac{13}{24}$
B $9\frac{8}{15}$
C $19\frac{4}{5}$ or $19\frac{8}{10}$
D $2\frac{1}{4}$ or $2\frac{3}{12}$
E $10\frac{7}{24}$
F $17\frac{11}{15}$ or $17\frac{22}{30}$
G $6\frac{5}{12}$
H $8\frac{13}{24}$

(block 6)

A $2\frac{1}{6}$
B $\frac{1}{2}$ or $\frac{5}{10}$
C $9\frac{13}{24}$
D $\frac{3}{16}$
E $1\frac{1}{2}$ or $1\frac{3}{6}$
F $7\frac{5}{6}$ or $7\frac{10}{12}$
G $\frac{2}{5}$ or $\frac{4}{10}$
H $1\frac{23}{30}$

(block 7)

A $\frac{3}{20}$
B $10\frac{1}{3}$ or $10\frac{4}{12}$
C $1\frac{9}{10}$
D $14\frac{5}{12}$
E $2\frac{9}{16}$
F $3\frac{11}{24}$
G $18\frac{19}{20}$
H $8\frac{11}{12}$
I $6\frac{1}{3}$

(block 8)

A 26,400 ft.
B 8 gal.
C 900 sec.
D 8 yd.
E 48 tbs.
F 9 pk.
G 3520 yd.
H 72 mo.
I 23 lb.
J 168 hr.

(block 9)

A 24
B 9
C 72
D 54
E 12
F 63
G 11
H 16

Keeping skillful 1

A $64791+3295=n$
B $n+874=1063$
C $12236-3531=n$
D $n-3865=975$
E $54891+n=70000$
F $4127+8684=n$
G $32481-n=18791$
H $6005-398=n$
I $n-76951=53824$
J $57739+12064=n$
K $8937+1605=n$

A $756\times69=n$
B $3692\times83=n$
C $48\times5647=n$
D $824\times175=n$
E $909\times725=n$
F $185\times85=n$
G $5280\times68=n$
H $72\times3600=n$
I $912\times348=n$
J $33\times7914=n$
K $4006\times81=n$

If there is a remainder, write a fraction numeral in the answer.

A $2682\div36=n$
B $27334\div79=n$
C $5000\div32=n$
D $24548\div646=n$
E $7854\div66=n$
F $75992\div826=n$
G $16929\div72=n$
H $600500\div875=n$

A $734\times97=n$
B $24905\div n=85$
C $5934\times630=n$
D $38750\div120=n$
E $350\times n=67550$
F $648\times317=n$
G $17004\div n=436$
H $n\times116=10672$
I $5036\times84=n$

A $\frac{3}{8}+\frac{1}{6}=n$
B $2\frac{4}{5}+\frac{1}{3}+6\frac{2}{5}=n$
C $11\frac{3}{10}+8\frac{1}{2}=n$
D $\frac{3}{4}+\frac{2}{3}+\frac{5}{8}=n$
E $7\frac{7}{8}+2\frac{5}{12}=n$
F $1\frac{9}{10}+12\frac{1}{2}+3\frac{1}{3}=n$
G $\frac{5}{6}+5\frac{7}{12}=n$
H $\frac{2}{3}+7\frac{1}{4}+\frac{5}{8}=n$

A $3\frac{1}{2}-1\frac{1}{3}=n$
B $\frac{4}{5}-\frac{3}{10}=n$
C $11\frac{5}{12}-1\frac{7}{8}=n$
D $\frac{15}{16}-\frac{3}{4}=n$
E $2\frac{1}{6}-\frac{2}{3}=n$
F $13\frac{7}{12}-5\frac{3}{4}=n$
G $\frac{9}{10}-\frac{1}{2}=n$
H $2\frac{3}{5}-\frac{5}{6}=n$

A $\frac{9}{10}-\frac{3}{4}=n$
D $6\frac{5}{12}+3\frac{11}{12}=n$
G $2\frac{1}{2}-n=\frac{3}{5}$
D $n-6\frac{2}{3}=7\frac{3}{4}$
E $6+n=8\frac{9}{16}$
F $4\frac{1}{3}-\frac{7}{8}=n$
G $\frac{1}{5}+18\frac{3}{4}=n$
H $n-8\frac{1}{2}=\frac{5}{12}$
I $n+\frac{2}{3}=7$

A 5 mi. = ▢ ft.
B ▢ gal. = 64 pt.
C 15 min. = ▢ sec.
D ▢ yd. = 288 in.
E 3 c. = ▢ tbs.
F ▢ pk. = 72 qt.
G 2 mi. = ▢ yd.
H 6 yr. = ▢ mo.
I ▢ lb. = 368 oz.
J 1 wk. = ▢ hr.

2

A $\frac{6}{15}=\frac{n}{60}$
B $\frac{26}{18}=\frac{13}{n}$
C $\frac{5}{18}=\frac{20}{n}$
D $\frac{3}{2}=\frac{n}{36}$
E $\frac{48}{72}=\frac{8}{n}$
F $\frac{4}{9}=\frac{28}{n}$
G $\frac{22}{14}=\frac{n}{7}$
H $\frac{2}{6}=\frac{n}{48}$

231

1 Assign only as much of this work at a time as you think the children can handle easily.
2 Provide answers for all the exercises, and let the children verify their work. Children who need help should restudy the original teaching on the exercises missed.

232-233 Moving forward

Exponded Notes ore not considered necessory for this lesson.

Objectives

The child learns to read a scale to determine size of an object or distance between two points from a scale drawing or map.

Vocabulary

New words page 232 represent, already*, along*, edge*, real*; page 233 drug

Comments

All representational drawings are, in a sense, scale drawings, but we usually reserve the term *scale drawings* for drawings made for the specific purpose of showing dimensions, distances, relative sizes or positions, and the like. Thus, when we think of scale drawings, we usually think of diagrams and blueprints, maps, and graphs. Scale drawings provide a convenient way of measuring indirectly.

Picture A is a scale drawing of a room with no dimensions shown. The scale provided is a continuous scale representing units of one foot with range from zero to twenty, numbered at intervals of four feet. This scale is really a number line divided into 20 equal parts. We are told that each one of these parts represents 1 foot of length in the picture. By using this scale, or one exactly like it marked off on a piece of paper, we can find dimensions in feet of the room and the furniture in the picture. The children should see that this type of scale drawing shows what things look like from only one point of view—in this case looking straight down on the library floor—and the relative positions of these things.

Picture B shows a simple map. The scale is a line representing 1 mile. If we measure this line with a ruler, we find that it is 1 inch long. So we can say that 1 inch on the map represents 1 mile of real distance. Since only 1 unit of the scale is given, it is necessary to repeat the unit to determine distances in excess of 1 mile.

Discuss the fact that the scale used in each drawing is one in which a small unit in the drawing represents a large unit in the actual object, so that the drawing is reduced to a miniature.

The children will benefit from bringing to school as many examples of scale drawings as they can find. (Good sources are road maps, atlases, "how-to-do" articles in magazines such as *Popular Mechanics, Popular Science, House and Garden,* etc. Some children may be able to bring blueprints from home.) This material may then be made the basis of many additional activities. For example, pupils might use the scale on a map or drawing to determine distances on the ground.

Answers

Pages 232-233:

A 20; 1, 2, 3, 5, 6, 7, 9, 10, 11, 13, 14, 15, 17, 18, 19
B 18 ft. long, 15 ft. wide
C 13 ft.; no
D 1 ft. wide; yes
E A, B, and C are each 5 ft. long, 3 ft. wide; 8 ft. long, 2 ft. wide
F 4 ft.
G 3 ft.
H 1; 1
I 2 mi.
J $1\frac{1}{2}$ mi.
K 3 mi.; $1\frac{1}{2}$ mi. farther
L 3 mi. ($1\frac{1}{2}$ mi. each way)
M $\frac{1}{2}$ mi. farther
N The drug store; It is 1 mi. nearer. (The drug store is $1\frac{1}{2}$ mi. from Tony's house. The grocery store is $2\frac{1}{2}$ mi. from Tony's house.)
O 8 mi.

Keeping skillful:

(block 1)
A 5
B 6
C 4
D 8
E 1
F 24

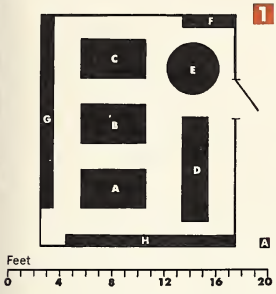
(block 2)
A $1\frac{4}{5}$
B 10
C $3\frac{1}{16}$
D $17\frac{11}{20}$
E $4\frac{2}{3}$ or $4\frac{4}{6}$
F $7\frac{3}{10}$
G $17\frac{13}{24}$
H $19\frac{7}{15}$
I $17\frac{9}{20}$
J $11\frac{3}{4}$ or $11\frac{9}{12}$
K $8\frac{5}{16}$

(block 3)
A $3\frac{1}{2}$ or $3\frac{2}{4}$
B $7\frac{1}{2}$ or $7\frac{5}{10}$
C $8\frac{2}{15}$
D $6\frac{7}{12}$
E $1\frac{3}{10}$
F $9\frac{5}{6}$
G $5\frac{17}{24}$
H $1\frac{13}{15}$
I $\frac{5}{16}$
J $\frac{9}{20}$
K $15\frac{5}{8}$
L $3\frac{13}{30}$
M $11\frac{19}{24}$

(block 4)
A 91 and 4 rm.
B 213
C 540 and 2 rm.
D 150
E 605
F 891
G 8
H 33 and 2 rm.
I 43
J 56 and 4 rm.
K 47
L 68
M 705 and 19 rm.

Moving forward

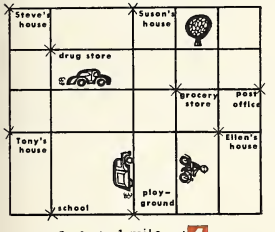
In this lesson you will learn about scales and scale drawings.



A

Feet

0 4 8 12 16 20



Scale: 1 mile

B

1

A Picture A is a scale drawing of the library at Downs School. The scale below the drawing shows how to measure lengths on the drawing in feet. How many feet in all does the scale represent? What numerals belong with the marks that are not already numbered?

B Put a paper along the scale. Put marks and numerals on the edge of the paper just as they are on the scale. Now you can find real distances represented on the scale drawing by putting your scale beside the line you want to measure. Find the real length and width of the library in this way.

2

C How long is Bookcase H? Is it the same length as Bookcase G?

3

D How wide is Bookcase F? Are all three bookcases the same width?

E What are the length and width of Tables A, B, and C? Of Table D?

F What is the diameter of Table E?

G How wide is the doorway?

H Now look at Picture B. The scale is shown below the map. The line is 1 inch long. So ▪ inch on the map represents ▪ mile of real distance.

1

I If you measure a distance of 2 in. on the map, how many miles will this distance represent?

J The distance between Steve's house and Susan's house is $1\frac{1}{2}$ in. on the map. How many miles apart are their houses?

K How far is the school from Steve's house? This is how much farther than the school is from Tony's house?

L One day Susan's mother asked her to ride to the drug store on her bicycle. How far did Susan have to ride in going to the drug store and back home again?

M Tony's house is how much farther from the playground than Ellen's house is?

2

N Tony's mother asked him to get some ice cream for dinner. He could buy the ice cream either at the drug store or at the grocery store. Which store would be nearer? How much nearer would it be?

O One day during lunch time the fifth-grade teacher, Mrs. West, drove from school to the post office and back to school. How far in all did Mrs. West drive?

3

Now you should know how to use a scale to measure distances on a scale drawing or map.

Keeping skillful

4

A $\frac{6}{1}=\frac{30}{n}$

B $\frac{2}{7}=\frac{n}{21}$

C $\frac{12}{3}=\frac{n}{1}$

D $\frac{20}{16}=\frac{10}{n}$

E $\frac{18}{6}=\frac{3}{n}$

F $\frac{8}{11}=\frac{n}{33}$

Find the sum.

A $\frac{2}{5}, \frac{4}{5}, \frac{3}{5}$

B $6\frac{7}{12}, 3\frac{1}{4}, \frac{1}{6}$

C $\frac{9}{16}, \frac{5}{16}, 2\frac{3}{16}$

D $1\frac{3}{4}, 7\frac{1}{8}, 8\frac{5}{8}$

E $\frac{1}{9}, \frac{5}{6}, 3\frac{1}{2}$

F $4\frac{1}{2}, 2\frac{3}{5}, \frac{1}{5}$

G $\frac{3}{8}, 7\frac{5}{8}, 9\frac{1}{2}$

H $4\frac{3}{5}, 6\frac{1}{5}, 8\frac{3}{5}$

I $\frac{9}{10}, 5\frac{1}{4}, 11\frac{3}{10}$

J $7\frac{1}{4}, 2\frac{5}{8}, 1\frac{2}{3}$

K $\frac{7}{8}, 6\frac{3}{4}, \frac{11}{16}$

A $n+3\frac{3}{4}=7\frac{1}{4}$

B $12\frac{1}{10}-4\frac{3}{5}=n$

C $n-5\frac{1}{3}=2\frac{4}{5}$

D $11-n=4\frac{5}{12}$

E $6\frac{1}{2}+n=7\frac{4}{5}$

F $18\frac{5}{6}-9=n$

G $14\frac{7}{8}-9\frac{1}{8}=n$

H $n+4\frac{1}{3}=6\frac{1}{6}$

I $1\frac{7}{8}+n=2\frac{3}{16}$

J $\frac{3}{4}-n=\frac{9}{10}$

K $n-7=8\frac{5}{8}$

L $n+\frac{9}{10}=4\frac{1}{3}$

M $16\frac{2}{3}-4\frac{7}{8}=n$

A $732\div 8=n$

B $1065\div n=5$

C $3782\div 7=n$

D $900\div 6=n$

E $5445\div n=9$

F $6237\div 7=n$

G $272\div 34=n$

H $926\div 28=n$

I $2795\div n=65$

J $4092\div 73=n$

K $2773\div 59=n$

L $1156\div n=17$

M $59239\div 84=n$

232

1 Direct attention to Picture A. Then have the pupils read Exercise A and answer the questions.

2 Have the class read the directions in Exercise B and follow them.

3 Let the pupils take turns answering the questions in Exercises C to G. Bring out the idea that if a drawing is accurate, the pupils can measure the dimensions fairly accurately and much more easily than they could measure the real objects.

4 Let the children measure the line on the map marked "1 mile." Have them read Exercise H. Ask: "How could you use this scale if you had no ruler?" [They could make an extended scale, marking off 1-mile units. Shorter distances shown on the map would be found to be one half the unit and represent $\frac{1}{2}$ mile. The scale could have small marks at $\frac{1}{2}$-mile intervals. They will find that they do not have to extend their scale beyond 3 miles.]

233

1 Let the pupils take turns in answering the questions in Exercises I to O.

2 Let the pupils discover that there are really three questions to be answered here.

3 Have the pupils turn to pages 108 and 109 of their books and discover that diagrams such as these are scale drawings.

4 Follow your usual procedures. Assign only as much work as can easily be completed in one working period.

234-237 Moving forward

Expanded Notes are not considered necessary for this lesson.

Objectives

The child learns how to interpret pictographs and bar graphs. He also learns to use scales when reading graphs.

Vocabulary

New words page 234 graphs, pictograph; page 235 absences, absent*, facts*; page 236 bottom*, Ohio*, Minnesota, Indiana*, Illinois, list*; page 237 faster*

Comments

This lesson introduces children to two types of graphs —pictographs and bar graphs. You can lead the children to see that graphs are used to present information about collections of numbers, and that when such information is arranged in the form of a graph, it is possible to compare many quantities with a glance. The child should understand that graphs use scales. Have the pupils turn back to page 232, and let someone tell what information the scale drawing of the library presents. (It shows the *amount* of space occupied by the objects in the room and gives a true account of the *position* of each object in relation to the other objects.)

Introduce the idea that a graph is a convenient method for presenting quantitative information for the purpose of visual comparison. Without going into detail, you could then discuss pictographs and bar graphs. The child should be taught, when observing any given graph, to note whether or not a scale is given.

You may wish to explain that the pictures of objects in pictographs on page 234 are merely symbols and not realistic reproductions (see page 234). You could then discuss each pictograph in turn, letting children discover how Pictograph C differs from Pictographs A and B. (Each line in Pictograph C represents a different kind of animal—that is, a different kind of data.)

Explain that the three bar graphs on page 235 are about the same thing (the number of absences from the Case School during one week). Let the children discover what can be learned from Graph D and compare this with the information provided by Graphs E and F. Have the pupils decide which of the latter two graphs is the easier to read.

You may also want to discuss the fact that the bars representing the number of pupils vary in length in these graphs, but that the relation between one bar and another on each graph is the same on all the graphs; that is, the bar representing 12 pupils is always twice as long as the bar representing 6 pupils.

Have the children note that on page 236 the bars for Graph G are horizontal, while those for Graph H are vertical. Make sure, too, that they understand that on Graph G the 125 means 125,000, the 250 means 250,000, and so on.

Able pupils might be asked to bring in bar graphs or pictographs from magazines, newspapers, advertising booklets, etc. They also could prepare sets of questions about the graphs.

Answers

A Bill

B The answer is a matter of opinion and judgment, rather than fact. The first two lines, and the first and third lines, might be said to be "nearly the same length," but the last line is almost twice as long as the second line.

C The length of each line in relation to the other lines shows this at a glance; count the cars

D 3 quarts

E Because the line of bottles is longer; first week

F The different kinds of pets and the number of each kind owned by the fifth graders; fish; dogs

G Dogs, cats and rabbits, birds, turtles, fish

H No

I The bar for Monday is the longest bar. Wednesday

J 24

K By measuring each bar and either multiplying the number of quarter inches by 6, or by adding as many sixes together as there are quarter inches

L Yes; 6

M 6

N No, different scales may be used.

O Iowa

P 250,000 bu.; 375,000 bu.; 500,000 bu.; 625,000 bu.

Q Fewer than; more

R Opinions will probably vary; nearer to 500,000 bu. is the better answer.
S Iowa, Illinois, Minnesota, Indiana, Ohio
T Year C
U Year A; more than
V C, B, E, D, A
W Average speed of passenger airplanes in the U.S. for 4 years; 1945, 1950, 1953, 1954; miles per hour
X Yes
Y 180 miles per hour; about 200 miles per hour
Z Yes; because the speed of airplanes has increased since 1954 (If the members of the class do not have recent information about speeds of new types of planes, assign someone to collect such data.)

Keeping skillful:

(block 1)

A 1747
B 3553
C 39,337
D 10,747
E 31,887
F 13,272
G 4655
H 2212
I 56,126
J 7278
K 65,348
L 1930
M 5600
N 12,895

(block 2)

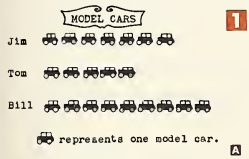
A 21,035
B 30,192
C 65
D 709
E 225,656
F 116,380
G 214
H 268,928
I 96
J 208
K 340,725
L 170,980
M 546
N 400,323

(block 3)

A 167 and 1 rm.
B 878 and 7 rm.
C 853
D 8 and 18 rm.
E 21 and 26 rm.
F 9
G 73
H 318
I 61 and 9 rm.
J 167 and 11 rm.
K 378
L 462 and 11 rm.
M 783
N 508

Moving forward

In this lesson you will learn about graphs.



1

A Picture A is a graph that compares the model cars that Jim, Tom, and Bill have. This kind of graph is called a pictograph. Each picture of a car shown on the graph represents one model car. Find the longest line of cars. Which boy has the most cars?

B Are all the lines of cars nearly the same length? Do the boys have nearly the same number of cars apiece?

C How would this pictograph help you compare the number of cars the boys have without your knowing exactly how many cars each boy has? What would you have to do if you wanted to know exactly how many cars each boy has?

2

D Pictograph B shows how many quarts of milk the Springer family used in each of 4 weeks. Below the graph you are told that each bottle of milk on the graph represents ▯ quarts of milk really used.

E How do you know that they used the most milk during the third week? They used the least milk during the ~~ week.

3

F What does Pictograph C show? The fifth graders have more ~~ than any other pet. They have fewer ~~ than any other pet.

G Name the pets in the order of the number the children have. Begin with the smallest number of pets.

4

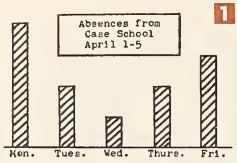
234

1 Exercise A should be read and discussed in relation to Pictograph A. Have Exercises B and C read and the questions answered.

2 Proceed in much the same way with Exercises D and E and Pictograph B. Get the children to observe the economy of effort expressed by using one symbol to represent 3 quarts. Explain how this is an application of the idea of scale.

3 Ask how Pictograph C differs from the others on the page. [Each line represents a different kind of animal.] When the pupils have answered Exercises F and G, point out that a graph may be used to compare different kinds of data.

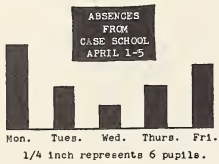
4 Review the graphs on this page and emphasize that graphs make visual comparisons easy and sometimes striking.



D

H Picture D shows another kind of graph, called a bar graph. This graph compares the number of absences at the Case School for the 5 school days during one week. Can you tell how many pupils were absent on any one day?

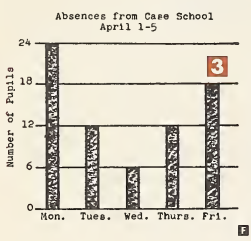
I How do you know that the most pupils were absent on Monday? On which day were the fewest pupils absent?



E

J The graph in Picture E also compares the same absences, but the bars are not the same length. You are told how the bars were measured when the graph was made. The bar for Monday is 1 in. long. How many pupils does it represent?

K How can you find out how many pupils each of the other bars represents?

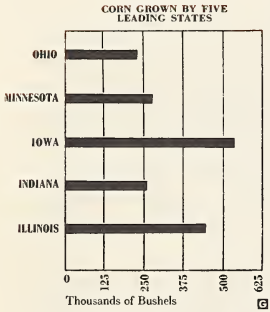


F

L From Graph D you could not tell how many more pupils missed school on Tuesday than on Wednesday. Can you now find out exactly how many more pupils were absent on Tuesday than on Wednesday? How many more were absent?

M Another graph about the absences is shown in Picture F. Find the scale at the left of the graph. Use it to answer Question L.

N Imagine that you are going to make two different bar graphs to show the same facts. Must the bars that represent the same facts be the same length on both graphs?



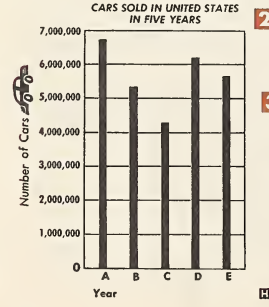
G

O Graph G shows the approximate number of bushels of corn grown in one year in the five states that grew the most corn. Which state grew the most corn?

P Look at the scale at the bottom of the graph. The 125 means 125,000 bu. What does each of the other numerals on the scale mean?

Q Did Ohio grow more than or fewer than 250,000 bushels of corn? Did Minnesota grow more or fewer bushels of corn than Indiana?

R Was the amount of corn grown in Illinois nearer to 375,000 bushels or to 500,000 bushels?



H

S List the states in the order of the amount of corn grown. Begin with the state that grew the most corn.

T Now look at Graph H. It shows the approximate number of cars sold in the United States in each of five years. In which year were the fewest cars sold?

U Were more cars sold in Year A or in Year E? Were more than 6,000,000 cars or fewer than 6,000,000 cars sold in Year D?

V List the years in the order of the number of cars sold. Begin with the year in which the fewest were sold.

235

1 Show pupils that when no scale is shown for a graph (see Graph D) you can still make comparisons, but you cannot be specific about them. Allow time for discussion of Exercises H and I in relation to Graph D.

2 Exercises J, K, and L point out important differences between Graphs D and E. Help pupils see that different scales can be used with the same data and that more precise comparisons can be made when the scale is given.

3 Point out that in Graph D the scale is not visible. In Graph E the scale is given, but precise comparison is not possible unless we measure. In Graph F, however, the scale makes it possible to read the quantities directly and compare them numerically.

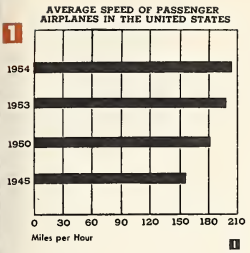
4 Be sure the children understand that the same set of data can be graphed in various ways. They should realize, too, that the relation of the lengths of the bars remains the same, no matter what scale is used.

236

1 Have the pupils read, discuss, and answer Exercises O to S in relation to Graph G. Stress the fact that each numeral in the scale means *thousands* of bushels. For Exercise S, write the names of the states on the chalkboard as the pupils call them in order.

2 Direct attention to the vertical arrangement of the bars and the position of the scale in Graph H. You also may wish to point out that rounded numbers have been used in Graphs G and H and let the class talk about why it is convenient to use such numbers in making graphs.

3 Ask the pupils to use Graph H in answering Exercises T to V.



1

W What does Graph I show? For what years does it show this? What do the numerals on the scale below the graph represent?

X Would you expect planes to be faster in 1954 than they were in 1945? Look at the graph to see if your answer is correct.

Y What was the approximate average speed of airplanes in 1950? In 1954?

Z If the average speed of airplanes now were shown, do you think the bar on the graph would be longer than the bar for 1954? Why or why not?

Now you should know how to read graphs to compare many things.

Keeping skillful

2

A	$n+4261=6008$	A	$7\times3005=n$	A	$836\div5=n$
B	$9359-5806=n$	B	$48\times629=n$	B	$7031\div8=n$
C	$30874+n=70211$	C	$80\times n=5200$	C	$5118\div n=6$
D	$6539+4208=n$	D	$n\times9=6381$	D	$378\div45=n$
E	$89640-n=57753$	E	$536\times421=n$	E	$593\div27=n$
F	$n-2683=10589$	F	$23\times5060=n$	F	$567\div n=63$
G	$n+77350=82005$	G	$n\times37=7918$	G	$2190\div30=n$
H	$6900+n=9112$	H	$704\times382=n$	H	$22896\div n=72$
I	$n-52346=3780$	I	$41\times n=3936$	I	$924\div15=n$
J	$10061-n=2783$	J	$n\times76=15808$	J	$8361\div50=n$
K	$74312-8964=n$	K	$525\times649=n$	K	$6804\div n=18$
L	$n+6070=8000$	L	$83\times2060=n$	L	$43901\div95=n$
M	$93002-n=87402$	M	$28\times n=15288$	M	$10962\div n=14$
N	$50689-37794=n$	N	$77\times5199=n$	N	$43688\div n=86$

237

237

1 The pupils should observe how the graph shows the increased speed of airplanes and note the marked increase between 1945 and 1954. They should be able to understand that, as long as the scale remains the same, greater speeds will be shown by longer bars.

2 You may use all these exercises, or you may assign only those blocks that will give the children the type of practice they need most. Be sure to provide answers, so that the children can verify their work, and discuss any exercises that cause trouble.

238-239 Thinking straight

Expanded Notes for this lesson are on page 344.

Objectives

The child reviews what he knows about areas and perimeters. He also learns how areas are related to dimensions.

Vocabulary

New words page 238 describe*, diagrams, dimensions, depth

Comments

The children should learn from this lesson what is meant by the term *dimensions* as applied to a rectangle. They should also discover that many different words, such as *high, long, deep, wide,* etc., are used to describe rectangular objects, and they should begin to develop some judgment about the use of these words.

A number of interesting and important ideas about measurement are introduced in this lesson. One idea is that two rectangles may have equal areas but different perimeters (hence, different bases and altitudes). Thus you cannot know the area of a rectangle if you know only its perimeter. If you know the base and altitude of the rectangle, however, you can find its area. These ideas can be further developed through discussion in connection with Exercise O. When the children have finished this lesson, they should be well prepared for the lesson on computing the area of a rectangle, which follows.

Answers

A 80 ft., 50 ft.; 50 ft., 80 ft.

B 80 ft. and 50 ft.; 50 ft. and 80 ft.

C 50 ft., 80 ft.; Children must accept the fact that lots are commonly described as so many feet deep, and that this is a special meaning attached to the word *deep*.

D No; We do not usually associate the word *high* with the dimensions of a piece of land.

E Yes; A lot can be described as wide and long. The width of a lot is usually the side along the street.

The width of a rectangle is usually the shorter side, but not always.

F 1 in. and 1 in.; 1 in. and 1 in.

G 1 sq. in.

H Yes

I Yes; 4 sq. in.; because it takes 4 one-inch squares to cover the 2-inch square; Children must understand that a 2-inch square is not the same as 2 square inches.

J 4 in.; 8 in.; 2 times

K 1 sq. in.; 4 sq. in.; 4 times

L 4 in. and 4 in.; 16 in.; 16 sq. in.

M 8 in. and 2 in.; 20 in.; 16 sq. in.

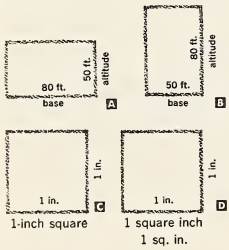
N No; yes; no; yes

O No; The perimeter of a rectangle does not tell you anything about the area, and you cannot tell which of two rectangles is larger unless you know their dimensions. This is shown by Rectangle H and Square G on page 239. Rectangle H has a greater perimeter than Square G, but the same area as Square G.

Keeping skillful:

(block 1)	(block 2)	(block 3)	
A $1\frac{17}{24}$	A 3	A $2\frac{1}{2}$	J $2\frac{5}{10}$ or $2\frac{1}{2}$
B $21\frac{13}{24}$	B $\frac{1}{24}$	B $6\frac{5}{12}$	K $3\frac{3}{8}$
C $5\frac{2}{12}$ or $5\frac{1}{6}$	C $3\frac{3}{8}$	C $3\frac{7}{24}$	L $6\frac{4}{12}$ or $6\frac{1}{3}$
D $23\frac{22}{30}$ or $23\frac{11}{15}$	D $\frac{4}{6}$ or $\frac{2}{3}$	E $15\frac{4}{6}$ or $15\frac{2}{3}$	M $2\frac{1}{10}$
E $3\frac{11}{16}$	E $1\frac{2}{4}$ or $1\frac{1}{2}$	F 10	N $2\frac{1}{4}$
F 2	F $11\frac{5}{16}$	G 3	O $\frac{3}{16}$
G $14\frac{9}{24}$ or $14\frac{3}{8}$	G $\frac{7}{15}$	H $9\frac{3}{8}$	P $2\frac{4}{12}$ or $2\frac{1}{3}$
H $33\frac{7}{16}$	H $36\frac{6}{10}$ or $36\frac{3}{5}$	I $1\frac{8}{12}$ or $1\frac{2}{3}$	Q $21\frac{2}{3}$
I 24	I $1\frac{7}{20}$		R $8\frac{1}{8}$

Thinking straight



A Rectangles A and B are diagrams of two lots. The base shown for Rectangle A is ■ ft., and the altitude shown is ■ ft. The base shown for Rectangle B is ■ ft., and the altitude shown is ■ ft. 1

B The lengths of the base and altitude of a rectangle are its dimensions. What dimensions are shown for Rectangle A? For Rectangle B?

C If you were to describe Lot A, you might say that it is 80 ft. wide and 50 ft. deep. What is the width of Lot B? What is the depth of Lot B? 2

D Would you describe Lot A as 80 ft. long and 50 ft. high? Why or why not?

E Would you describe Lot B as 50 ft. wide and 80 ft. long? Why or why not? How do you decide which dimension is the width?

F What are the dimensions of Square C? Of Square D? 3

G Square C and Square D have the same area. This area is ■ sq. in.

H Squares E and F are scale drawings of 2-inch squares. Do they have the same dimensions?

I Do Square E and Square F have the same area? The area of a 2-in. square is ■ sq. in. Why is this area 4 sq. in. and not 2 sq. in.? 4 5

238

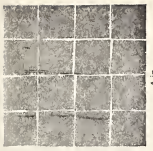
1 Discuss Exercises A and B and Pictures A and B with the class. Remind them that the altitude of a rectangle is measured at right angles to the base. They should observe that Rectangles A and B are just alike, but that different sides have been selected as the base.

2 Use Exercises C to E to acquaint pupils with some terminology often used in giving the dimensions of a physical object such as a piece of land.

3 Remind the pupils of the difference between a 1-inch square and a square inch. They should recall that area is measured in terms of the number of square units in a surface.

4 Be sure pupils understand why Pictures E and F are called scale drawings.

5 Square F is represented as covered by one-inch squares. Have a pupil count the squares to find the area of Square F. The class should see that Squares E and F have the same area because they have the same dimensions.



G

H

J The perimeter of Square D is ■ in. The perimeter of Square F is ■ in. The perimeter of Square F is how many times the perimeter of Square D? **1**

K The area of Square D is ■ sq. in. The area of Square F is ■ sq. in. The area of Square F is how many times the area of Square D? **2**

L The dimensions of Square G above are ■ in. and ■ in. Its perimeter is ■ in. Its area is ■ sq. in. **3**

M The dimensions of Rectangle H are ■ in. and ■ in. Its perimeter is ■ in. Its area is ■ sq. in. **4**

N Do Square G and Rectangle H have the same dimensions? Are both of them rectangles? Do they have equal perimeters? Are their areas equal? **4**

O If you know only the perimeters of two lots that are in the shape of rectangles, can you tell which of the lots is larger? Why or why not? **5**

Keeping skillful **6**

A $\frac{3}{4}+\frac{1}{8}+\frac{5}{8}=n$	A $14\frac{2}{3}-11\frac{2}{3}=n$	A $n+2\frac{1}{2}=5$	J $11\frac{1}{5}-8\frac{7}{10}=n$
B $11\frac{2}{3}+9\frac{2}{6}=n$	B $\frac{7}{8}-\frac{5}{8}=n$	B $14\frac{2}{3}-n=8\frac{1}{4}$	K $7\frac{3}{8}-n=4$
C $2\frac{1}{2}+1\frac{1}{12}+1\frac{3}{4}=n$	C $6\frac{1}{8}-2\frac{3}{4}=n$	C $7\frac{3}{8}+n=10\frac{2}{3}$	L $5\frac{11}{12}+\frac{5}{12}=n$
D $14\frac{2}{10}+8\frac{5}{6}=n$	D $12\frac{1}{2}-11\frac{5}{8}=n$	D $9\frac{5}{8}-2\frac{1}{4}=n$	M $n-\frac{9}{10}=1\frac{1}{2}$
E $\frac{5}{8}+\frac{11}{16}+2\frac{3}{8}=n$	E $4\frac{1}{4}-2\frac{3}{4}=n$	E $n-3\frac{1}{6}=12\frac{1}{2}$	N $18\frac{3}{4}+n=21$
F $\frac{7}{12}+\frac{3}{4}+\frac{2}{3}=n$	F $19\frac{11}{16}-8\frac{3}{8}=n$	F $8\frac{7}{10}+1\frac{1}{10}=n$	O $n+\frac{15}{16}=1\frac{1}{8}$
G $\frac{11}{12}+3\frac{7}{8}+9\frac{7}{12}=n$	G $\frac{4}{5}-\frac{1}{3}=n$	G $n+1\frac{1}{2}=4\frac{1}{2}$	P $12\frac{7}{12}-n=10\frac{1}{4}$
H $4\frac{9}{16}+28\frac{7}{8}=n$	H $48\frac{1}{2}-11\frac{9}{10}=n$	H $n-2\frac{3}{8}=7$	Q $13+8\frac{2}{3}=n$
I $20\frac{1}{4}+3\frac{3}{4}=n$	I $13\frac{3}{8}-12\frac{1}{4}=n$	I $6\frac{7}{12}+n=8\frac{1}{4}$	R $24\frac{1}{8}-n=16$

239

1 Discuss how to make an equation to compare the perimeters of Squares D and F on page 238 ($8/4=n/1$). Have a pupil write the equation on the board and then find the number that n represents.

2 Ask the pupils how they know that the area of Square F is 4 times as large as the area of Square D.

3 Pupils should understand that the small squares in Pictures G and H represent one-inch squares.

4 These questions should help pupils see that although two rectangles have equal areas, they do not necessarily have equal perimeters.

5 Get the pupils to see that the perimeter of a rectangle that is not a square does not tell you what the base and altitude are, so you do not have enough information to compare its size with that of another rectangle.

6 Handle these exercises in your usual way. Supply answers so that the pupils can verify theirs.

240-244 Exploring problems

Expanded Notes for this lesson are on pages 345-347.

Objectives

The child learns to compute the area of a rectangle in square inches.

Vocabulary

There are no new words.

Comments

By now the children know what the measurement of area is—namely, finding the number of square units that cover a given surface. They also understand what a square inch is. They are well prepared to find the area of a rectangle by computation.

The usual way of finding the area of a rectangle by multiplying the number of units in the length (base) by the number of units in the width (altitude) has always raised an issue about what the numbers really represent. The child has no clear idea of why he can multiply to find area or why his answer means square units, and he finds areas by rule. (See "Charting the Course" on page 345.) The traditional formula or equation is a short cut, and, like most short cuts, it suppresses meaning.

Actually, two ratios are involved in finding area. The one ratio expresses the *number of square inches per row*, and this ratio equals the ratio that expresses the *number of square inches in all the rows*. Once the child has made the equation, he solves it just as he does all the other equations involving ratios he has encountered in the previous lessons.

The traditional short cut will be introduced in Book 6, where all the short cuts involving ratios will be studied.

Answers

Page 244:

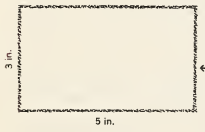
(block 1)

A $8/1=[40]/5$; 40 sq. in.	E $7/1=[28]/4$; 28 sq. in.
B $2/1=[12]/6$; 12 sq. in.	F $4/1=[16]/4$; 16 sq. in.
C $4/1=[40]/10$; 40 sq. in.	G $9/1=[54]/6$; 54 sq. in.
D $15/1=[45]/3$; 45 sq. in.	H $6/1=[36]/6$; 36 sq. in.

(block 2)

A 11/1=[121]/11; 121 sq. in.
B 7/1=[63]/9 or 9/1=[63]/7; 63 sq. in.
C 11/1=[88]/8 or 8/1=[88]/11; 88 sq. in.
D 29/1=[406]/14 or 14/1=[406]/29; 406 sq. in.
E 22/1=[374]/17 or 17/1=[374]/22; 374 sq. in.
F 12/1=[120]/10 or 10/1=[120]/12; 120 sq. in.
G 20/1=[400]/20; 400 sq. in.
H 8/1=[64]/8; 64 sq. in.
I 36/1=[468]/13 or 13/1=[468]/36; 468 sq. in.
J 9/1=[108]/12 or 12/1=[108]/9; 108 sq. in.
K 26/1=[442]/17 or 17/1=[442]/26; 442 sq. in.

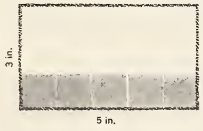
Exploring problems



A A piece of paper is in the shape of a rectangle 3 in. wide and 5 in. long. What is its area in square inches? 1

Look at the diagram.

Imagine that 1-inch squares are laid in rows to cover the rectangle. There will be 5 of these squares in each row because the base of the rectangle is 5 in. There are 5 square inches in one row. 2

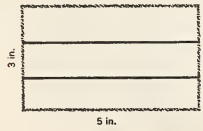


B This is a rate of 5 square inches per row. Write this rate as a ratio.

$\frac{5}{1}$ ← Number of square inches in each row

← Number of rows

Now you must find how many rows of square inches there will be. The altitude is 3 in., and each row will use 1 in. of the altitude. So there will be 3 rows.



C You do not know how many square inches will be used for the 3 rows, but you do know that they will be used at the rate of 5 square inches per row. You can express the rate as n square inches per 3 rows. 3

$\frac{n}{3}$ ← Total number of square inches

← Total number of rows

240

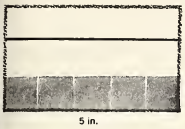
1 Have the pupils read the problem. Ask them how they would go about finding the area of this rectangle.

2 Have the pupils read this text in connection with Picture B. Be sure they understand that "5 square inches per 1 row" is a rate, just as "3¢ per 1 pencil" or "35 miles per hour" are rates. They must see that the 1 always means 1 row of square units on the side of the rectangle that is considered the base.

3 Be sure the pupils see that, since the altitude is 3 inches and since each row of units takes up 1 inch of the altitude, there are 3 rows of units in the whole rectangle. Now the rate is expressed as n square inches per 3 rows. Ask questions to get pupils to observe that the first term in both ratios means number of square inches, and the second term in both ratios means number of rows.

The rate at which square inches will be used will be the same for the 3 rows as for 1 row. Look at the equation.

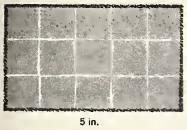
D



1 $\frac{5}{1} = \frac{n}{3}$ ← You must find the numeral that replaces n.

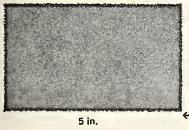
5 square inches are used for 1 row.
15 square inches are used for all 3 rows.

E



2 $\frac{5}{1} = \frac{n}{3}$ ← 3 can be found by multiplying 1 by 3. So the numeral that replaces n can be found by multiplying 5 by 3. $n = 15$

F



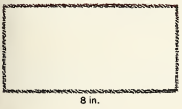
3 $\frac{5}{1} = \frac{15}{3}$ ← The 15 was found by multiplying the 5 by 3. Why was 5 multiplied by 3?

The area of the paper rectangle is 15 square inches.

G

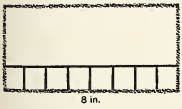
A piece of paper is in the shape of a rectangle 4 in. wide and 8 in. long. What is its area in square inches? 1

Look at the diagram.



H

The base of the rectangle is 8 in. long. So in one row there will be ▩ sq. in. The rate is ▩ square inches per row. Write the rate as a ratio. 2



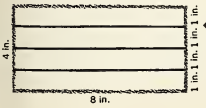
3 What does this numeral stand for?

$\frac{8}{1}$

What does this numeral stand for?

I

The altitude of the rectangle is 4 in. So there will be ▩ rows of square inches. Why can the rate also be expressed as n square inches per 4 rows? Write this rate as a ratio. 4



5 What will the numeral that replaces n stand for?

$\frac{n}{4}$

What does this numeral stand for?

241

1 Be sure the pupils understand that the two ratios express the same rate and can therefore be written as members of an equation. Picture D shows the 5 square inches per 1 row and also the unknown number of square inches per 3 rows.

2 Have the pupils read the text and study Picture E. Ask: "How do we get 15? What does the 15 mean? How do we know that 15 is the number we want?" Let pupils count the squares in Picture E to verify the answer.

3 Picture F suggests the unbroken surface enclosed by the rectangle. It can be compared with Picture A, in which the surface is not indicated.

242

1 Have the pupils read the problem and look at Picture G.

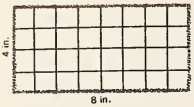
2 Be sure the pupils understand in Picture H that the divisions represent 1-inch squares and that there are eight of them across the base. Then they should be able to supply the missing numerals in the text.

3 Have the pupils tell what the numerals in this ratio represent in Picture H.

4 Have the pupils read this text in connection with Picture I. Be sure they understand why there are 4 rows of inch squares.

5 Be sure the pupils understand that this ratio expresses the same rate as 8/1. They should observe that the first term in both ratios means number of square inches.

J



Square inches will be used at the same rate for 4 rows as for 1 row.

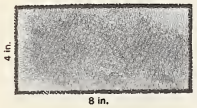
$$\frac{8}{1} = \frac{n}{4}$$ 1

What do the 8 and the n stand for?

What do the 1 and the 4 stand for?

In 4 rows there will be ■ times as many square inches as in 1 row.

K



$$\frac{8}{1} = \frac{n}{4}$$ 2

Why do you multiply 8 by 4 to find the numeral that replaces n? $n = ■$

The area of the paper rectangle is ■ sq. in.

L

The base of Square L is 5 in. The altitude is 5 in. Find its area in square inches. 3

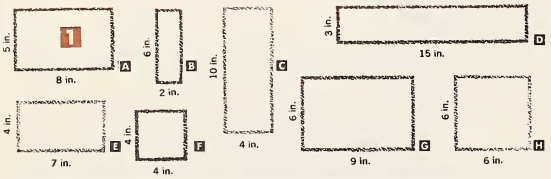
$$\frac{5}{1} = \frac{n}{5}$$

This ratio means ■ sq. in. per 1 row.

When you have found the numeral that replaces n, you will know how many square inches there are in ■ rows.

Why do you multiply 5 by 5 to find the numeral that replaces n?

The area of Square L is ■ sq. in.



Each diagram above represents a piece of board that is a rectangle. Find the area of each piece of board. 2

A A side of a square piece of glass is 11 in. long. Find the area of the piece of glass in square inches. 3

THINK The base is ■ in. long. The altitude is ■ in. long.

B The pages in a book are 7 in. wide and 9 in. high. Find the area of a page.

THINK The base is 7 in. What is the altitude?

C A piece of drawing paper is 8 in. wide and 11 in. long. What is its area in square inches?

THINK If you use 11 in. for the base, what do you use for the altitude?

D The length of a piece of cardboard is 29 in., and the width is 14 in. What is the area of the piece of cardboard in square inches?

E A table top is 22 in. long and 17 in. wide. Find the area of the table top in square inches.

F A puzzle is a rectangle 12 in. long and 10 in. wide. What is its area in square inches?

G A side of a square piece of glass is 20 in. long. Find its area in square inches.

H A picture is 8 in. square. Find the area of a piece of glass that will cover the picture.

I A piece of glass in a window is 36 in. wide and 13 in. high. What is the area of this piece of glass?

J The width of a page in a magazine is 9 in., and the height is 12 in. What is the area of the page in square inches?

K What is the area of the top of a box that is 26 in. long and 17 in. wide?

Now you should know how to solve problems about the areas of rectangles.

243

1 Be sure pupils understand how the equation is formed and that they can answer the questions about it.

2 After the pupils have read the text and answered the questions, ask: "What does the 32 mean? How do we know that 32 is the number we want?" Pupils can go back to Picture J and count the squares to verify the computation.

3 Have the pupils read Problem L. Ask a pupil to make a 5-inch square on the chalkboard and then show by drawing inch squares that there are 5 square inches in one row. Ask another pupil to tell how the n and the 5 are used in $n/5$. Then have the pupils read the sentences and supply the missing numerals.

244

1 Discuss Rectangle A with the class. If any of the pupils have difficulty making the first ratio (8/1), tell them to visualize a row along the base marked off into 8 one-inch squares and then to think of the rate as 8 square inches per 1 row. If they have trouble making the second ratio, have them imagine the height of the rectangle as marked off into 5 equal parts. Then they can see that there are 5 rows of square inches. The rate can also be expressed as n square inches per 5 rows. Next put the equation on the board and let a pupil tell how to find the number that n represents.

2 Let the pupils work independently to find the areas of Rectangles B to H. Instruct them to write an equation for each, do the necessary computation, and rewrite the equation with the correct numeral in place of n. Then they should write the answer as "12 square inches."

3 Follow the procedure suggested in Note 2 above for Problems A to K.

245-248 Moving forward

Exponded Notes for this lesson are on pages 347-349.

Objectives

The child learns to think of the square foot and square yard as units of square measure. He also learns how to reduce units of square measure.

Vocabulary

New words page 247 floor*

Comments

The "Charting the Course" section (page 345) and the "Comments" for the preceding lesson indicate why area is taught in this book as an application of rate. In the previous lesson on computing area (pages 240-244), the square inch was used exclusively as the measuring unit of the areas found. By doing this, certain complexities were avoided, and major general principles were established in the simplest way possible.

Now, however, it is desirable to extend the computation of area to include the ideas of the square foot and the square yard. And, since in real-life situations it is frequently necessary to change from one unit of square measure to another, upward and downward reduction of square measures is included in this lesson. Since such reduction depends to some extent upon a knowledge of square measure equivalents, the idea of the square foot is introduced in terms of its area in square inches (see page 245). The square yard is introduced in terms of its area in square feet (see page 246). The child thus learns a meaningful computational way to find these equivalents and is not forced to depend upon his memory alone.

With the relationships established among square inches, square feet, and square yards, computational reduction is now possible. This is illustrated and explained on page 247. Practice on finding areas and on reduction of square measures is provided on page 248.

Answers

Pages 247-248:

(block 1)

I 4/1=[8]/2

J 144; 144/1=[1152]/8; because 8 is 8 times 1

K 9/1=[567]/63; number of square feet per 1 square yard; total area in square yards

L 5; 9; 5/1=[45]/9

M Yes; yes; number of square yards per 45 sq. ft.; 9/1=45/[5]

N 42 sq. yd.; 378 sq. ft.; number of square feet per 1 sq. yd.; by multiplying 1 by 42 because 9 can be multiplied by 42 to get 378

O 18/1=[288]/16; 144; 144/1=288/[2]; because 288 is 2 times 144

P 36; 36/1=[864]/24; 864 sq. in.

Q 144/1=864/[6] or 3/1=[6]/2; 6 sq. yd.

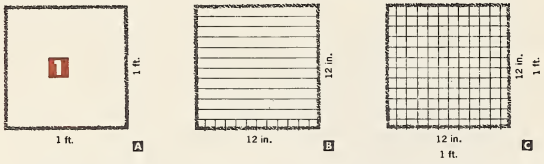
R [B]9/1=[27]/3, 27 sq. ft. or 3/1=[3]/1, 3 sq. yd.
[C] 72/1=[1728]/24, 1728 sq. in. or 6/1=[12]/2, 12 sq. yd.
[D] 24/1=[288]/12, 288 sq. in.
[E] 36/1=[432]/12, 432 sq. in. or 3/1=[3]/1, 3 sq. ft.

(block 2)

A 2448 sq. in.
B 9 sq. yd.
C 3 sq. ft.
D 99 sq. ft.
E 5 sq. ft.
F 1152 sq. in.
G 14 sq. yd.
H 162 sq. ft.
I 19 sq. yd.
J 1872 sq. in.

Moving forward

In this lesson you will learn how to measure area in square feet and square yards.



1 Diagram A represents a square piece of cardboard. Each side of the square is ▩ ft. The area of the cardboard is 1 square foot. The work below shows how to find the area of the cardboard in square inches.

2 A Why can you think of each dimension of the cardboard square as 12 in.? Look at Diagram B. If you mark off the cardboard in square inches, there will be ▩ sq. in. in one row. This is a rate of 12 sq. in. per 1 row.

$\frac{12}{1}$ ← What does 12 represent?
← What does 1 represent?

D How do you know that there will be 12 rows? Now the rate can also be expressed as n square inches per 12 rows.

3 What will the numeral that $\frac{n}{12}$ ← replaces n represent?
← What does this 12 represent?

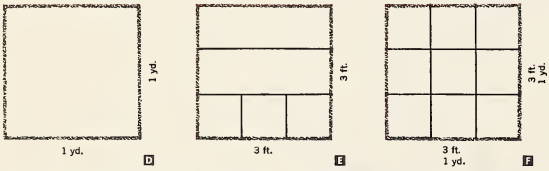
C Look at Diagram C. Square inches will be used at the same rate for 12 rows as for 1 row.

4 $\frac{12}{1} = \frac{n}{12}$ ← When you find the numeral that replaces n, you will know how many square inches there are in 12 rows.

Why do you multiply 12 by 12 to find the numeral that replaces n?

The area of the piece of cardboard is ▩ sq. in.

1 square foot = 144 square inches



1 Diagram D represents another square of cardboard. The length of each side of this square is 1 yd. The area is 1 square yard. The work below shows how to find the area in square feet.

2 D Look at Diagram E. In one row there are ▩ sq. ft. The rate is ▩ sq. ft. per 1 row.

3 E How many rows are there? The rate can be expressed now as n sq. ft. per ▩ rows.

F Now look at Diagram F. Square feet will be used at the same rate for the 3 rows as for 1 row.

$\frac{3}{1} = \frac{n}{3}$ ← When you find the numeral that replaces n, you will know how many square feet there are in 3 rows.

Why do you multiply 3 by 3 to find the numeral that replaces n?

1 square yard = 9 square feet

4 G Diagram G on page 247 represents a large piece of glass in the shape of a rectangle. Its dimensions are ▩ yd. and ▩ yd. You can express its area as 30 sq. yd. How do you get the 30?

5 H You can also express the area of the glass in square feet. To do this, change the 30 sq. yd. to square feet.

6 The rate of square feet per square yards is always the same. The rate of square feet for 1 sq. yd. is expressed as 9 per 1. Look at Diagram H. The rate of square feet for 30 sq. yd. can be expressed as n per 30.

$\frac{9}{1} = \frac{n}{30}$ ← When you find the numeral that replaces n, you will know how many square feet there are in 30 sq. yd.

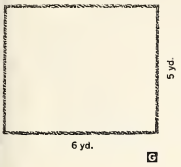
To find the numeral that replaces n, multiply 9 by ▩. Why do you multiply by 30? 30 sq. yd. = 270 sq. ft.

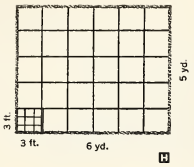
245

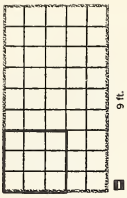
1. Have the pupils examine Picture A and read and discuss the statements immediately below the picture that define 1 square foot as the area of the foot square.
2. Have the pupils read and discuss this exercise in relation to Picture B. Be sure they can explain what the 12 and the 1 represent in the ratio 12/1.
3. Be sure the children understand why the ratio $n/12$ can be used to express the rate for all the rows.
4. Emphasize that we know the ratios 12/1 and $n/12$ are equal because they are different expressions of the same rate. When the children have found the numeral that replaces n, develop the idea that now they know how many square inches are equal to 1 sq. ft.

246

1. Before these statements are read, you may wish to show a paper model (or a full-scale drawing) of an actual 1-yd. square to help the children realize that Picture D is a scale drawing of 1 sq. yd.
2. Make clear that 1 sq. ft. is the unit of square measure now being used.
3. Demonstrate on the square-yard model that square feet are used at the same rate in each row. Be sure the pupils understand that the ratios 3/1 and $n/3$ express the same rate. Point out that the numeral that replaces n shows how many square feet are equal to 1 sq. yd.
4. Have the children write the equation and find the area of Diagram G (on page 247).
5. Point out that to change 30 sq. yd. to square feet, we think of the number of square feet per 1 sq. yd. Show this on the model.
6. Emphasize that the number of square feet for the 30 square yards can be expressed as $n/30$.







I A table top is 4 ft. long and 2 ft. wide. Find its area in square feet.

J What is the area in square inches? In 1 sq. ft. there are ▩ sq. in.

$\frac{144}{1}=\frac{n}{8}$ **2**

Why do you multiply 144 by 8 to find the numeral that replaces n?

K A floor is 63 sq. yd. in area. Find its area in square feet.

$\frac{9}{1}=\frac{n}{63}$

What does the 9 represent? What does the 63 represent?

L Diagram I represents a door. The dimensions of the door are ▩ ft. and ▩ ft. The area can be expressed as 45 sq. ft. How do you get the 45?

M You can also express the area of the door in square yards. To do this, change the 45 sq. ft. to square yards.

Does 9 per 1 express the rate of square feet per square yards? There are 45 sq. ft. in the door. Does 45 per n also express the rate of square feet per square yards? What does n represent?

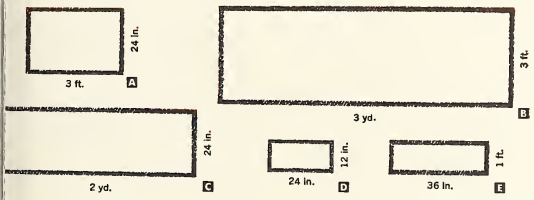
$\frac{9}{1}=\frac{45}{n}$ **6** When you find the numeral that replaces n, you will know how many square yards there are in 45 sq. ft.

Why do you multiply 1 by 5 to find the numeral that replaces n?

N A wall is 27 ft. long and 14 ft. high. What is its area in square yards? **7**

$\frac{9}{1}=\frac{378}{n}$

What is the area of the wall in square feet? What does the 9 represent? How do you find the numeral that replaces n?



O The dimensions of a picture are 18 in. and 16 in. Find the area of the picture in square feet.

First find the area in square inches. This area is 288 sq. in. How do you get the 288?

Next you change 288 square inches to square feet. ▩ sq. in. will be used for each square foot.

$\frac{144}{1}=\frac{288}{n}$

Why do you multiply 1 by 2 to find the numeral that replaces n?

P Look at Diagram A. The length of the base is shown in feet, and the length of the altitude is shown in inches. Find the area in square inches. Change 3 ft. to inches. Then what do you do?

Q You can also find the area of Rectangle A in square feet. To do this, you can change 864 square inches to square feet. Or you can first change 24 in. to feet. Then what do you do? **3**

R Find the area of each of the other rectangles above. You may have to change one dimension before you make your equation. **4**

A 17 sq. ft. = ▩ sq. in.
B 81 sq. ft. = ▩ sq. yd.
C 432 sq. in. = ▩ sq. ft.
D 11 sq. yd. = ▩ sq. ft.
E 720 sq. in. = ▩ sq. ft.
F 8 sq. ft. = ▩ sq. in.
G 126 sq. ft. = ▩ sq. yd.
H 18 sq. yd. = ▩ sq. ft.
I 171 sq. ft. = ▩ sq. yd.
J 13 sq. ft. = ▩ sq. in. **5**

Now you know how to measure areas in square inches, square feet, and square yards and how to change square measures when you need to.

247

1 Have the pupils write the proper equation and find the numeral that replaces n.
2 Get the children to explain why 144 and 8 are used in the ratios 144/1 and n/8. Be sure they interpret the answer as meaning square inches.
3 Get the pupils to see that this is a type of reduction they have been studying. Be sure they can explain the equation and can interpret their answer.
4 Tell the children to write the necessary equation and find the numeral that replaces n.
5 Have them observe Diagram I and note that 9 of the square feet are used per 1 sq. yd.
6 Help the pupils understand that since n represents the unknown number of square yards, it must be in the same position as the 1 [for 1 sq. yd.]. Ask someone to tell the area of the door in square yards.
7 Help the pupils see that they are to change to a larger unit of square measure. Have them find the area in square yards.

248

1 In this exercise, be sure the pupils understand that 144 is used because square inches are used at the rate of 144 sq. in. per 1 sq. ft. and interpret the answer as meaning sq. ft.
2 Diagram A illustrates how dimensions can be given in different units of measure, one of which must be changed before the area can be computed. The children should write the proper equation and find the area in sq. in.
3 Get the pupils to observe the different ways of using the data they have to find the area of Rectangle A in square feet. Have them write the equation suggested by each way and observe the results. They should note that it is much simpler first to change 24 in. to feet.
4 Observe the children closely as they find these areas, and assist any child who needs help. Drawing diagrams may help some children.
5 Most of the children should be able to work independently on these exercises, but continue to give help wherever it is needed.

249-250 Using arithmetic

Expanded Notes are not considered necessary for this lesson.

Objectives

The child learns what an acre and a square mile are and how to use these measures in problem solving.

Vocabulary

New words page 249 land*, acre*; page 250 section

Comments

It is useful for anyone to be able to recognize, in a rough way, what size a piece of land is—about an acre, very much more than an acre, or considerably less than an acre. The best way for the child to acquire judgment about the size of an acre is to become familiar with the size of pieces of land (measured in acres) in his immediate environment—the amount of land occupied by the school buildings and grounds, for example, or a nearby playground, or his own yard. Most property owners know what their acreage is (lots occupied by one-family houses are usually well under an acre), and town officials can supply the dimensions or area of publicly owned property, such as parks or land occupied by public buildings. Able pupils might like to pace off the school yard and compute the area.

Children should realize that an acre does not have a particular shape, but that it may take various shapes.

Answers

(block 1)

A 209; 209/1=[43,681]/209, 43,681 sq. ft.

B More; yes

C 100 yd.; 160 ft.; (300)/1=[48,000]/160, 48,000 sq. ft.

D 48,000−43,560=[4440], 4440 sq. ft. more

E 160 ft.; yes; 272/1=[43,520]/160, 43,560−43,520=[40], 40 sq. ft. less

F Yes; 75/1=[13,500]/180. Each lot is 13,500 sq. ft. 3×13,500 sq. ft. is 40,500 sq. ft., which is just a little less than 1 acre.

G —

H 660/1=[435,600]/660, 435,600 sq. ft.

I 43,560/1=[435,600]/10, 435,600 sq. ft., exactly 10 A.

J 2640 ft.; less; 5280−2640=[2640], 2640 ft., or $\frac{1}{2}$ mile

K 1; 1; 640 A.

L 640 A.

M 640÷[160]=4, 160 A.

N 4 mi.

O 640/1=[3840]/(6), 3840 A.

(block 2)

A 325/1=[58,500]/180, 58,500 sq. ft.; more

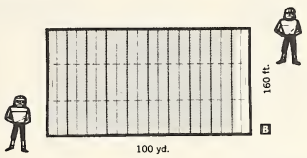
B 260/1=[44,200]/170, 44,200 sq. ft.; more

C (600)/1=[42,000]/70, 42,000 sq. ft.; less

D (210)/1=[18,270]/(87), 18,270 sq. ft.; less

E (336)/1=[50,400]/150, 50,400 sq. ft.; more

Using arithmetic



A Diagram A represents a square piece of land. Each side of the square is ▒ ft. long. Find its area.

B Land is usually measured in acres. An acre is 43,560 sq. ft. Is the piece of land in Diagram A more than or less than 1 acre? Should you say that it is approximately 1 acre?

C Diagram B represents the playing part of a football field. How long is it? How wide is it? What is its area?

D "A." is the abbreviation for "acre." The area of the football field is ▒ sq. ft. more than 1 A.

E The part of the playing field that is green in the picture is 272 ft. long. How wide is it? Is the green part approximately 1 acre? It is how much more or how much less than 1 acre?

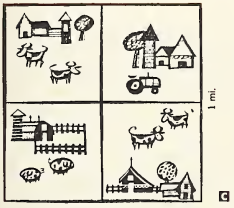
F Mr. West is building a house on a lot that is 75 ft. wide and 180 ft. long. Would 3 lots of this size be about 1 A.?

G Find the approximate dimensions of your school yard. What is its approximate area? Is it more or less than 1 acre? It is about how much more or less than 1 acre?

H Each block in one large city is a square that is 660 ft. on a side. What is the area of each block?

I How many square feet are there in 10 A.? Is the city block described in Problem H more than, less than, or exactly 10 A.?

J Find the perimeter of the city block in Problem H. Is the perimeter more than or less than 1 mi.? It is how much more than or less than 1 mi.?



1 square mile = 640 acres

K Diagram C represents a square piece of land. Each side is ▒ mi. long. The area of this piece of land is ▒ sq. mi., or ▒ A.

L A square mile of land may also be called a section. How many acres are in a section?

M The 4 farms shown on Diagram C are equal in size. How many acres are there in each farm?

N Find the perimeter of the section of land in Diagram C.

O A large farm is in the shape of a rectangle 3 mi. long and 2 mi. wide. Find the area of the farm in acres.

The dimensions of five fields that are in the shape of rectangles are given below. Find the area of each field. Is the area of each field more than or less than 1 acre?

- **A** 325 ft. long; 180 ft. wide
- **B** 260 ft. long; 170 ft. wide
- **C** 200 yd. long; 70 ft. wide
- **D** 70 yd. long; 29 yd. wide
- **E** 112 yd. long; 150 ft. wide

Thinking straight

A Picture A on page 251 shows a square whose sides are 1 inch in length. What is the area of this square?

B In Picture B, the inch square is being cut into four strips that are equal in width. What will be the dimensions of each strip?

C Is the total area of the four strips 1 sq. in.? How do you know?

D In Picture C the strips from Picture B have been arranged to form a polygon. What kind of polygon is this? Is the area of Polygon C the same as the area of Square A? What are the dimensions of Polygon C?

E What kind of polygon do the strips form in Picture D? What is the area of Polygon D? What are the dimensions of this polygon?

249

1 Discuss Exercise A and Diagram A with the class. Have a pupil write the equation on the board and tell how to find the numeral that replaces *n*.

2 Pupils should understand that the acre is a unit of square measure and is used only in connection with area. Ask: "Could we speak of a farm as being 10 acres long? Why not?"

3 Exercises C, D, and E should be read and discussed in connection with Diagram B. Pupils who have seen an official football field will be interested to know that the field is slightly larger than an acre and can use this knowledge to help them visualize 1 acre.

4 Compute the area of the 3 lots on the board, following directions given by members of the class. Then let a pupil answer the question.

5 Able pupils might be assigned to this task. Their findings should be discussed in class.

6 Let pupils work Problems H, I, and J independently and then compare their answers.

250

1 Discuss Exercises K and L in connection with Diagram C. Ask: "Do you think a square mile always has the shape of a square? Why or why not?"

2 Let pupils work Problems M, N, and O independently; then discuss the equations and the answers.

3 Problems A to E should be assigned as written work. Remind the class that they will have to change yards to feet before they make their equations for some of the problems. Ask why. Provide answers so that pupils may verify their work.

250-251 Thinking straight

Expanded Notes are not considered necessary for this lesson.

Objectives

The child learns that a square inch is not necessarily an inch square, but may have various shapes.

Vocabulary

New words page 250 whose*

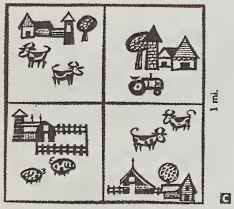
Comments

Because it is convenient to use the inch square in developing the concept of area, there is always the danger that the child will believe that a square inch must be an inch square. This lesson has been designed to show that a square inch need not be square shaped.

Provide each child with several 1-inch squares of different colored paper. Have each child cut up and rearrange each square into a different shaped polygon. This activity will supplement the pictures on page 251, since only a few of many possibilities have been illustrated. The squares need not be cut into equal parts, or even into parts whose edges form straight lines. The parts of each cut-up square, however, must be rearranged into a shape having no gaps showing between adjoining pieces. It is important for pupils to understand that no matter how irregular a shape the combined pieces of one of the original squares may make, the area will still be 1 square inch.

Answers

A 1 sq. in.
B 1 in. long, $\frac{1}{4}$ in. wide
C Yes; Nothing has been added or taken away.
D Rectangle; yes; 4 in. by $\frac{1}{4}$ in.
E Rectangle; 1 sq. in.; 2 in. by $\frac{1}{2}$ in.
F Six; yes; Side a, $\frac{3}{4}$ in.; Side b, $1\frac{1}{4}$ in.; Sides c and f, 1 in.; Sides d and e, $\frac{1}{4}$ in.
G Six; 1 sq. in.; Sides a, c, and d, $\frac{1}{2}$ in.; Sides b and e, 1 in.; Side f, $1\frac{1}{2}$ in.
H Triangle
I Triangle; 1 sq. in.; 2 in.; no; 1 in.
J Six; 1 sq. in.; $\frac{1}{2}$ sq. in.
K 1 sq. in.
L Yes; yes



1 square mile = 640 acres

K Diagram C represents a square piece of land. Each side is ▨ mi. long. The area of this piece of land is ▨ sq. mi., or ▨ A.

L A square mile of land may also be called a section. How many acres are in a section?

M The 4 farms shown on Diagram C are equal in size. How many acres are there in each farm?

N Find the perimeter of the section of land in Diagram C.

O A large farm is in the shape of a rectangle 3 mi. long and 2 mi. wide. Find the area of the farm in acres.

The dimensions of five fields that are in the shape of rectangles are given below. Find the area of each field. Is the area of each field more than or less than 1 acre?

A 325 ft. long; 180 ft. wide
B 260 ft. long; 170 ft. wide
C 200 yd. long; 70 ft. wide
D 70 yd. long; 29 yd. wide
E 112 yd. long; 150 ft. wide

Thinking straight

A Picture A on page 251 shows a square whose sides are 1 inch in length. What is the area of this square? 1

B In Picture B, the inch square is being cut into four strips that are equal in width. What will be the dimensions of each strip? 2

C Is the total area of the four strips 1 sq. in.? How do you know?

D In Picture C the strips from Picture B have been arranged to form a polygon. What kind of polygon is this? Is the area of Polygon C the same as the area of Square A? What are the dimensions of Polygon C? 3

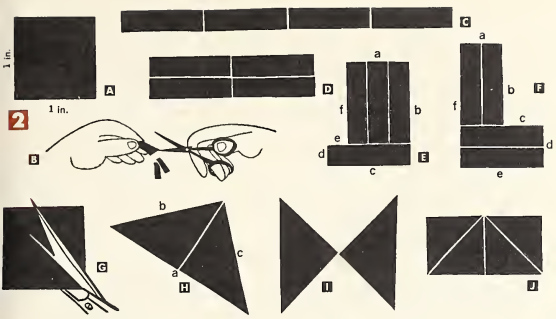
E What kind of polygon do the strips form in Picture D? What is the area of Polygon D? What are the dimensions of this polygon?

250

1 Have pupils read Exercise A on page 250 in connection with Picture A on page 251.

2 Call on pupils to answer the questions in Exercises B and C.

3 In Pictures C and D, the strips should be thought of as making an unbroken shape. The pupils should use Pictures C and D to answer the questions in Exercises D and E. They should be able to give the dimensions of the polygons without measuring the sides, but you may let them use their rulers to verify their answers.



1 F In Picture E the strips have been arranged in another way. How many sides does this polygon have? Is the area still 1 sq. in.? What is the length of each side of this polygon?

G How many sides has Polygon F? What is its area? What is the length of each side?

H Picture G shows an inch square that is being cut into halves. What kind of polygon will each half be?

I The polygon in Picture H has been formed from the two pieces shown in Picture G. What kind of polygon is it? What is its area? How long is side a? Can you tell how long sides b and c are without measuring them? Line d is the altitude. What is its length? 3

J Polygon I has also been formed from the two pieces of the inch square in Picture G. How many sides does Polygon I have? What is its area? What is the area of each triangle?

K Polygon J was formed by putting together 4 equal triangles. These triangles were made by cutting the two triangles in Polygon I into halves. What is the area of Polygon J? 4

L Can a triangle have an area of 1 sq. in.? Can two rectangles that do not have the same dimensions each have an area of 1 sq. in.?

251

251

1 The exercises on this page should be read and discussed in class in connection with Pictures E to J.

2 In all the pictures, the pieces should be thought of as making unbroken shapes. Pupils will be interested to learn that Pictures E and F are hexagons, because they have 6 straight sides and 6 angles. Pupils are used to thinking of the hexagon in terms of the regular hexagon.

3 Ask someone to explain what the altitude of a triangle is. Be sure pupils understand that Line *d* forms right angles with Side *a* of the triangle, which may be taken as the base.

4 Some pupils will ask whether or not Polygon J is a rectangle. It is, although the pupils are not expected to know that all four "corners" are right angles.

252-253 Moving forward

Exponded Notes for this lesson are on page 349.

Objectives

The child learns that certain rates are customarily expressed as per cents.

Vocabulary

New words page 252 sprouted

Comments

Some of the difficulties the child has had with per cent in the past have arisen because he has thought of per cents as whole numbers or as fractions instead of as number pairs that express rates. It is important for the child to learn from the outset that per cent is a rate. The confusion between per cents and fraction numerals will disappear if the child understands, for example, that the 3 of the symbol 3% expresses a rate in which the second term of the ratio is 100.

Draw attention to the fact that in the table at the bottom of page 252, we first write ratios to express the information in the pictures (Column A). Each ratio represents the rate at which one group of beans sprouted. In Column B we write equal ratios with 100 as the second term. Be sure the children understand that the ratios in Column B express the same rates as the ratios in Column A. For example, 72 per 100 is the same rate as 18 per 25. In Column D the child is shown how to read the ratios by substituting the word "cent" for 100. In Column E the child is shown how to write per cents by using the per cent sign.

Answers

Pages 252-253:

(block 1)

A John had 25, Paul 20, Bill 10. Of sprouted beans, John had 18, Paul 15, and Bill 9.

B John; Bill

C 18/25; 15/20; 9/10

D 72/100; 75/100; 90/100

E The three rates can easily be compared because each ratio has 100 as its second term. 90/100 shows

a higher rate than 72/100 or 75/100. John's beans sprouted at the lowest rate.

F —

G John, 72 per cent; Paul, 75 per cent; Bill, 90 per cent

H Bill; John

(block 2)

A 25%	C 3%	E 16/100	G 86/100
B 14%	D 99%	F 2/100	H 39/100

(block 3)

Accept any correct ratios.

A Another possibility: 50% = 10/20 = 2/4 = 1/2.

B 18/100 = 9/50

C 25/100 = 5/20 = 1/4

D 44/100 = 22/50 = 11/25

E 75/100 = 15/20 = 3/4

F 40/100 = 20/50 = 10/25 = 2/5 or 40/100 = 4/10 = 2/5

G 12/100 = 6/50 = 3/25

H 60/100 = 12/20 = 6/10 = 3/5 or 60/100 = 30/50 = 15/25 = 3/5

I 35/100 = 7/20

J 80/100 = 40/50 = 20/25 = 4/5 or 80/100 = 8/10 = 4/5

K 90/100 = 45/50 = 9/10

L 65/100 = 13/20

M No; no

N No; yes; yes

O Yes

(block 4)

The higher rate is shown by the larger per cent.

A 50%, 60%	D 50%, 60%	G 28%, 90%
B 40%, 75%	E 15%, 16%	H 55%, 38%
C 50%, 30%	F 70%, 75%	I 95%, 64%

Keeping skillful:

(block 1)

A $4\frac{3}{8}$	G $18\frac{2}{30}$ or $18\frac{1}{15}$	
B 1	H $45\frac{11}{24}$	
C $2\frac{3}{10}$	I $8\frac{4}{5}$	
D $80\frac{1}{12}$	J $11\frac{7}{24}$	
E $124\frac{1}{2}$	K $26\frac{1}{3}$	
F $5\frac{1}{2}$	L $55\frac{2}{3}$	

(block 2)

A $10\frac{7}{24}$	
B $11\frac{2}{12}$ or $11\frac{1}{6}$	
C $13\frac{13}{20}$	
D 23	
E $11\frac{5}{12}$	
F $17\frac{1}{24}$	

Moving forward

Now you will study a new way to express rates.

1 A Look at the pictures above. Each boy had how many beans? How many of each boy's beans sprouted?

B Which boy had the most sprouted beans? Which boy had the fewest?

2 C You can show the rate at which each boy's beans sprouted by writing a ratio. Look at Column A below. What is the rate for John's beans? For Paul's beans? For Bill's beans?

3 D To find which boy had beans that sprouted at the highest rate, you can express each rate with a ratio that shows how many per 100. Look at Column B. What is the rate per 100 for John's beans? For Paul's beans? For Bill's beans?

4 E How does Column B help you decide that Bill's beans sprouted at the highest rate? Which boy had beans that sprouted at the lowest rate?

5 F Column C shows how to read the ratios that express how many per 100. Read these ratios.

6 G Column D shows another way to say each rate. When a rate is expressed as so many per 100, the rate is called a per cent. Per cent means per hundred. What per cent of each boy's beans sprouted?

	A	B	C	D	E
John	$\frac{18}{25}$	$\frac{72}{100}$	72 per 100	72 per cent	72%
Paul	$\frac{15}{20}$	$\frac{75}{100}$	75 per 100	75 per cent	75%
Bill	$\frac{9}{10}$	$\frac{90}{100}$	90 per 100	90 per cent	90%

252

1 The pupils should use Pictures A, B, and C to answer the questions in Exercises A and B.

2 Exercise C should be read and discussed. The pupils should read the rates in Column A as 18 per 25, 15 per 20, and 9 per 10.

3 In connection with discussion of Exercise D, call on someone to explain how he knows that the ratio 72/100 is equal to 18/25. Make sure the pupils understand that each ratio in Column B is equal to a ratio in Column A and expresses the same rate.

4 Be sure the pupils understand that it is now possible and easy to compare the rates of sprouting because the comparison is made on the basis of each boy's having 100 beans.

5 The children should understand that Column C merely shows how to read the ratios in Column B.

6 Establish the idea that the word "cent" merely takes the place of "100" in Column C.

H The sign at the right is called the per cent sign. You read it "per cent." Column E on page 252 shows how to write the ratios with the per cent sign. Which boy had the highest per cent of sprouted beans? Which boy had the lowest per cent? 0/0 1

Write each of the ratios below, using the per cent sign. 2

A $\frac{25}{100}$ B $\frac{14}{100}$ C $\frac{3}{100}$ D $\frac{99}{100}$

Write each of the following as a ratio without using the per cent sign.

E 16% F 2% G 86% H 39%

For each rate given at the top of the next column, write one or more equal ratios without using the per cent sign. The work you might write for 50% is shown below. 3

A 50% = $\frac{50}{100} = \frac{25}{50} = \frac{5}{10} = \frac{1}{2}$

A 50% B 18% C 25% D 44% E 75% F 40% G 12% H 60% I 35% J 80% K 90% L 65%

M Jim said that he spelled 90% of the words in a test correctly. Do you know how many words were in the test? Do you know how many words he missed? 4

N Did there have to be 100 words in the test? Could there have been 50 words? Could there have been 10?

O Can you say that Jim spelled 9 out of 10 words correctly?

Express the ratios below as per cents, and for each pair decide which ratio shows the higher rate. 5

A $\frac{2}{4}, \frac{3}{5}$ D $\frac{1}{2}, \frac{3}{5}$ G $\frac{7}{25}, \frac{9}{10}$

B $\frac{2}{5}, \frac{3}{4}$ E $\frac{3}{20}, \frac{4}{25}$ H $\frac{11}{20}, \frac{19}{50}$

C $\frac{1}{2}, \frac{3}{10}$ F $\frac{7}{10}, \frac{3}{4}$ I $\frac{19}{20}, \frac{16}{25}$

Now you should understand that a per cent is a way of expressing a rate so that it shows how many per 100.

Keeping skillful 6

A $n + 11\frac{1}{8} = 16\frac{1}{4}$
B $13\frac{2}{3} - 12\frac{5}{8} = n$
C $8\frac{7}{10} + n = 11$
D $60\frac{1}{4} + 19\frac{5}{8} = n$
E $n - 44 = 80\frac{1}{2}$
F $18 - n = 12\frac{1}{2}$
G $46\frac{9}{10} - 28\frac{2}{5} = n$
H $11\frac{5}{8} + 33\frac{2}{8} = n$
I $27 + n = 35\frac{4}{8}$
J $60\frac{1}{8} - n = 48\frac{7}{8}$
K $n + 14\frac{2}{3} = 41$
L $n - 14\frac{2}{3} = 41$
A $1\frac{5}{8} + 7\frac{3}{4} + 1\frac{1}{2} = n$
B $4\frac{3}{4} + \frac{7}{12} + 5\frac{5}{6} = n$
C $8\frac{1}{5} + 1\frac{7}{10} + 3\frac{3}{4} = n$
D $9\frac{1}{2} + 10 + 3\frac{1}{2} = n$
E $6\frac{3}{5} + \frac{7}{12} + 4\frac{1}{2} = n$
F $5\frac{3}{8} + 6\frac{1}{2} + 5\frac{1}{8} = n$

253

1 Be sure the pupils understand that 72% is just a special way of writing 72 per 100.

2 The pupils should work Exercises A to H independently.

3 Before having the pupils write the equal ratios for the per cents in Exercises A to L, discuss the example shown in handwriting. Make sure everyone understands that at the moment these pairs of numerals are not to be considered fraction numerals, but ratios, and hence to be read as "50 per 100," "25 per 50," etc.

4 Let pupils take turns reading Exercises M, N, and O and answering the questions.

5 Assign these exercises as written work. Supply answers so that the pupils can verify their work.

6 Assign these exercises as written work.

254 Checking up; Keeping skillful

Expanded Notes are not considered necessary for these lessons.

Objectives

The child reviews his knowledge of finding areas and changing units of square measure.

Vocabulary

There are no new words.

Comments

Use the "Checking up" tests to discover how well the pupils understand terminology employed in giving dimensions, finding areas, and reducing square measures. If Tests 1 and 2 reveal weakness, review the lessons on pages 240-248. If Test 3 causes trouble, review the lessons on pages 245-250.

Answers

Test 1

A 576 sq. in.
B 864 sq. in.
C 16 sq. ft.
D 405 sq. ft. or 45 sq. yd.
E 6 sq. mi.

Test 2

A 294 sq. in.
B 225 sq. ft.
C 405 sq. ft. or 45 sq. yd.
D 64 sq. mi.
E 1584 sq. in.
F 8100 sq. ft. or 900 sq. yd.

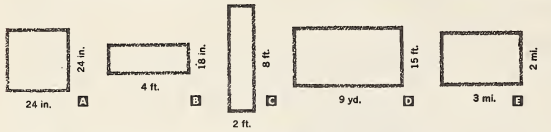
Test 3

A 1008 sq. in. D 1920 A. G 4480 A. J 14 sq. yd.
B 4 sq. yd. E 72 sq. ft. H 5 sq. ft. K 1728 sq. in.
C 4 sq. ft. F 3 sq. mi. I 99 sq. ft.

Keeping skillful:

(block 1)	(block 2)	(block 3)
A $3\frac{3}{4}$	A $85\frac{24}{66}$ or $85\frac{4}{11}$	A 51,427
B 64	B 17	B 216,716
C 9	C $314\frac{76}{89}$	C $34\frac{56}{72}$ or $34\frac{7}{9}$
D $8\frac{1}{24}$	D 602	D 7658
E $6\frac{1}{4}$	E 827	E 210,910
F $18\frac{1}{2}$	F 87	F 11,578
G 2	G $63\frac{95}{110}$ or $63\frac{19}{22}$	G 50
H $10\frac{11}{16}$	H $6\frac{314}{847}$	H 156
	I 53	I 147,780

Checking up



Test 1 **1**
Find the area of each rectangle above.

Test 2
2 Find the area of each rectangle whose dimensions are given below.

A base, 21 in.; altitude, 14 in.
B 15 ft. square
C 27 ft. long; 5 yd. wide
D 8 mi. square
E 66 in. long; 2 ft. high
F depth, 36 yd.; width, 75 ft.

Test 3
3

A 7 sq. ft. = ▒ sq. in.
B 36 sq. ft. = ▒ sq. yd.
C 576 sq. in. = ▒ sq. ft.
D 3 sq. mi. = ▒ A.
E 8 sq. yd. = ▒ sq. ft.
F 1920 A. = ▒ sq. mi.
G 7 sq. mi. = ▒ A.
H 720 sq. in. = ▒ sq. ft.
I 11 sq. yd. = ▒ sq. ft.
J 126 sq. ft. = ▒ sq. yd.
K 12 sq. ft. = ▒ sq. in.

Keeping skillful

4

A $12\frac{1}{4} - n = 8\frac{1}{2}$
B $46\frac{8}{10} + 17\frac{7}{10} = n$
C $10\frac{5}{8} - 1\frac{5}{8} = n$
D $n + 7\frac{7}{8} = 15\frac{1}{12}$
E $21\frac{3}{4} + n = 28$
F $n - 5\frac{1}{2} = 13$
G $\frac{4}{5} + \frac{3}{10} + \frac{5}{8} + \frac{1}{10} = n$
H $15\frac{7}{16} - 4\frac{3}{4} = n$

5

A $5634 \div 66 = n$
B $7446 \div n = 438$
C $28022 \div 89 = n$
D $28896 \div n = 48$
E $344032 \div 416 = n$
F $8265 \div n = 95$
G $7025 \div 110 = n$
H $5396 \div 847 = n$
I $5194 \div n = 98$

6

A $46381 + 5046 = n$
B $68 \times 3187 = n$
C $2504 \div 72 = n$
D $17006 - 9348 = n$
E $230 \times 917 = n$
F $80001 - 68423 = n$
G $35850 \div 717 = n$
H $74 + 19 + 23 + 40 = n$
I $60 \times 2463 = n$

254 End-of-block tests on square measure

254

1 Pupils should work independently, make equations, and find the area of each rectangle. They should label their answer in terms of the square unit in which the area was found. Remind them that when the dimensions are in different units they must change one of the dimensions before computing. Discuss the various ways in which areas were found and stated. [For example, the area of Rectangle D can be stated as 45 sq. yd. or as 405 sq. ft.]

2 Again the pupils should write equations, compute, and label the answers.

3 The pupils should write the equations and compute to find the answers.

4 Have the children find the numeral that replaces *n* for each equation.

5 Tell the children to use remainders to form fractions. Because of the large numbers, you should not require the children to reduce the fraction numerals to simplest form.

6 Direct the pupils to find the answers.

255-256 Moving forward

Expanded Notes are not considered necessary for this lesson.

Objectives

The child extends his knowledge of the number system by learning to use the decimal notation through tenths and hundredths.

Vocabulary

New words page 255 rain*, decimal

Comments

There is no intention of treating decimal notation exhaustively here. This lesson is merely an introduction to decimal fraction numerals, and it is included so that the child will be equipped with knowledge he may need to understand decimals properly in his other school work or as he meets them in activities outside of class.

Answers

Page 256:

G 1.5; 1.9 H .2 in. I .02 in. J .1 in. K .11 in.

(block 2)

A 3 tenths
B 6 hundredths
C 5 and 4 tenths
D 2 tenths
E 1 tenth
F 2 and 6 tenths
G 9 tenths
H 88 hundredths
I 6 and 7 tenths
J 18 hundredths
K 1 hundredth
L 4 and 63 hundredths
M 9 and 4 hundredths
N 75 hundredths
O 2 and 36 hundredths
P 7 hundredths

(block 3)

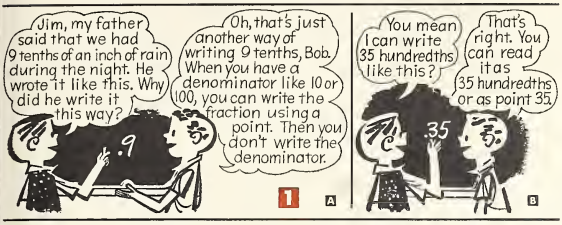
A 3.5
B .48
C .25
D 98.6
E .35
F .65
G 2.9
H 6.75
I .02
J 3.4
K .95
L 16.08
M 1.06
N .9
O 2.28
P 3.8

(block 4)

C $\frac{35}{100} = \frac{7}{20}$
D $1\frac{4}{10} = 1\frac{2}{5}$
E $7\frac{75}{100} = 7\frac{3}{4}$
F $\frac{6}{100} = \frac{3}{50}$
G $\frac{55}{100} = \frac{11}{20}$
H $10\frac{9}{100}$
I $4\frac{36}{100} = 4\frac{9}{25}$
J $\frac{9}{10}$
K $3\frac{2}{100} = 3\frac{1}{50}$
L $\frac{60}{100} = \frac{3}{5}$
M $1\frac{5}{10} = 1\frac{1}{2}$
N $7\frac{99}{100}$

Moving forward

In this lesson you will learn a different way to write fraction numerals.


Jim, my father said that we had 9 tenths of an inch of rain during the night. He wrote it like this. Why did he write it this way?
.9
Oh, that's just another way of writing 9 tenths, Bob. When you have a denominator like 10 or 100, you can write the fraction using a point. Then you don't write the denominator.
You mean I can write 35 hundredths like this?
.35
That's right. You can read it as 35 hundredths or as point 35.

1 A B

3 $\frac{1}{5}=\frac{2}{10}=\frac{20}{100}$

.2 .20 C

$\frac{3}{4}=\frac{75}{100}$

.75 D

5 $\frac{2}{25}=\frac{8}{100}$

.08 E

2

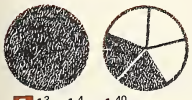
A Fraction numerals written in the way that Jim explained are called decimal fraction numerals. Where have you seen decimal fraction numerals used?

B Do the three black numerals under Picture C express the fraction of the rectangle that is green? The green numerals show how to write $\frac{2}{10}$ and $\frac{20}{100}$ as decimal fraction numerals. You read them in the same way you read the numerals above them.

4

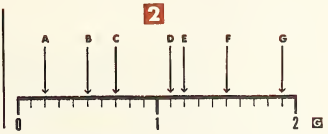
C Some fraction numerals can be changed to hundredths but not to tenths. Look at Picture D. Do both the black and the green numerals express the fraction of the circle that is green?

D Look at Picture E. What do the black numerals under the rectangle express? Look at the green numeral. This shows how to write $\frac{8}{100}$ as a decimal fraction numeral. The 0 between the point and the 8 shows that the decimal fraction numeral means hundredths, not tenths. How do you read .08? How do you read .8? How do you read .80?



1 $1\frac{2}{5}=1\frac{4}{10}=1\frac{40}{100}$

1.4 1.40 F



2 G

E Look at Picture F. Does each of the black numerals tell how much of the two circles is red? The green numerals show how to write $1\frac{4}{10}$ and $1\frac{40}{100}$ using decimal fraction numerals. To read the first green numeral, say "1 and 4 tenths." Read the second numeral.

F Picture G shows a number line. The marks between 0 and 1 divide this part of the number line into tenths. You can write the fraction numerals $\frac{2}{10}$, .2, or $\frac{1}{5}$ for Mark A. Now write two or more different fraction numerals for Mark B. For Mark C.

G The marks between 1 and 2 divide this part of the number line into tenths.

3 For Mark D, you can write either the numeral $1\frac{1}{10}$ or 1.1. For Mark E, you can write $1\frac{2}{5}$, or $1\frac{2}{10}$, or 1.2. What decimal fraction numeral can you write for Mark F? For Mark G?

H Which is longer, .1 in. or .2 in.?

4 I Which is longer, .01 in. or .02 in.?

J Which is longer, .1 in. or .01 in.?

K Which is longer, .1 in. or .11 in.?

4

Read each numeral below.

5

A	.3	E	.1	I	6.7	M	9.04
B	.06	F	2.6	J	.18	N	.75
C	5.4	G	.9	K	.01	O	2.36
D	.2	H	.88	L	4.63	P	.07

Write each of the numerals below as a decimal fraction numeral.

6

A	$3\frac{1}{2}$	E	$\frac{35}{100}$	I	$\frac{1}{50}$	M	$1\frac{3}{50}$
B	$\frac{12}{25}$	F	$\frac{13}{20}$	J	$3\frac{6}{8}$	N	$\frac{9}{10}$
C	$\frac{1}{4}$	G	$2\frac{9}{10}$	K	$\frac{19}{20}$	O	$2\frac{7}{25}$
D	$98\frac{3}{5}$	H	$6\frac{3}{4}$	L	$16\frac{2}{25}$	P	$3\frac{4}{5}$

Write each numeral below without using a decimal fraction numeral. Be sure to show the simplest form. The first two examples are done for you.

7

A	$.16=\frac{16}{100}=\frac{4}{25}$	G	.55	K	3.02
B	$3.8=3\frac{8}{10}=3\frac{4}{5}$	H	10.09	L	.60
C	.35	E	7.75	I	4.36	M	1.5
D	1.4	F	.06	J	.9	N	7.99

Now you know that decimal fraction numerals are another way of writing certain fraction numerals.

255

1 Direct attention to the cartoon conversations in Pictures A and B.

2 Discuss Exercise A. Examples that might be mentioned are batting averages and team standings in baseball, readings on thermometers, odometers, weather reports, etc.

3 Have pupils look at Picture C and relate it to Exercise B. You might also ask such questions as: "Into how many parts has the rectangle been divided? How many parts are green? This represents what fraction of all the parts? Can you change $\frac{1}{5}$ into tenths and hundredths? How?" Then let pupils read the decimal fraction numerals aloud.

4 Discuss Exercise C and Picture D.

5 Have pupils look at Picture E and relate it to Exercise D. You may need to explain that .08 is sometimes read as "point-oh-eight."

256

1 Have children look at Picture F and relate it to Exercise E.

2 Direct attention to Picture G. Have pupils count the spaces between 0 and 1. Then let them read and discuss Exercise F. Draw a line on the chalkboard and write the suggested numerals on it.

3 Discuss Exercise G and let pupils volunteer to answer the questions. Include the decimal fraction numerals on the chalkboard number line.

4 Use the number line to illustrate the answers for these questions.

5 This is an oral exercise. Pupils can take turns reading each numeral.

6 Have children write the decimal fraction numerals on their papers. (Provide answers so they may verify their work.) Then discuss those that proved troublesome.

7 Provide answers for these exercises so that children may verify their work.

257-258 Using arithmetic

Exponded Notes are not considered necessary for this lesson.

Objectives

The child practices solving problems.

Vocabulary

New words page 257 freight, ago*, Boston, great*, locomotive, differ, narrower*, tracks*; page 258 coach*, baggage*, dining

Answers

Using arithmetic:

A (8)×2,000=[16,000] or (108,000)−(92,000) =[16,000]

B 41,721−37,251=[4470]

C 420/70=[6]/1

D 12/1=[72]/6

E 790,144−221,098 =[569,046]

F 75/3=[25]/1

G 12×5280=[63,360]

H $56\frac{1}{2}-(36)=[20\frac{1}{2}]$

I 6/5=[72]/60

J 19/1=[152]/8 or 8×19=[152]

K $156÷[$3]=52 or 156/52=[3]/1; $78

L 2/1=156/[78]; $78

M 4/1=[208]/52 or 52×4=[208]

N 4/1=[220]/55

O $45,616+$46,646 +$110,423+$173,267 =[$375,952]

P $500,000+$45,616 +$46,646+($441,692) +($748,880)+ $145,289+$173,267 =[$2,101,390]

Keeping skillful:

(block 1)

A 183 D 301 G 1136 J 2362 M 11,790

B 134 E 2338 H 1763 K 8478

C 1038 F 10,686 I 1839 L 14,729

(block 2)

A 2617 D 85,446 G 19,979 J 14,286 M 4607

B $357\frac{5}{7}$ E 32,640 H 38 K 235,312 N 200,544

C 48,924 F $31\frac{12}{23}$ I 70 L 57

(block 3)

A 7 sq. yd. F 3840 A. K 5904 sq. in.

B 432 sq. in. G 8 sq. ft. L 144 sq. yd.

C 90 sq. ft. H 108 sq. ft. M 12,800 A.

D 12 sq. yd. I 324 sq. ft. N 432 sq. yd.

E 216 sq. ft. J 4 sq. mi.

Using arithmetic 1

A Tony and Paul have been reading about railroads in the United States. Today the average freight car carries about 54 T. of freight. The average freight car of 25 years ago carried about 46 T. The freight car of today carries about how many more pounds than the freight car of 25 years ago?

B In 1954, United States railroads used 37,251 locomotives. In 1940, they used 41,721 locomotives. They used how many fewer locomotives in 1954 than in 1940?

C Passenger and freight locomotives of today differ in weight from about 70 tons to about 420 tons. The heaviest locomotive in use today weighs about how many times as much as the lightest locomotive?

D One locomotive used in the United States 100 years ago weighed 6 T. One of the lightest locomotives in use today weighs 12 times as much as this old one. How much does this new light locomotive weigh?

E There are 790,144 mi. of railroad in the world. 221,098 mi. are in the United States. There are how many miles of railroad in the rest of the world?

F In 1845, before mail was carried by the railroads, it cost 75¢ to send a 3-page letter from Boston to Washington, D.C. In 1957, it cost 3¢. It cost how many times as much to send a letter from Boston to Washington in 1845 as it did in 1957?

G The longest railway bridge in the United States crosses Great Salt Lake. The bridge is 12 miles long. How many feet long is the bridge?

H The distance between the parallel rails of a standard railroad track is $56\frac{1}{2}$ in. On a few short railroads in the United States the distance between the rails is 3 ft. These 3-foot tracks are how many inches narrower than the standard tracks?

I A train traveled 6 mi. in 5 min. How many miles per hour was this train traveling?

THINK 1 hr. = 60 min.

J Today a freight train travels an average distance of about 19 miles in an hour. About how far can a freight train travel in 8 hours?

K Tony's father has a model railroad. Last year he spent $156 on his hobby. He spent an average of how much per week?

257

257

1 Assign Problems A to P (L to P on the next page) as written work without any prior discussion. Direct the children to write equations as usual. Problems C, D, F, I, J, K, L, M, and N involve rate or comparison. Pupils should use ratios in making their equations for these problems. If any difficulties are encountered, refer them to the lessons on pages 215-226.

L Last year Tony's father spent twice as much money on his model railroad as Paul's father did on his. How much did Paul's father spend on his model railroad last year?

M Tony helps his father with the model railroad for about 4 hours every week. Tony spends about how many hours in a year helping with the model railroad?

N Paul's father now has 55 feet of track for his model railroad. He hopes to have 4 times as much track some day. How many feet of track does he hope to have?

O The table below shows the average amount a railroad spends for new equipment for a passenger train. What is the total cost of 1 mail car, 1 coach, 1 baggage car, and 1 club car?

Locomotive	$500,000
Mail car	45,616
Baggage car	46,646
Coach	110,423
Sleeping car	149,776
Club car	173,267
Dining car	145,289

P Find the total cost of a new train that is made up of a locomotive, a mail car, a baggage car, 4 coaches, 5 sleeping cars, a dining car, and a club car.

Keeping skillful

Find the sum.

A	37, 60, 41, 29, 16	A	7006 − 4389 = n
B	8, 55, 34, 9, 28	B	2504 ÷ 7 = n
C	205, 83, 118, 632	C	9 × 5436 = n
D	65, 90, 7, 53, 86	D	94252 − 8806 = n
E	451, 205, 718, 964	E	64 × 510 = n
F	3395, 820, 6471	F	725 ÷ 23 = n
G	86, 470, 518, 62	G	n + 30041 = 50020
H	119, 383, 736, 525	H	1710 ÷ n = 45
I	229, 641, 802, 167	I	n × 52 = 3640
J	955, 387, 226, 794	J	n − 8337 = 5949
K	620, 4008, 2917, 933	K	764 × 308 = n
L	9207, 439, 5083	L	205 × n = 11685
M	8375, 991, 2346, 78	M	12475 − n = 7868
		N	96 × 2089 = n

A	63 sq. ft. = ▭ sq. yd.
B	3 sq. ft. = ▭ sq. in.
C	10 sq. yd. = ▭ sq. ft.
D	108 sq. ft. = ▭ sq. yd.
E	24 sq. yd. = ▭ sq. ft.
F	6 sq. mi. = ▭ A.
G	1152 sq. in. = ▭ sq. ft.
H	12 sq. yd. = ▭ sq. ft.
I	36 sq. yd. = ▭ sq. ft.
J	2560 A. = ▭ sq. mi.
K	41 sq. ft. = ▭ sq. in.
L	1296 sq. ft. = ▭ sq. yd.
M	20 sq. mi. = ▭ A.
N	3888 sq. ft. = ▭ sq. yd.

258

1. Have the children solve Problems L to P.
2. Provide answers, and discuss the problems that caused difficulty for more than a small number of children.
3. Assign these exercises as separate blocks of written work.
4. Provide answers and discuss those exercises that caused the most difficulty.

259 Thinking straight

Expanded Notes are not considered necessary for this lesson.

Objectives

The child learns about some of the quantitative thinking that is necessary in buying and selling.

Vocabulary

There are no new words.

Comments

This last lesson before the review lessons on "Looking back" and "Checking up" is a very challenging one. It presents a situation about buying, selling, and pricing for profit that will require the children to do some sound thinking. It also presents many possibilities for classroom discussion about other pricing plans and procedures for selling the cards in the problem. Exercise E in particular presents a good basis for discussion and quantitative thinking.

Answers

A ($.78) + ($.40) + ($.60) + ($.20) = [$1.98]

B They would have to sell the cards for enough to cover the total cost and the desired profit. $1.98 + $5.00 = [$6.98]

C 10/$.35 = 2.00/[$7.00]; Her plan would provide enough money.

D 1/4 = 200/[$8.00]; Ann's plan would provide even more money.

E Yes; Get the pupils to see that if all the cards were sold at the cheaper rate of 10 for 35¢, the total would be 2¢ more than the amount needed. If any are sold at the higher rate of 4¢ each, more money will be available.

F ($7.10) − $1.98 = [$5.12]; yes

Thinking straight

A Picture A shows the materials Carol and Ann used to make 200 birthday cards to sell at the school fair. How much did all of these materials cost? **1**

B The girls wanted to sell the cards for enough money to have a profit of $5.00. For how much would they have had to sell all the cards? **2**

C Look at Picture B. Would Carol's plan have brought in enough money? **3**

THINK She planned to sell the cards at the rate of 10 per 35¢, or 200 per n. Write the equation.

D Look at Picture C. Would Ann's plan have brought in enough money?

THINK Ann planned to sell the cards at the rate of 1 per 4¢, or 200 per n.

E Look at Picture D. If they had sold all the cards by either plan, would they have had the profit they wanted? Explain your answer. **5**

F The girls sold 100 cards at the rate of 10 cards for 35¢ and 90 cards for 4¢ each. Did they make a profit of $5.00? **6**

259

259

1. Ask the pupils to read Problem A. Have them study Picture A and figure the cost of the materials.
2. Let the class read Problem B silently. Discuss the answer.
3. Discuss Picture B and Problem C. Be sure the pupils see that Carol's plan will produce 2¢ more than the desired amount.
4. Discuss Picture C and Problem D. Ask: "Which plan is better? Why?" Develop the idea that all the cards may not be sold.
5. Develop the idea that there will be enough money no matter how few of the cards are sold at the higher rate of 4¢ each if all the others are sold at the rate of 10 per 35¢.
6. Ask, "If the 190 cards had been sold at the rate of 10 per 35¢, would there have been enough money?"

260-263 Looking back

Exponded Notes ore not considered necessory for this lesson.

Objectives

The child reviews concepts necessary for understanding the number system, fractions, measurement, ratio, bar graphs, and scale drawing.

Vocabulary

New words page 260 space*; page 263 sentences

Comments

Use the following reteaching chart to locate topics on which the pupils show weakness.

Reteaching chart

The pages listed in the chart below indicate the pages in the pupils' book that can be used for reteaching.

Page 260:

Block	Pages	Block	Pages
1	12-15	4	255-256
2	12-15	5	255-256
3	23, 204	6	255-256

Page 261:

Block	Pages	Block	Pages	Block	Pages
1	126-131	3	126-131,	5	143-146
2	126-131,		148-150	6	158-159,
	148-150	4	151-153		160-162,
					170

Page 262:

(block 1)

Exercises	Pages	Exercises	Pages
A	3-7	G	3-7
B-F	107-109	H-K	240-248

(block 2) 245-248, 249-250

Page 263:

(block 1)

Exercises	Pages	Block	Pages
A-B	232-233	2	112-117, 119-123,
C-D	215-220,		132-133
	232-233	3-4	252-253
E-G	234-237		

Answers

Page 260:

(block 2)

A 580,012	C 100,000,036	E 410,005
B 17,023,690	D 144,001,000	

(block 3)

	Higher hundred thousand	Lower hundred thousand	Nearer million
B	196,900,000	196,800,000	197,000,000
C	9,600,000	9,500,000	10,000,000
D	3,200,000	3,100,000	3,000,000
E	499,800,000	499,700,000	500,000,000
F	1,900,000	1,800,000	2,000,000

(block 4)

A nine tenths
B eighteen hundredths
C two and five hundredths
D sixty-seven hundredths
E thirty and three tenths
F forty-seven and thirty hundredths
G two hundredths

(block 5)

A .16
B 4.7
C .86
D .4
E 2.01
F .1
G .20

(block 6)

A $\frac{3}{5}$	D $\frac{1}{2}$	G $5\frac{33}{100}$	J $\frac{9}{10}$
B $6\frac{12}{25}$	E $3\frac{3}{4}$	H $\frac{1}{100}$	K $\frac{3}{20}$
C $\frac{3}{100}$	F $\frac{4}{5}$	I $7\frac{1}{10}$	L $4\frac{1}{5}$

Page 261:

(block 1)

A $\frac{2}{6}$, $\frac{1}{3}$
B $\frac{3}{5}$
C $\frac{4}{16}$, $\frac{1}{4}$
D $\frac{4}{8}$, $\frac{1}{2}$
E $\frac{2}{3}$
F $\frac{4}{10}$, $\frac{2}{5}$
G $\frac{1}{4}$
H $\frac{9}{12}$, $\frac{3}{4}$

(block 2)

A six and five eighths
B nine sixteenths
C sixteen and three fourths
D five sevenths
E eleven twelfths
F nine and seven ninths
G thirteen tenths
H nineteen and seven fifteenths
I fifteen tenths
J forty fifths

(block 3)

A $2\frac{4}{5}$
B $17\frac{1}{3}$
C $\frac{18}{7}$
D $5\frac{3}{5}$
E $\frac{11}{3}$
F $6\frac{5}{8}$
G $\frac{29}{12}$
H $15\frac{1}{9}$

(block 4)

A $3\frac{1}{3}$ or $3\frac{2}{6}$	D $5\frac{4}{5}$	G $3\frac{9}{10}$	J $3\frac{2}{3}$
B $1\frac{1}{3}$	E $1\frac{1}{8}$ or $1\frac{2}{16}$	H $3\frac{1}{4}$	K $2\frac{1}{2}$ or $2\frac{6}{12}$
C $2\frac{1}{4}$ or $2\frac{2}{8}$	F $3\frac{1}{2}$	I $1\frac{1}{5}$	L $2\frac{1}{6}$
M $2\frac{1}{4}$	P $2\frac{1}{3}$ or $2\frac{4}{12}$	S $5\frac{1}{6}$	
N $2\frac{7}{8}$	Q $5\frac{1}{2}$	T $3\frac{3}{5}$	
O $1\frac{3}{16}$	R $1\frac{7}{10}$	U $6\frac{1}{2}$ or $6\frac{2}{4}$	

(block 5)

A $\frac{16}{20}$	C $\frac{9}{27}$	E $\frac{7}{28}$	G $\frac{25}{30}$
B $\frac{22}{24}$	D $\frac{14}{16}$	F $\frac{9}{18}$	H $\frac{18}{20}$

(block 6)

A $\frac{7}{8}$, $\frac{3}{4}$, $\frac{2}{3}$	E $\frac{3}{5}$, $\frac{1}{3}$, $\frac{3}{10}$	I $\frac{2}{3}$, $\frac{3}{5}$, $\frac{1}{2}$
B $\frac{5}{6}$, $\frac{1}{2}$, $\frac{2}{5}$	F $\frac{5}{6}$, $\frac{2}{3}$, $\frac{5}{8}$	J $\frac{3}{4}$, $\frac{5}{8}$, $\frac{9}{16}$
C $\frac{3}{4}$, $\frac{11}{16}$, $\frac{3}{8}$	G $\frac{9}{10}$, $\frac{5}{6}$, $\frac{4}{5}$	K $\frac{5}{12}$, $\frac{1}{3}$, $\frac{1}{6}$
D $\frac{5}{6}$, $\frac{7}{12}$, $\frac{1}{3}$	H $\frac{5}{12}$, $\frac{3}{8}$, $\frac{1}{3}$	L $\frac{3}{4}$, $\frac{7}{10}$, $\frac{3}{5}$

Page 262:

(block 1)

A [C, F, J]; D; C; 2
B 64 in.
C less than
D 31 mi.
E greater than
F 10 yd.; 30 ft.
G 4 in.
H 36 sq. ft.
I 60 sq. in.
J less than
K 12 sq. yd.; 108 sq. ft.; greater than

(block 2)

A 108 sq. ft.	D 6 sq. ft.	G 15 sq. mi.
B 576 sq. in.	E 1728 sq. in.	H 3200 A.
C 8 sq. yd.	F 12 sq. yd.	I 1296 sq. in.

Page 263:

(block 1)

A 100 miles
B 100 miles
C 30/1=150/[5]; 5 hr.
D 35/1=[175]/5; 175 mi. (The scaled map also shows the distance to be 175 mi.)
E number of pupils; Grade 1; Grade 6
F 5
G Grades 3 and 5

(block 3)

A thirty-five per cent
B one per cent
C ninety-six per cent
D ten per cent
E forty-two per cent

(block 4)

A 87%
B 3%
C 30%
D 12%
E 21%

Looking back

hundreds	tens	ones	hundreds	tens	ones	hundreds	tens	ones
	7	9	6		5			8
	millions			thousands				

1 The table above has space for only one large numeral. Make a table like this one, but allow space for eight numerals. Write the numerals below in your table.

A 79,605,008
B 33,600
C 427,001,955
D 1,963,017
E 804,126
F 15,000,742
G 633,984,182
H 48,000,000

2 Write with numerals.

A five hundred eighty thousand twelve
B seventeen million twenty-three thousand six hundred ninety
C one hundred million thirty-six
D one hundred forty-four million one thousand
E four hundred ten thousand five

3 Round off the numbers below. The first example has been done for you.

		Higher hundred thousand	Lower hundred thousand	Nearer million
A	38,482,710	38,500,000	38,400,000	38,000,000
B	196,832,336			
C	9,531,875			
D	3,137,424			
E	499,701,300			
F	1,895,369			

4 Read.

A .9
B .18
C 2.05
D .67
E 30.3
F 47.30
G .02

5 Write as decimal fraction numerals.

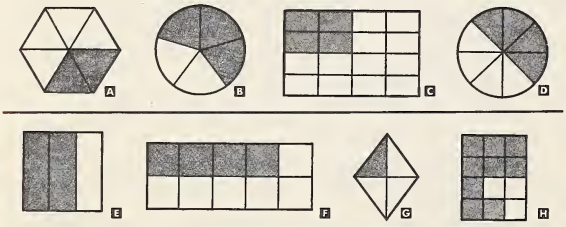
A sixteen hundredths
B four and seven tenths
C eight-six hundredths
D four tenths
E two and one hundredth
F one tenth
G twenty hundredths

6 Write the numerals below using fraction numerals without the decimal point. Write them in simplest form.

A .6
B 6.48
C .03
D .5
E 3.75
F .80
G 5.33
H .01
I 7.1
J .9
K .15
L 4.2

7

Looking back



1 Look at Diagram A. You can use either $\frac{2}{6}$ or $\frac{1}{3}$ to tell what fraction of the diagram is red. Use as many fraction numerals as you can to tell what fraction of each of the other diagrams is red.

2 Read.

A $6\frac{5}{8}$
B $\frac{9}{16}$
C $16\frac{3}{4}$
D $\frac{5}{7}$
E $\frac{11}{12}$
F $9\frac{7}{9}$
G $\frac{13}{10}$
H $19\frac{7}{15}$
I $\frac{15}{10}$
J $\frac{40}{5}$

3 Write with numerals.

A two and four fifths
B seventeen and one third
C eighteen sevenths
D five and three fifths
E eleven thirds
F six and five eighths
G twenty-nine twelfths
H fifteen and one ninth

4 Write as mixed numbers.

A $\frac{20}{6}$
B $\frac{4}{3}$
C $\frac{18}{8}$
D $\frac{29}{5}$
E $\frac{18}{16}$
F $\frac{7}{2}$
G $\frac{39}{10}$
H $\frac{13}{4}$
I $\frac{6}{5}$
J $\frac{11}{3}$
K $\frac{30}{12}$
L $\frac{13}{6}$
M $\frac{9}{4}$
N $\frac{23}{8}$
O $\frac{19}{16}$
P $\frac{28}{12}$
Q $\frac{11}{2}$
R $\frac{17}{10}$
S $\frac{31}{6}$
T $\frac{18}{5}$
U $\frac{26}{4}$

5

A $\frac{4}{5} = \frac{\square}{20}$
B $\frac{11}{12} = \frac{22}{\square}$
C $\frac{1}{3} = \frac{\square}{27}$
D $\frac{7}{8} = \frac{\square}{16}$
E $\frac{1}{4} = \frac{7}{\square}$
F $\frac{1}{2} = \frac{9}{\square}$
G $\frac{5}{6} = \frac{\square}{30}$
H $\frac{9}{10} = \frac{18}{\square}$

6 Write in order of size. Begin with the largest fraction.

A $\frac{2}{3}, \frac{7}{8}, \frac{3}{4}$
B $\frac{1}{2}, \frac{5}{6}, \frac{2}{5}$
C $\frac{11}{16}, \frac{3}{8}, \frac{3}{4}$
D $\frac{1}{3}, \frac{5}{6}, \frac{7}{12}$
E $\frac{3}{5}, \frac{1}{3}, \frac{3}{10}$
F $\frac{5}{8}, \frac{2}{3}, \frac{5}{6}$
G $\frac{4}{5}, \frac{9}{10}, \frac{5}{6}$
H $\frac{5}{12}, \frac{3}{8}, \frac{1}{3}$
I $\frac{1}{2}, \frac{2}{3}, \frac{3}{5}$
J $\frac{9}{16}, \frac{5}{8}, \frac{3}{4}$
K $\frac{1}{6}, \frac{5}{12}, \frac{1}{3}$
L $\frac{3}{5}, \frac{7}{10}, \frac{3}{4}$

7

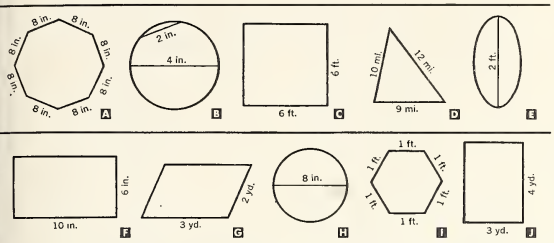
260

1 Tell the children to make a table similar to the illustration at the top of the page, and have them make spaces downward for 8 numerals. Then ask them to write the numerals in the proper places.
2 Have the children write these exercises.
3 Tell the children to write three column headings on their papers entitled "Higher hundred thousand," "Lower hundred thousand," and "Nearer million." Then ask them to round the numbers labeled A to F and write the new numerals opposite their identifying letters.
4 Have the children take turns giving the answers for these exercises.
5 Have the pupils write these exercises.
6 Ask the children to write these using ordinary, or common, fraction numerals.
7 Refer to the Reteaching Chart for any references you need after you have gone over the work on this page.

261

1 Assign this exercise as written work.
2 Use these exercises for oral work.
3 Ask the pupils to write these numerals on their papers.
4 Tell the pupils to write the letters A to U on their papers and then write the correct mixed number after the appropriate letter. If you want the fraction numerals written in simplest form, direct the pupils to do so.
5 Tell the pupils to write the equal fraction numerals on their papers.
6 Ask the pupils to write the fraction numerals in order according to size of the fractions they express.
7 Use the pages in the pupils' book that are listed in the Reteaching Chart for any reteaching that you feel is necessary.

Looking back



1 A Look at Pictures A to J. Which of the polygons are rectangles? Are there any triangles? Are there any squares? How many circles are there?

B Find the perimeter of Polygon A.

C Is the perimeter of Polygon C less than or greater than the perimeter of Polygon J?

D Find the perimeter of Polygon D.

E Is the perimeter of Polygon I less than or greater than the perimeter of Polygon A?

F What is the perimeter of Polygon G? How many feet is this?

G What is the length of the diameter in Circle B?

H Find the area of Polygon C.

I Find the area of Polygon F.

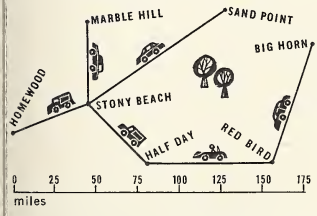
J Is the area of Polygon F greater than or less than one square foot?

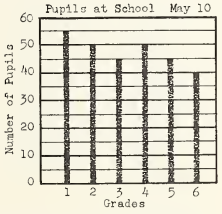
K Find the area of Polygon J. How many square feet is this? Is the area of Polygon J greater than or less than the area of Polygon C?

2
A 12 sq. yd. = ☐ sq. ft.
B ▨ sq. in. = 4 sq. ft.
C 72 sq. ft. = ▨ sq. yd.
D 864 sq. in. = ▨ sq. ft.
E 12 sq. ft. = ▨ sq. in.
F ▨ sq. yd. = 108 sq. ft.
G 9600 A. = ▨ sq. mi.
H 5 sq. mi. = ▨ A.
I 1 sq. yd. = ▨ sq. in.

3

Looking back





1 A Use the scale to find the distance between Sand Point and Stony Beach on the map. How many miles is it?

B How many miles is it from Half Day to Marble Hill?

C If you drive at the rate of 30 mi. per hour, about how many hours will it take to drive from Half Day to Big Horn?

D It takes Mr. Stone 5 hours to drive from Red Bird to Homewood when he averages 35 mi. per hour. About how far is it from Red Bird to Homewood? Now measure the distance on the map.

E Look at the bar graph. What do the numerals on the scale at the left of the graph represent? Which grade had the most pupils at school on May 10? Which grade had the fewest?

F There were how many more pupils in the second grade than in the third?

G Which grades had the same number of pupils at school that day?

2 Write sentences using the ratios below. For Exercise A you might write: 6 apples cost 25¢.
John has 6 books to Bill's 25 books.

A $\frac{6}{25}$ B $\frac{3}{5}$ C $\frac{18}{26}$ D $\frac{2}{25}$ E $\frac{6}{15}$ F $\frac{12}{27}$ G $\frac{5}{27}$ H $\frac{9}{15}$

3 Read.
A 35%
B 1%
C 96%
D 10%
E 42%

4 Write with numerals.
A eighty-seven per cent
B three per cent
C thirty per cent
D twelve per cent
E twenty-one per cent

5

262

1 Ask the pupils to find the answers to Exercises A to K. For the problems on area, instruct them to write the necessary equations.

2 For these exercises on changing measures, instruct pupils to write the equivalent measures [108 sq. ft., etc.].

3 Again, use the pages in the pupils' book that are listed in the Reteaching Chart for any reteaching that you feel is necessary.

263

1 Assign Exercises A to G as written work. Tell the children to use the scale drawing and graph as necessary.

2 Tell the children that for the ratios they may write sentences about either rate or comparison.

3 Treat these exercises orally.

4 Have the children write the answers to these exercises.

5 If you feel that any reteaching is necessary, refer to the Reteaching Chart.

264-267 Checking up

Expanded Notes are not considered necessary for this lesson.

Objectives

The child tests his ability to solve all types of problem situations and equations presented in this book. He also tests his ability to compute with whole numbers, using all four processes. He shows his understanding of adding and subtracting fractions and mixed numbers and of reducing measures.

Vocabulary

New words page 265 slower*, snowfall*

Comments

These 18 tests are to be used diagnostically to determine the child's ability to understand and solve problems, to work with the four processes, fractions, and measures, and to interpret and solve equations. They are not speed tests, and only as many of them as the group can reasonably be expected to complete should be used in one class period. It is best to give the tests one at a time and to follow each by any discussion and reteaching that may be necessary.

Reteaching chart

The pages listed below tell you where to turn for reteaching materials if you need them. All references are to Book 5 unless otherwise noted.

Test 1

Problem	Pages	Problem	Pages
A	25, 215-220	H	25, 215-220
B	19	I	24, 215-200
C	20, 196	J	20
D	19, 183	K	21-22
E	43-45	L	207-208
F	20, 196	M	20, 196
G	40-42, 183		

Test 2

Problem	Pages	Problem	Pages
A	101-104, 215-220	H	101-104, 215-220
B	96-100	I	227-230
C	221-226	J	215-220
D	227-230	K	221-226
E	215-220	L	96-100, 215-220
F	227-230	M	215-220
G	96-100		

Test 3

Reteaching references are not considered necessary for these questions on quantitative thinking.

Test	Pages	Test	Pages
4	Book 4: 24-27	7	61-65, 77
5	Book 4: 36-41, 45, 54	8	68-74, 91
		9	78-84, 91, 203
6	48-52, 53-59, 164		

Test 10

Exercises	Pages	Exercises	Pages
A	Book 4: 24-27	D, F, and H	Book 4: 45
B	Book 4: 45		Book 5: 20
C and I	Book 4: 24-27, Book 5: 40-42	E and G	Book 4: 45 Book 5: 43-45

Test 11

Exercises	Pages	Exercises	Pages
A	78-84, 138-139	F	24, 203
B	203	H	78-84
C	24, 78-84	I	164
D and G	138-139, 203	J	91, 138-139
E	77		

Test 12 171-176

Test 13 177-182

Test 14 186-189

Test 15 190-195

Test 16

Exercises	Pages
A	177-182
B and H	177-182, 183
C, D, and E	190-195, 196
F	186-189
G	186-189, 196

Test 17

Exercises	Pages
A, F, and I	61-64
B and C	Book 4: 80-81
D and G	Book 4: 82-83
E	86-88, 91
H	86-88

Test 18

Exercises	Pages
A, B, C, E, G, H, and I	92-95
D and F	245-248

Answers

Test 1

A $13 \times 28 = [364]$ or $28/1 = [364]/13$

B $\$20.00 - \$7.89 = [\$12.11]$

C $[3\frac{4}{5}] + 1\frac{1}{10} = 4\frac{9}{10}$; $[3\frac{8}{10}]$

D $2\frac{1}{4} + 3\frac{1}{2} + 1\frac{1}{2} = [7\frac{1}{4}]$

E $84 - [68] = 16$

F $[5\frac{1}{4}]+1\frac{3}{4}=53$

G $[5\frac{1}{8}]-1\frac{7}{8}=3\frac{1}{4}$

H $\$2.75\div\$.25=[11]$ or $25/1=275/[11]$

I $\$2.94\div[\$.14]=21$ or $294/21=[14]/1$; $\$.14$

J $\$9.30+[\$8.65]=\$17.95$

K $110-83=[27]$

L $11\frac{1}{4}-10\frac{1}{2}=[\frac{3}{4}]$; $\frac{3}{4}$ ft.

M $28\frac{1}{2}+[71\frac{1}{2}]=100$

Test 2

A $(5)\div[1]=5$ or $(5)/5=[1]/1$

B $(\$2.36)+(\$.98)+(\$1.65)=[\$4.99]$

C $150/135=300/[270]$

D $186/62=[3]/1$

E $2/20=10/[100]$; 1 hr. 40 min.

F $60/15=[4]/1$

G $(\$.60)+\$1.25\div\$.79=[\$2.64]$

H $(102)\div[17]=6$ or $(102)/6=[17]/1$

I $4/1=[384]/96$

J $3/29=[15]/145$

K $24/20=[6]/5$

L $(6)\times\$.50=[\$3.00]$ or $50/1=[300]/6$; $\$3.00$

M $3/25=12/[100]$; $1.00

Test 3

A Yes, unless one of the numbers is zero

B No

C No; You can go on counting or writing the figures for a number for the rest of your life.

D No. See page 239.

E No; It can be many different shapes. See page 251.

F Yes. See page 232.

G Yes; The scale may be different.

Test 4

A	7136	F	2186
B	15,067	G	15,536
C	16,187	H	15,260
D	1994	I	1423
E	10,188		

Test 5

A	4658	F	13,456
B	127	G	40,444
C	12,522	H	33,913
D	3946	I	74,134
E	3922	J	70,731

Test 6

A	2952	F	3900
B	78,669	G	14,037
C	665	H	54,282
D	2252	I	22,470
E	1328	J	9873

Test 7

A	27,306	F	148,570
B	693,360	G	318,528
C	483,636	H	545,440
D	115,444	I	102,446
E	306,832	J	516,306

Test 8

A	131	E	$94\frac{1}{9}$
B	$33\frac{1}{8}$	F	$53\frac{1}{4}$ or $53\frac{2}{8}$
C	129	G	$103\frac{1}{6}$
D	114	H	$995\frac{1}{2}$ or $995\frac{2}{4}$
		I	280
		J	$296\frac{5}{7}$

Test 9

A	213	E	$76\frac{8}{15}$ or $76\frac{64}{120}$
B	$140\frac{13}{64}$	F	306
C	$46\frac{7}{13}$	G	674
D	30	H	$22\frac{86}{223}$
		I	$119\frac{50}{203}$ or $119\frac{100}{406}$
		J	517

Test 10

A	12,251	F	1149
B	6339	G	4254
C	6349	H	3541
D	371	I	8732
E	4208	J	8129

Test 11

A	27	F	21
B	$26\frac{114}{245}$	G	17
C	125	H	$211\frac{4}{37}$
D	6	I	83,915
E	423,620	J	408

Test 12

A	$1\frac{1}{12}$	F	$2\frac{23}{24}$
B	1	G	$2\frac{2}{5}$ or $2\frac{6}{15}$
C	$\frac{9}{16}$	H	$1\frac{3}{4}$ or $1\frac{9}{12}$
D	$\frac{9}{10}$		
E	2		

Test 13

A	9	F	$14\frac{1}{2}$ or $14\frac{5}{10}$
B	$27\frac{7}{15}$	G	$14\frac{1}{2}$ or $14\frac{6}{12}$
C	$22\frac{5}{6}$	H	35
D	$36\frac{13}{16}$	I	$45\frac{1}{4}$ or $45\frac{3}{12}$
E	$15\frac{7}{12}$		

Test 14

A	$\frac{5}{12}$	F	$\frac{7}{12}$
B	$\frac{3}{8}$	G	$\frac{2}{15}$
C	$\frac{2}{5}$ or $\frac{4}{10}$	H	$\frac{5}{24}$
D	$\frac{13}{24}$	I	$\frac{11}{24}$
E	$\frac{5}{12}$		

Test 15

A	$3\frac{1}{8}$	F	$13\frac{7}{16}$
B	$3\frac{2}{15}$	G	8
C	$7\frac{5}{6}$	H	$10\frac{5}{8}$ or $10\frac{10}{16}$
D	$\frac{1}{2}$ or $\frac{5}{10}$	I	$23\frac{1}{2}$ or $23\frac{5}{10}$
E	$20\frac{13}{24}$		

Test 16

A	$23\frac{5}{12}$	F	$\frac{5}{24}$
B	$7\frac{5}{8}$	G	$\frac{13}{24}$
C	$\frac{13}{15}$	H	11
D	$11\frac{2}{3}$ or $11\frac{4}{6}$		
E	$8\frac{7}{20}$		

Test 17

A	$173.91	F	$61.11
B	$4.43	G	$667.53
C	$67.31	H	39
D	$85.25	I	$116.84
E	210		

Test 18

A	48 pt.	F	3 sq. ft.
B	108 pk.	G	23 yd.
C	12 doz.	H	8 c.
D	135 sq. ft.	I	26,000 lb.
E	2208 hr.		

Checking up

Test 1 **1**

A A parking lot has 13 rows for cars. Each row holds 28 cars. How many cars are in the lot when all the rows are filled?

B Mrs. Long bought $7.89 worth of groceries. She paid for the groceries with a $20 bill. How much money should she have received in change?

C Sue lives $4\frac{9}{10}$ mi. from the lake. One day she rode the bus part of the way to the lake. Then she walked the rest of the way. She walked $1\frac{1}{10}$ mi. How far did she ride on the bus before she started to walk to the lake?

D Mrs. Long used $2\frac{1}{4}$ c. of flour for a cake, $3\frac{1}{2}$ c. of flour for bread, and $1\frac{1}{2}$ c. of flour for coffee cake. How much flour did she use in all?

E John had 84 papers to deliver one morning. He delivered some and then counted the papers he had left. There were 16 papers left. How many papers had he delivered?

F When Jim's father measured him, he said, "You are exactly 53 in. tall. You have grown $1\frac{3}{4}$ in. since your birthday." How tall was Jim on his birthday?

G A grocer sold $1\frac{7}{8}$ lb. from a whole cheese. Then $3\frac{1}{4}$ lb. of the cheese were left. How much had the whole cheese weighed?

H Sue paid $2.75 for some party favors. They cost 25¢ apiece. How many party favors did she buy?

I Mrs. Case paid $2.94 to mail 21 small packages that were all the same weight. How much did it cost to mail each package?

J The skates that Don wants to buy cost $17.95. He has saved $9.30. How much more money does he need before he can buy the skates?

K Don and Bob collect shells. Don has 110 shells, and Bob has 83 shells. Bob has how many fewer shells than Don?

L Mary's room is $11\frac{1}{4}$ ft. wide, and Sue's room is $10\frac{1}{2}$ ft. wide. What is the difference in the widths of the two rooms?

M Mary needs 100 ft. of crepe paper for decorations. She has about $28\frac{1}{2}$ ft. About how much more crepe paper does she need? **2**

Test 2 **1**

A One week Carol worked for $1\frac{1}{2}$ hr. on Monday, $\frac{1}{2}$ hr. on Tuesday, $\frac{3}{4}$ hr. on Wednesday, $1\frac{1}{4}$ hr. on Thursday, and 1 hr. on Friday. She worked an average of how many hours a day?

B At the store one day, Mrs. West bought a 4-lb. chicken at 59¢ a pound, 2 lb. of ground meat at 49¢ a pound, and 3 lb. of other meat at 55¢ a pound. How much did she have to pay for all this meat?

C Two small planes left at the same time to go from Redstone to Casetown, a distance of 300 mi. One plane was traveling 150 miles per hour. The other was traveling 135 miles per hour. How far had the slower plane gone when the faster plane had arrived at Casetown?

D Jim's father weighs 186 lb., and Jim weighs 62 lb. Jim's father weighs how many times as much as Jim?

E Tom and his friends went on a 10-mi. bicycle trip. They rode the first 2 mi. in 20 min. If they kept riding at this rate, how long should the 10-mi. trip have taken?

F On Monday Don practiced playing his cornet for 1 hr. Pat practiced playing his cornet for only 15 min. Don practiced how many times as long as Pat that day?

G Sue bought 20 three-cent stamps, a box of paper and envelopes for $1.25, and a pen for 79¢. How much did she pay for all these things?

H The snowfall for 6 years in one town was 16 in., 17 in., 16 in., 15 in., 20 in., and 18 in. What was the average snowfall?

I Nancy and Ann collect postcards. When Nancy counted hers, she found she had 96. Ann said, "I have 4 times as many postcards as you do." How many postcards did Ann have?

J Mrs. Case paid $1.45 for soap that was on sale at 3 cakes for 29¢. How many cakes of soap did she buy?

K Mrs. Miner uses 24 oranges each week, and Mrs. Parks uses 20 oranges each week. How many oranges will Mrs. Miner use during the time that Mrs. Parks uses 5 oranges?

L Tom worked in his father's store for 2 days. He worked $2\frac{3}{4}$ hr. the first day and $3\frac{1}{4}$ hr. the second day. His father paid him 50¢ an hour. How much did Tom earn in the two days?

M When grapefruit was selling at 3 for 25¢, Mrs. Parks bought 1 dozen grapefruit. How much did she have to pay for the grapefruit? **2**

264

1 Ask the children to work the problems in Test 1. Direct them to write the equation with the letter *n* for the unknown number (and a screen or screens for any preliminary numbers that must be found). Then have them rewrite the equation with the answer in place of the letter *n*. Ask them to label the problems with the identifying letters used in the book.

2 Provide a set of answers for the children to use in verifying their work. Any discussion or reteaching that is necessary should be done before going on to Test 2.

265

1 Administer this problem test in the same way as Test 1.

2 Provide a set of answers so that the children can verify their own work.

Test 3 1

A If you add 2 numbers, will the sum be larger than each of the numbers?

B Can the difference of 2 numbers be larger than each of the numbers?

C Is there a limit to how large a number can be? How do you know?

D Do all rectangles with the same perimeter have the same area?

E Does 1 square foot have to be 1 foot square? How do you know?

F Do all distances of 1 inch on the same map represent the same real distance?

G Can 1 inch on one map represent a longer distance than 1 inch on another map? Why?

Test 4 2

Find the sum.

A 4821, 1719, 202, 394
B 653, 9160, 5254
C 7005, 893, 149, 8140
D 927, 360, 580, 127
E 2645, 6597, 946
F 198, 767, 407, 814
G 6015, 3421, 6100
H 4899, 2743, 7618
I 865, 177, 381

Test 5 2

A $8562 - 3904 = n$
B $1275 - 1148 = n$
C $15482 - 2960 = n$
D $4755 - 809 = n$
E $8600 - 4678 = n$
F $37002 - 23546 = n$
G $42341 - 1897 = n$
H $69164 - 35251 = n$
I $83923 - 9789 = n$
J $76503 - 5772 = n$

Test 6 2

A $6 \times 492 = n$
B $9 \times 8741 = n$
C $95 \times 7 = n$
D $4 \times 563 = n$
E $166 \times 8 = n$
F $5 \times 780 = n$
G $3 \times 4679 = n$
H $6 \times 9047 = n$
I $7 \times 3210 = n$
J $9 \times 1097 = n$

Test 7 2

A $74 \times 369 = n$
B $810 \times 856 = n$
C $492 \times 983 = n$
D $98 \times 1178 = n$
E $508 \times 604 = n$
F $166 \times 895 = n$
G $79 \times 4032 = n$
H $560 \times 974 = n$
I $283 \times 362 = n$
J $647 \times 798 = n$

Test 8 2

A $786 \div 6 = n$
B $265 \div 8 = n$
C $903 \div 7 = n$
D $570 \div 5 = n$
E $847 \div 9 = n$
F $426 \div 8 = n$
G $619 \div 6 = n$
H $3982 \div 4 = n$
I $840 \div 3 = n$
J $2077 \div 7 = n$

Test 9 2

A $5751 \div 27 = n$
B $8973 \div 64 = n$
C $605 \div 13 = n$
D $6450 \div 215 = n$
E $9184 \div 120 = n$
F $58752 \div 192 = n$
G $39092 \div 58 = n$
H $4992 \div 223 = n$
I $48414 \div 406 = n$
J $16544 \div 32 = n$

266

1 Use this test orally, and give the children ample time for discussions.

2 Direct the pupils to write the numerals in computational form and compute. Be sure to evaluate the work on each test, and do any reteaching necessary before going on to the next test.

Test 10 1

A $3462 + 8789 = n$
B $6918 - 579 = n$
C $n - 2347 = 4002$
D $986 + n = 1357$
E $4291 - n = 83$
F $n + 5634 = 6783$
G $8010 - n = 3756$
H $n + 1369 = 4910$
I $5472 + 3260 = n$
J $n - 1855 = 6274$

Test 11

A $n \times 36 = 972$
B $6484 \div 245 = n$
C $5875 \div n = 47$
D $706 \times n = 4236$
E $590 \times 718 = n$
F $9240 \div n = 440$
G $n \times 342 = 5814$
H $7811 \div 37 = n$
I $1291 \times 65 = n$
J $59 \times n = 24072$

Test 12

Find the sum.

A $\frac{3}{4}, \frac{1}{3}$
B $\frac{7}{12}, \frac{5}{12}$
C $\frac{5}{16}, \frac{1}{4}$
D $\frac{3}{10}, \frac{3}{5}$
E $\frac{1}{2}, \frac{2}{3}, \frac{5}{6}$
F $\frac{7}{8}, \frac{5}{6}, \frac{3}{4}, \frac{1}{2}$
G $\frac{3}{5}, \frac{1}{4}, \frac{2}{3}, \frac{4}{5}$
H $\frac{5}{12}, \frac{1}{2}, \frac{5}{6}$

Test 13

A $2\frac{3}{8} + 6\frac{5}{8} = n$
B $14\frac{2}{5} + 12\frac{4}{5} = n$
C $17\frac{1}{2} + 5\frac{1}{3} = n$
D $25\frac{7}{8} + 10\frac{15}{16} = n$
E $8\frac{3}{4} + 6\frac{5}{6} = n$
F $9\frac{9}{10} + 4\frac{3}{5} = n$
G $5\frac{1}{3} + 8\frac{1}{4} + 1\frac{11}{12} = n$
H $\frac{7}{10} + 32\frac{1}{2} + 1\frac{4}{5} = n$
I $12\frac{7}{12} + \frac{1}{6} + 32\frac{1}{2} = n$

Test 14

A $\frac{3}{4} - \frac{1}{3} = n$
B $\frac{7}{8} - \frac{1}{2} = n$
C $\frac{9}{10} - \frac{1}{2} = n$
D $\frac{11}{12} - \frac{3}{8} = n$
E $\frac{5}{6} - \frac{5}{12} = n$
F $\frac{3}{4} - \frac{1}{6} = n$
G $\frac{4}{5} - \frac{2}{3} = n$
H $\frac{1}{3} - \frac{1}{8} = n$
I $\frac{5}{6} - \frac{3}{8} = n$

Test 15

A $5\frac{7}{8} - 2\frac{3}{4} = n$
B $21\frac{4}{5} - 18\frac{2}{3} = n$
C $15\frac{2}{3} - 7\frac{5}{8} = n$
D $10\frac{3}{10} - 9\frac{4}{5} = n$
E $27\frac{1}{8} - 6\frac{7}{12} = n$
F $32 - 18\frac{9}{16} = n$
G $12\frac{5}{8} - 4\frac{3}{8} = n$
H $21\frac{13}{16} - 11\frac{3}{16} = n$
I $37\frac{1}{10} - 13\frac{3}{5} = n$

Test 16

A $17\frac{2}{3} + 5\frac{3}{4} = n$
B $n - \frac{1}{8} = 7\frac{1}{2}$
C $4\frac{1}{5} - n = 3\frac{1}{3}$
D $27\frac{5}{6} + n = 39\frac{1}{2}$
E $n + 3\frac{9}{10} = 12\frac{1}{4}$
F $\frac{5}{6} - \frac{3}{8} = n$
G $\frac{11}{12} - n = \frac{3}{8}$
H $n - 3\frac{2}{5} = 7\frac{3}{5}$

Test 17

A $93 \times \$1.87 = n$
B $\$4.25 + \$.18 = n$
C $\$17.86 + \$49.45 = n$
D $\$94.17 - \$8.92 = n$
E $\$52.50 \div \$.25 = n$
F $63 \times \$.97 = n$
G $\$816.18 - \$148.65 = n$
H $\$28.08 \div \$.72 = n$
I $46 \times \$2.54 = n$

Test 18

A 24 qt. = ▢ pt.
B 27 bu. = ▢ pk.
C 144 things = ▢ doz.
D 15 sq. yd. = ▢ sq. ft.
E 92 da. = ▢ hr.
F 432 sq. in. = ▢ sq. ft.
G 69 ft. = ▢ yd.
H 64 fl. oz. = ▢ c.
I 13 T. = ▢ lb.

267

1 Assign these tests as written work. Do any reteaching necessary before going on to the next test.

Your notes

This section of the *Teaching Guide* contains detailed suggestions for teaching the lessons. The Expanded Notes are really enriched lesson plans that go beyond the minimal essentials described in the Lesson Briefs (pages 19-241). Before using these Expanded Notes, read the Comments and examine the reproduced pages and keyed notes in the Lesson Briefs section. For your convenience, page references are made to the Lesson Briefs at the beginning of each set of notes in this section.

Expanded notes

The Expanded Notes for each lesson include:

Overview

A very brief survey of the lesson as it is developed from page to page in the pupil's book

Teaching the whole class

A plan of procedure to be used for the class as a whole before the children are separated into ability groups

Providing for the able pupil

Special suggestions for activities to extend the ideas of the lesson for abler students

Helping the slow learner

Suggestions to use in making clear the ideas of the lesson and for motivating interest therein

A survey tour

of theory and method as applied to the arithmetic content of this course is provided in the Charting the Course sections included in the Expanded Notes.

Expanded notes

CHARTING THE COURSE

Geometric ideas — shapes, area concept

Most children start the new school year with interest and enthusiasm. After a long vacation they are ready to learn new things. For this reason *Seeing Through Arithmetic 5* begins with content that is new to the pupils. The book does not dampen their interest at the outset by requiring them to review computational arithmetic learned in preceding grades. There is time enough for this review after the class is off to a good start.

Pupils in Grade 5 need certain simple geometric ideas to carry on their activities both in and out of school. For example, they should be able to recognize and name such geometric figures as the square, rectangle, triangle, and circle. They should understand what is meant by the terms *length, width, height, base, altitude,* and *diameter*. They should also understand the basic idea of area, and at this point know how to find it, at least approximately, by counting the number of units of area that will cover a closed geometric figure, such as a triangle. Computational procedures for finding the area of a rectangle will be taught later in this book.

More than sixty years ago an influential committee (The Committee of Ten, 1893) recommended that informal geometry be taught beginning in Grade 5. The

desirability of doing this has never been seriously challenged, but relatively little has been done to make the recommendation effective. Today it is even more urgent that progress be made in this direction. *Seeing Through Arithmetic 5,* therefore, devotes pages 3-11 to teaching some of these concepts. Later sections will use these concepts in finding perimeters and areas by computation.

3-7 Moving forward

Lesson Briefs for this lesson are on pages 20-24.

Overview

The work in this lesson has been planned to give the children some basic generalizations about certain geometric forms (polygons and circles) and their measurement. Proper ways to measure dimensions are emphasized, and the concepts expressed by the words *length, width, height, base, altitude,* and *diameter* are developed.

Page 3 may be used to stimulate discussion about how and where geometric shapes occur in the children's environment. Page 4 formalizes these shapes and relates them to ordinary objects. The work on page 5 illustrates correct ways of measuring dimensions. Pages 6 and 7 are concerned with the geometric presentation of plane shapes discussed on previous pages and the measurement of their dimensions.

Teaching the whole class

Children need to become acquainted with various geometric ideas, and this lesson should familiarize them with the names and characteristics of geometric shapes to be found in their environment. As preparation for developing the work on pages 3-7, have ready for use rulers, yardsticks, and large cardboard figures in the shape of rectangles, other parallelograms, circles, and triangles.

The teaching suggestions for pages 3-7 may seem formal, but keep in mind that the approach is essentially intuitive. Relate all the teaching to the children's environment and experience as much as possible. Remember, too, that the use of correct terminology is important from the very beginning of the study of geometric ideas. If geometric terms are used loosely, confusion is certain to result later on. The authors feel that the meaning of such terms as *base* and *altitude* should be established as soon as the need for their use arises. This principle has been followed in developing the teaching suggestions.

No time limit is set for this lesson or for those that follow. However, you should not attempt to cover all the work on these pages in one class period. Let the needs of your pupils govern the length of time devoted to this lesson. Spend enough time so that pupils will assimilate the ideas presented before they are asked to go on to something new. On page 389 you will find an approximate time schedule for the work in this book.

Present the new words in this lesson just as you would in a reading lesson—that is, write the words on the chalkboard, pronounce them, discuss them, and use them meaningfully in oral situations before you ask the children to read them. Notice that, to lighten the reading load for the children, complicated directions for the work are not given in the child's book. It is the teacher's responsibility to give such directions.

As you know, any classroom contains individuals who differ in capacity for achievement. Some pupils need many concrete experiences to master arithmetical ideas. Abler pupils quickly tire of such activities. They need, instead, more challenging—usually more abstract—activities to keep them interested. The notes for this block of work and for those that follow are arranged accordingly.

This section, "Teaching the whole class," tells you what to do in working with the group as a whole. "Providing for the able pupil" and "Helping the slow learner" give activity suggestions arranged according to the ability of the pupils. (For this block of work, you will find these special sections on pages 250-251.) Be sure to look at these suggestions *before* you begin teaching the lesson. For those pupils with the most serious deficiencies in arithmetic, you may for a time need to provide work at the fourth-grade level. Copies of *Seeing Through Arithmetic 4** of this series and the *Teaching Guide* for Grade 4 will be very helpful.

Page 3: Observe the statement beside the heading "Moving forward." A statement similar to this one will be found with each block of work that introduces a new idea or ideas. These statements have been included because the authors feel it important for the children to know what they are going to study in a new block of work. Always have these statements read and discussed before the new work is taken up.

After the statement has been read and discussed, encourage free discussion of the pictures on page 3. If the children wish to talk about what is going on in the pictures, let them do so, but gradually lead them to discover as many of the geometric forms illustrated in the pictures as they can. This can be developed into quite an exercise in ingenuity, and some of the children will show considerable cleverness in discovering such forms as the ellipse in Picture A, the

**Seeing Through Arithmetic 4* by Maurice L. Hartung, Henry Van Engen, and Lois Knowles. Fourth-grade book in The Basic Mathematics Program. Scott, Foresman and Company.*

square in the road sign (often referred to as a "diamond") in the same picture, the six-sided kite in Picture B, the triangles formed by two legs of the music stands and an imaginary line on the floor between them in Picture C, etc. You may wish to use such terms as *polygon, parallelogram, square, triangle,* etc., in the discussion, but make no attempt at formal definition of these terms.

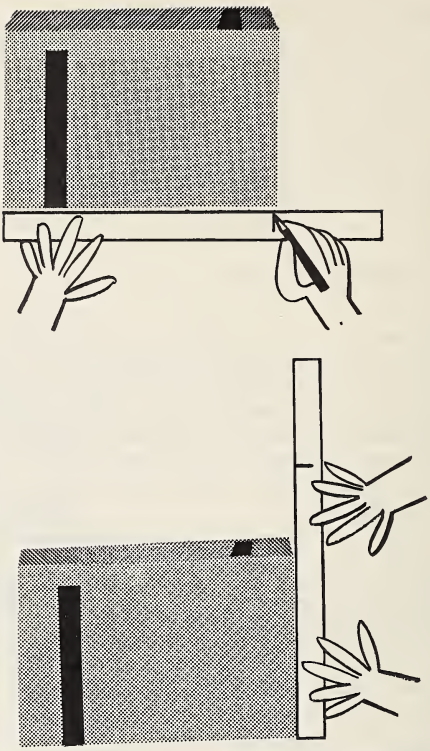
When you think the children have discovered as many geometric shapes as they can, they may find it interesting to play a game in which one pupil describes something in the classroom in terms of its shape and the other pupils guess what is being described. For example, one pupil might say, "I am thinking of something that has six straight sides that are all the same length." The answer might be the clock on the classroom wall. Another pupil might describe the chalkboard somewhat as follows: "The two long sides are equal. The two short sides are equal, too. All the corners are square corners." When you feel that the children are beginning to see that geometric shapes can be described in terms of equal or unequal lengths, straight or curved sides, square corners or corners that are not square, move on to page 4.

Page 4: Begin the work on the page by showing the children how to compare lengths by making marks on the edge of a piece of paper. The paper may then be moved to the next length to be "measured," and the two lengths can be compared. See the picture below. Explain to the children that the term *polygon* can be used to describe a shape whose sides are straight lines. Triangles, rectangles, hexagons, etc., are polygons. Tell the children to examine the polygons at the top of the page carefully. Then say:

> *Find one polygon whose sides are all the same length. Now find four more polygons with equal sides. Find one polygon whose sides are not all the same length. Find three more polygons whose sides are not equal. How are Polygons I and K different? What is the difference between Polygon I and Polygon J?*

Some of these questions can be answered without measuring. Have the children "measure" only when they are not sure of the answer without measuring. Now let the children find examples of four-sided polygons in the pictures below (L to W).

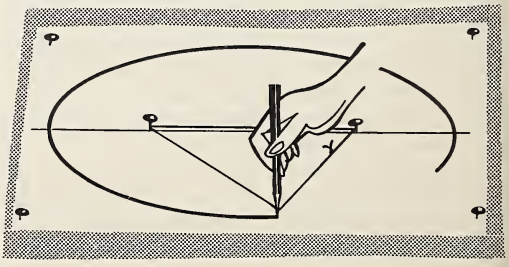
Next direct attention to the circle (G) and the ellipse (H). To help the children understand the difference between an ellipse and a circle, draw an ellipse on the board, and then with the aid of chalkboard compasses or chalk and string, draw a circle in a different color over the ellipse. By using the string



and chalk, let the children see that all distances from the center to the line that is the circle are equal, whereas the distances from the center to the line that is the ellipse are unequal.

You will probably also need to make models of ellipses and draw around them to reproduce them on the chalkboard. To make such model ellipses, follow the directions and observe the picture shown below.

Draw a straight line across a piece of heavy paper tacked to a board. Push two thumbtacks through the paper and into the board on the line. Tie two ends



of a piece of string together, making a loop, and lay the loop around the thumbtacks. Place a pencil point inside the loop and pull the string taut. Keeping the string taut at all times, move the pencil inside the string and around the tacks to draw a curved line. Note that the size of the ellipse is governed by the distance between the tacks and the length of the loop of string.

Perhaps with the clues suggested above, the children will be able to describe a circle and also explain why the curve in Picture H is not a circle.

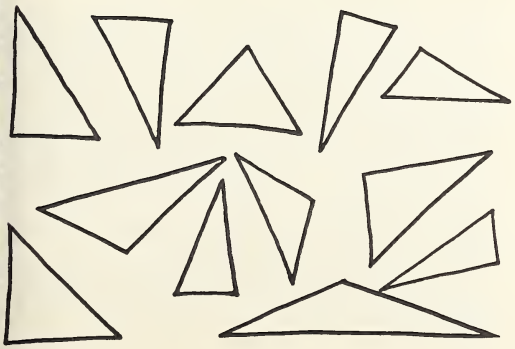
Let the children find as many circles as they can in the picture at the bottom of page 4. Let them find as many ellipses as they can.

To understand further why Polygons A, B, C, and D are different, even though each has four sides, the children not only must visualize these polygons as made up of straight lines but also must see that the lines form angles. Explain that whenever two straight lines meet, angles are formed. Point out, too, that not all angles are the same size. Also, be sure the pupils understand that the *length* of the sides of an angle has nothing to do with the size of the angle itself. Your explanation might follow this pattern:

In Polygon A the four corners are all the same size. We say that the square has four angles. These are square corners, or right angles. What kind of corners does Polygon B have? Do you think that all right angles are the same size? Why?

Now look at Polygons C and D. Do these polygons have square corners or right angles? Are all the angles in Polygon C the same size? In Polygon D? Now look at Polygons E and F. Are they alike in any way? How are they different?

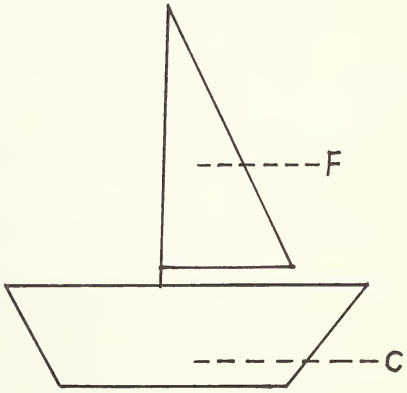
Suggest that the children draw (freehand) as many types of triangles as they can. Point out that triangles do not always have equal sides and need not have one right angle. (Various kinds and positions of triangles are shown below.) Then let the children find a triangle in the pictures on the bottom half of page 4 in the pupils' book.



When discussing Polygons I, J, and K, note that one hexagon has equal sides, while the other has sides that are unequal. The octagon is *regular,* that is, all its sides and angles are equal. It is not necessary to teach the terms *hexagon, octagon,* and *regular,* but some children may already know them, and many will want to know the names of the polygons. Again let the pupils find examples of these geometric forms in the pictures at the bottom of page 4.

By this time the children should recognize the square, rectangle, triangle, and circle. They should also have some understanding of the right angle. Be sure they understand that the square is a rectangle.

Some pupils might enjoy drawing pictures containing as many geometric shapes as possible. Encourage them to use their ingenuity in reproducing objects that are in their immediate environment and that they recall from their past experience. Let them use the pictures at the top of page 4 as a key for labeling the shapes in their pictures. A very simple example of labeling is shown below. Be sure to display the finished pictures.



The children could make another display by cutting from magazines and newspapers pictures that show rectangles, triangles, circles, etc.

An activity the whole class might enjoy is making designs for linoleum, wallpaper, tile, decorative windows, etc., on squared paper by using the geometric forms shown on page 4 and combinations of these forms.

Also encourage the children to bring to class a list of items they have noted outside the classroom that are examples of geometric forms, such as traffic signs,

clock frames, stove burners, flower beds, and the like. Careful observation of traffic signs, for example, will result in the discovery of several different geometric forms.

Page 5: In teaching this page, introduce only as many ideas at one time as you think the children will be able to assimilate.

First, review with the children how to use a ruler. Remind them that the beginning point on the scale must be held even with the beginning point of the line being measured. Demonstrate by measuring the length or width of a sheet of paper, a chalkboard eraser, or any other object near at hand.

Start this lesson with a brief discussion of how we determine how big an object is. Then hold up cardboard models of some geometric shapes and ask the class what you mean when you ask how big they are. Let the children take turns explaining how they would tell how big the square is, or how big the triangle is, etc. If no one suggests measuring the lengths of the sides, suggest it yourself. Then go on somewhat in this way:

Page 5 shows some children measuring different polygons and a circle. In which pictures are the children measuring rectangles? In which two pictures are they measuring triangles? [*E and F*] *Now look at Picture A. Two of the girls are measuring the top of the table, which is in the shape of a rectangle. The girl in the middle is finding the length of the table. The girl on the right is finding the width of the table.*

Confusion results from improper use of such terms as *length, base, height, width,* and *altitude*. In working with this and subsequent pages, try to associate the terms *length, width, height* (and later, *depth*) with physical objects. Associate the terms *base* and *altitude* with the geometric representation of these objects. Thus we can speak of the length and width of a table top, and the height and width of a window, etc.; but when we represent these objects by rectangles, the words *base* and *altitude* should be used. Proceed as follows:

Remember that we must always be careful not to let the beginning point of the scale on a foot ruler or yardstick extend beyond or fall short of the beginning point of the object we are measuring. Are the girls in Picture A using their yardsticks correctly? The girl on the left is measuring the height of the table. How is she holding her yardstick?

Show the class that when height is measured the yardstick or ruler must be held straight up and down; it should not slant. Have one of the pupils come to the front of the room. Demonstrate that his height is the length of a *straight line* from the top of his head to the floor.

To help the children increase their understanding of the terms *length, width,* and *height,* let them individually, or in groups, practice measuring the height, width, and length of their own desks, your desk, a chair, a piano, or any other suitable object in the classroom. Give them time to do this, but do not let the activity drag.

Now draw on the chalkboard a rectangle to represent the table top. Explain briefly how this rectangle can be described in terms of the length of its base and altitude (that is, the length of the table top is now the base of the rectangle; the width of the table top is now the altitude of the rectangle).

Discuss Picture B in a similar way. Get the children to explain why the boy is holding the yardstick as he is to measure the height of the window. (One end is even with the top of the window; the yardstick is at right angles to the window sill.) Have the children explain how the width of the window could be determined. Then draw on the chalkboard a rectangle that represents the window (or use a cardboard model). Get the children to identify the base and altitude and to explain how they can be measured.

Proceed in a similar way with Picture C. Emphasize correct measuring techniques. Some children may have discovered by now that any side of a rectangle may be thought of as its base if we choose to do so. Let them discuss this, if they wish to, but do not put too much emphasis on this idea.

Before discussing Picture D, hold up two circular pieces of cardboard and ask the pupils how they would proceed to find out how big the circles are. They may see immediately that the length of each diameter can be measured. Remind the children that all points of a circle are the same distance from its center. Consequently, any line passing through the center may be used to measure the diameter. Now have the children look at Picture D. Ask:

How is the girl measuring the size of the birdbath? Where is the beginning point of the scale on the yardstick? Notice how she holds the yardstick directly over the center while she measures the distance between two points on the circle. The line she is measuring is called the diameter of the circle. Could she measure the diameter in any other place on the birdbath?

Let the class study and discuss Picture E. Lead their discussion somewhat like this:

The children in this picture are measuring a flower bed. Is it a polygon? Is it a rectangle? Is it a triangle?

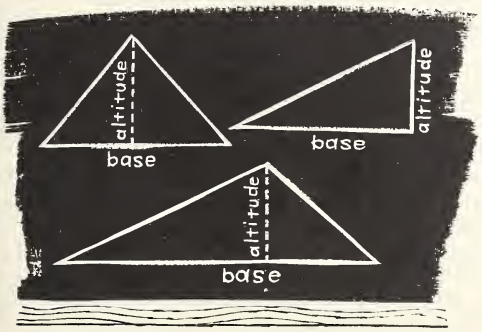
Point out that the children are measuring the flower bed from one side of the triangle (the base) to the opposite point. Help the children understand that this point (or "corner") must be the point farthest from the base. Go on to say:

Notice the yardstick that the boy is holding. It measures the distance from the base to the farthest point from it. How is it placed? Do you see how the boy's yardstick meets the girl's stick at right angles?

The terms *altitude* and *base* are sometimes difficult for children to understand, especially when applied to polygons that are not rectangles. To explain these terms in connection with Picture E, you might say:

The side of the triangle the girl is measuring is called the base. *The boy is measuring the* altitude *of the triangle. The altitude of a triangle is measured on a straight line from its base to the point farthest from the base. The terms* altitude *and* base *may also be used with some other polygons.*

If you wish, you may reproduce on the chalkboard triangles in various positions and indicate the base and altitude of each triangle with colored chalk. See the picture below.



When the children study Picture F, explain that the boys want to know how far the coconut fell. Get them to decide that the dotted line shows this. Also get them to see that this line suggests a triangle formed by the trunk of the tree, a line on the ground, and the dotted line. Some children may observe that this triangle has one right angle.

For practice, let the children draw several triangles on a piece of paper. Have them label the base and then draw a dotted line to show where to measure the altitude. The abler children could use for the base a side other than the side on which the triangle appears to rest. They may thus arrive at the generalization that any side of a triangle may be taken as its base.

Page 6: The children should now understand how certain dimensions of polygons and circles can be measured. The work on this page provides practice in the identification of base and altitude (extended to apply to the square, rectangle, and other parallelograms) and also presents the naming of lines by means of letters.

Show the children how to identify lines on this page by using letters. Let them practice finding specific lines, such as Line z in Picture E, Line y in Picture F, etc.

Make sure each child has a ruler with which to measure certain dimensions of the polygons and circles on page 6. Since the sides of the polygons and the diameters of the circles are one, two, or three inches, they can be measured easily by the children.

If you wish to use the term *parallelogram* in discussing pages 6 and 7, do so, but do not expect the children to use it accurately. (Any polygon in which opposite sides are parallel is a parallelogram. Thus rectangles and squares are special cases of parallelograms.)

The questions that follow suggest how you may direct the discussion. Be sure to identify the lines by their labeling letters before proceeding.

How many sides does Polygon A have? Which lines can you use to measure the base of the polygon? How long are they? Are they equal? Which line can you use to measure the altitude of the polygon? Why do you measure the altitude on Line z and not on Line v or Line x? Do Lines v and y meet at right angles? Do Lines x and y meet at right angles? Lines v and w? Lines x and w? Is this polygon a rectangle? Why or why not?

You may have to explain again that altitude can be measured only on a line that meets the base at right angles. Point out that for a rectangle or square, in which all angles are right angles, the sides can be used to measure the altitude. Sometimes there is no line of the polygon that can be used to measure altitude. Then we must draw a line that is at right angles to the base and extends to the point farthest from the base. Go on to ask:

What is the name of Polygon B? What kind of corners does a square have? What other polygon has four right angles? How is a square different from other rectangles? Which lines can be used to measure the base of Square B? Which lines can be used to measure the altitude? Find another square on this page. What lines can be used to measure the base? The altitude?

What is the curve in Picture C called? Should the diameter be measured along Line x or along Line y? Could more lines be drawn inside this circle on which to measure the diameter? Are there any other circles on page 6? What lines can be used to measure the diameter of Circle F? What is its diameter?

Is Polygon D a rectangle or a triangle? Is it also a square? How do you know? On which two lines can you measure the base? On which two lines can you measure the altitude? What is the length of the base? The altitude? Are there other rectangles on this page? What are they? [*You may need to point out that the two squares are also rectangles. Ask the children to explain why.*]

How many sides does Polygon E have? What is it called? How many angles (or "corners") does a triangle have? Which side is the base? Do Lines z and y meet at right angles? Can the altitude be measured on Line z? What is the length of the altitude of the triangle? Find another triangle on this page. What line shows the altitude of Triangle H? What is the length of the altitude?

For additional practice in finding base, altitude, and diameter, have available two work sheets. The first one should be similar to page 6 in the book (sides, altitudes, and diameters of the shapes lettered) and may include a square, a rectangle with larger dimensions in the vertical position (see Picture D, page 6), a rectangle with larger dimensions in the horizontal position (Picture J), a parallelogram that is not a rectangle, a triangle, and a circle. Let the children find the dimensions of the polygons and circles and have them answer a few questions for each geometric shape similar to the questions suggested for page 6.

The second work sheet should contain unlettered polygons and circles (with centers indicated). The children may label the shapes and indicate altitudes and diameters according to directions given. The directions may follow the following pattern:

For each polygon that is a square or a rectangle, put y *on the base and put* v *on the altitude.*

For each polygon that is not a square, a rectangle, or a triangle, put y *on the base and draw a line that shows where you measure the altitude. Label the altitude* z.

For each triangle, label the sides w, x, *and* y; *choose one side as the base; draw a straight line on which the altitude from that base can be measured and label it* z.

For each circle, draw two diameters, label them with different letters, and measure them.

Page 7: On this page the children should identify the base and altitude of various polygons and the diameters of circles. For the first time lines that are not altitudes, heights, or diameters are introduced.

You may want to discuss Polygons A to J and Circles K to N with the children before they do the exercises. Be sure they understand that the altitude is always a line at right angles to the base and passing through the point of the polygon farthest from the base. They must also understand that a diameter of a circle passes through its center. Give them plenty of time to do the exercises. When the work has been completed, be sure the children are able to explain each answer.

To conclude the lesson, let one or more pupils tell in their own words what they have learned. Then ask someone to read the statement beside the flag at the bottom of the page.

Providing for the able pupil

The abler pupils may enjoy making a notebook illustrating geometric shapes. The book may include:

1. Geometric shapes drawn by the pupil and labeled to tell what they are.

2. Sketches of familiar objects similar to those shown on page 4 of the pupils' book. Each picture should be labeled according to its shape.

3. Several geometric shapes on which the base, altitude, and diameter have been labeled.

4. One or two drawings that show how altitude can be found on parallelograms that are not rectangles and on triangles that do not contain a right angle.

5. A large design made with straight lines only. Any triangle within it may be colored red; any square, black; any other rectangle, green; any four-sided polygon that is not a rectangle or square, yellow; any polygon with five sides, blue; any polygon with six sides, purple; any polygon with eight sides, orange.

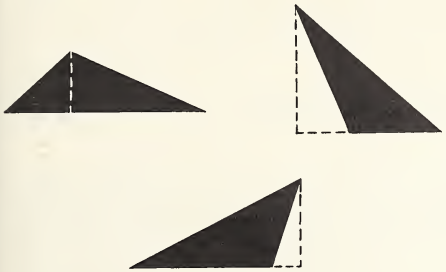
6. A large design drawn with compasses and using only curved lines. This design also may be colored if the children so desire.

7. A collection of geometric shapes cut from magazines, etc.

For another activity, give each child directions for drawing and labeling a large shape which he cuts from construction paper or wrapping paper. Instructions like the following may be given to the children. Rectangle: base, 24″; altitude, 12″; label each side with a different letter. Parallelogram that is not a rectangle: base, 20″; altitude, 15″; label the four sides and the altitude. Triangle: base, 8″; altitude, 12″; label the three sides. Circle: 6″ diameter; draw and label several lines on which diameter can be measured. Square: each side 9″; label the base and altitude.

Since the pupils have had little experience in drawing geometric shapes, do not demand great accuracy. Be sure to display the polygons and circles they draw and label.

An important generalization for abler pupils may be developed as follows. Cut out three large triangles and, using each side in turn as base, discuss the altitude. The pupils should conclude that a triangle may be thought of as having three bases and three altitudes. Be sure to use a triangle similar to the one shown below.



Have the abler children do the work suggested in Activity 1, page 356. They will find these experiments a challenge.

Activities 2 (page 358) and 3 (page 358) may also be used for these children, if you wish.

Helping the slow learner

Prepare work sheets containing carefully enlarged copies of the eleven polygons and circles at the top of page 4. Also cut from cardboard several sets of the shapes. Put the cardboard shapes into a box. Give each of the children a work sheet. Let them, in turn, take a cardboard shape from the box and place it on top of the matching polygon or circle on the work sheet. If a child has already used a polygon or circle he takes out of the the box, he should put it back and take another one. The object of the activity is to see who can fill his work sheet first.

Activity 2, page 358, may be used for these children. Activity 3, page 358, may also be used if you group the children so that there are abler and slower pupils in each group.

Some of these children may enjoy making a notebook as described in "Providing for the able pupil," since many slow learners have a particular liking for and are skillful at artwork. Allow them to use only as many of the seven suggestions as they are able to complete successfully. Suggestions 2 and 7 are activities that are within the ability of all pupils. Suggestion 6 may be more difficult.

8-11 Moving forward

Lesson Briefs for this lesson are on pages 24-27.

Overview

The work in this lesson introduces the child to the concept of the measurement of surface, or area. He is already aware of the need for standard units in the measurement of distance, time, weight, etc., and so can readily accept the fact that standard units of square measure are needed to determine area.

Note that in this lesson measurement is confined to using square inches. Note also that the work is noncomputational. Areas are found by covering surfaces to be measured with inch squares, which are then counted. To bring out the idea that surfaces with shapes other than squares and rectangles can be measured, methods of estimating and approximating are introduced with triangles, circles, and irregular polygons on pages 9-11.

It will be wise to see that each child has a supply of 25 to 30 inch squares that he can apply to various surfaces and count to find the area.

Teaching the whole class

It would be well to have available two rectangular surfaces cut from construction paper. The rectangles should have different dimensions but the same area, such as 15″ x 12″ and 20″ x 9″. Hold the 15″ x 12″ rectangular surface against the chalkboard and draw its outline. Then move a piece of chalk along the rectangle to indicate its dimensions. Shade the inside of it to show the surface to be measured. Say:

Suppose we painted the inside of the rectangle. The problem is, how could we measure how much surface we painted?

Next, use the 20″ x 9″ rectangle. Draw its outline on the board and then ask:

Does this rectangle have the same dimensions as the first rectangle? Look at both rectangles. Is the area of one rectangle greater than the area of the other? Or do you think the areas are the same?

Let a child measure the dimensions of the two rectangles. Discuss whether knowing the dimensions helps to judge the areas of the rectangles. The children should decide that it is difficult to judge the areas by sight. Direct the discussion toward the idea that in order to measure area, the unit of measure must be a kind that covers surface.

Page 8: Read and discuss the statement beside the heading "Moving forward." You may begin the work in the book somewhat in this way:

The pictures on this page show three doll tables with squares placed on top of each one to measure the area. Can you tell, just by looking, which table

top has the largest surface? Now count the squares. Which table top is the largest?

Do Table B and Table C have the same or different dimensions? Do they have the same area? If we were going to paint Tables A, B, and C, for which two would we use the same amount of paint?

We have said that the area of Table B and of Table C is 16 "squares." If one of you went to the store and told the clerk you wanted enough paint to cover a table whose area is 16 squares, do you think this would mean much to him?

Remind the children that to say the chalkboard is "5 sticks long" does not have as much meaning as when you say it is "60 inches long." Emphasize that a standard unit is needed for measuring area, just as it is for measuring length. Say:

Look at the first square at the bottom of page 8. What are its dimensions? Since it is a square, you know that each of the four sides measures 1 inch.

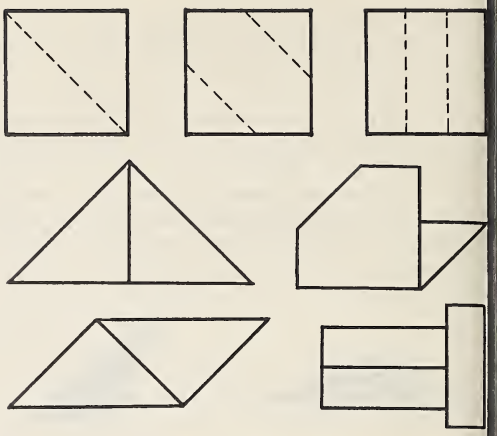
Explain that any square like this is a "1-inch square." However, when we think of an inch square as covering surface (that is, use it to measure area), we speak of it as "1 square inch." Have the children again examine the doll tables at the top of the page. This time, instead of saying, "The area of Table A is 15 squares," they should say, "The area of Table A is 15 square inches." Have them express the area of Tables B and C in square inches.

If you wish, let each child use 25 or 30 one-inch squares to measure the areas of small polygons. Insist that the children refer to the unit of measure as "1 square inch" and to the area as "15 (or so) square inches." Let one child measure the areas of two rectangles with different dimensions but equal areas to demonstrate that the areas are equal.

Page 9: The transparent squares laid on top of the polygons in the pictures on pages 9 and 10 are actual inch squares.

The square inches in Pictures A and B can be counted and the areas compared. Be sure this is done, because some of the children will jump to the conclusion that the area of Picture B is greater than that of Picture A. Upon counting, however, they will find that the area of Picture B is 8 square inches and that of Picture A is 9 square inches.

As preparation for judging the number of square inches in polygons having angles that are not right angles (such as the polygons in Pictures C, D, and E), let the children cut several inch squares into parts in various ways. Have them put the pieces of each square together again. The picture in the next column illustrates a few ways in which an inch square may be cut:



Cutting inch squares in these ways will help the children see that the two small shaded triangles in Picture C can be put together to make one square. This shows that the large triangle has an area of about 2 square inches. In like manner discuss Picture D. The children can see that the area must be between 1 square inch and 3 square inches. If they imagine the two end pieces as put together, the area of the polygon can be seen as approximately 2 square inches. Point out that the areas of the polygons in Pictures C and D are about the same, even though their shapes are quite different.

A similar explanation will help the children to conclude that the area of the polygon in Picture E is between 4 square inches and 6 square inches, or about 5 square inches. Picture F is neither a rectangle nor a square, but all its corners are right angles, and its area is 3 square inches.

Page 10: The procedures used with page 9 may also be used with this page. In estimating the area of the circle in Picture G, point out that it is between 5 and 9 square inches, or about 7 square inches. Since the amount of surface inside the circle in each corner square is approximately equal to one half of a square inch (cut diagonally), these four surfaces together would make about two square inches. Thus the circle covers 5 square inches plus approximately 2 more square inches, or a total of about 7 square inches.

Before having the children count and estimate for Pictures H and I, let them estimate which of the two polygons is larger. Do the same for Pictures J and K.

If you wish, you may follow the discussion for pages 9 and 10 with practice in measuring the surface area of objects in the classroom, such as small blotters, rulers, chalk erasers, the sides of small boxes,

etc. If no objects of irregular shape are available, cut small polygons (area should not exceed 25 or 30 square inches) from construction paper.

If the children bring into the discussion the square foot and other large units of square measure, you may mention examples (measuring rugs, swimming pools, rooms) where such large units would be used, but at this time do not have the children work with the "square foot" or attempt to visualize and estimate areas containing square feet. The larger square units will be taken up later, on page 245, when the children are ready to handle the changing of square measures.

Page 11: The procedures used for pages 9 and 10 can be applied here also.

To help the children understand why smaller squares are used here to represent square inches, have them count the 14 square inches in Picture F and with their paper inch squares lay out on their desks a rectangle 7 squares long and 2 squares wide. They will readily see, when they compare the rectangles on their desks with the size of a page in their books, that the squares must be smaller on a book page. Use a picture containing people to illustrate that imagining the real sizes of objects printed in books is not something new. Just as they have been imagining the real size of a person in a picture, they are now to imagine the real size of the square inch on page 11.

Again let the children judge whether the areas are about the same size. For example, the area of the polygons in Pictures A and G is about 16 square inches; that of the circle in Picture E is about 12 or 13 square inches; that of the rectangle in Picture F is 14 square inches; that of the polygons in Pictures D and I is 12 square inches.

Ask a child to read the summary beside the flag, and then review with the class what they have learned in this lesson on area.

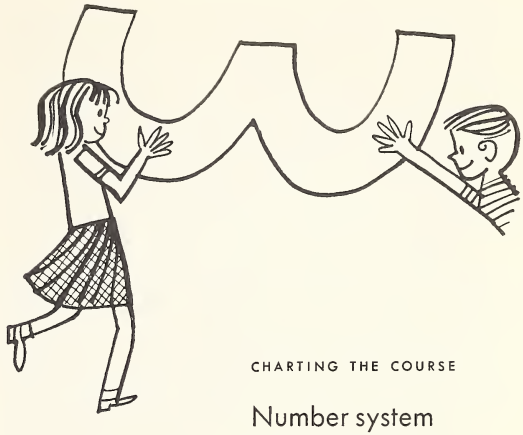
Providing for the able pupil

These children could cover small boxes of various shapes and sizes with different colored inch squares cut from construction paper. After all the sides have been covered, let the children decide whose box has the largest total surface area.

Activity 4, page 358, may be used for these children. They will estimate the number of square inches in polygons and then find the correct answer by placing square inches over them.

Helping the slow learner

Let these children paste transparent square inches on simple polygons cut from construction paper, then label each to tell the number of square inches in its area. Display this work on the bulletin board.



CHARTING THE COURSE

Number system

All of the materials for preceding grades in the *Seeing Through Arithmetic* program* provide many learning experiences centering on the number system. These activities emphasize the basic principles of grouping by tens and of place value. Visual methods show objects grouped in tens, hundreds, thousands, and so on up to millions. Children are shown how these groups may be represented by tallies or by numerals in which position indicates whether 2, for example, indicates 2 "ones," or 2 "tens," or 2 "hundreds," and so on. The aim, however, is not to teach merely how to read and write individual numerals. Instead the *system* is taught, so that the pupil can understand and deal with any numeral by using the principles of the system.

Special attention is given to methods of regrouping that are used in "carrying" in addition. Pupils are shown, for example, how 2 groups of one hundred ob-

*Numbers We See, by Maurice L. Hartung, Henry Van Engen, Anita Riess, and Catharine Mahoney. Scott, Foresman and Company. *Numbers in Action,* by Maurice L. Hartung, Henry Van Engen, and Catharine Mahoney. Scott, Foresman and Company. *Seeing Through Arithmetic 3,* by Maurice L. Hartung, Henry Van Engen, Lois Knowles, and Catharine Mahoney. Scott, Foresman and Company. *Seeing Through Arithmetic 4,* by Maurice L. Hartung, Henry Van Engen, and Lois Knowles. Scott, Foresman and Company. *Our Number Workshop 1* and *Our Number Workshop 2,* by Maurice L. Hartung, Henry Von Engen, and Catharine Mahoney. Scott, Foresman and Company. *Arithmetic Readiness Cards, Set 1: Grouping* and *Set 2: The Number System,* by Maurice L. Hartung, Henry Van Engen, and Helen Palmer. Scott, Foresman and Company.

The Meaning of Plus and Minus and *The Number System,* each eleven minutes in full color, available from Encyclopaedia Britannica Films, Inc., Wilmette, Illinois, or from regional EBF libraries in New York, Dallas, Hollywood, Atlanta. These films were produced by Encyclopaedia Britannica Films, Inc., in collaboration with the authors of *Numbers in Action.*

jects and 14 groups of ten objects can be regrouped as 3 groups of one hundred and 4 tens, and hence the number is 340. Similarly, the principle underlying "borrowing" in subtraction is shown by taking apart a group of 100 objects and arranging it as 10 groups of ten. Thus a total group of 425 objects can be arranged as 3 hundreds, 12 tens, and 5 ones. This is a preliminary step in a subtraction from 425 if the number to be taken away has more than 2 tens—as, for example, 32.

In *Seeing Through Arithmetic 5,* these ideas are reviewed on pages 12-15. The careful development of the number system in preceding grades should make the recall of the principles quite easy.

Roman numerals are also reviewed and greatly extended in a "Side Trip" on pages 16 and 17. Here also the emphasis is upon the system by which the Roman numerals are built up to represent larger and larger numbers.

There is a distinction between "numbers" and "numerals" which teachers should understand but which need not be emphasized with pupils. Numbers are abstract concepts. They cannot be seen, touched, heard, or otherwise be reached by the senses. They are purely mental constructs. Numerals, on the other hand, are concrete symbols for numbers. Numerals, if written or printed, can be seen and touched. Numerals in the form of number words (sounds) can be heard if uttered.

Strictly speaking, then, the discussion in the paragraphs above is primarily concerned with the numeral system in common use today. The Roman numeral system is also still used, but is needed much less frequently. The familiar numeral 5 and the Roman use of V as a numeral illustrate how quite different numerals represent the same number.

Throughout this book the authors have used the words "number" and "numeral" in ways that maintain the distinction whenever it is important or convenient to do so. There are, however, numerous places where the distinction has been reluctantly but deliberately disregarded. For example, the expression "a two-figure number" is to be regarded as a shortened form of the expression "a number written (in base 10) by means of a two-figure numeral." Again, when we write "a number which ends in zero" we mean "a number which is named by means of a numeral that ends in zero."

It is virtually impossible to maintain the distinction between numbers and numerals without using language that seems unduly wordy and awkward. People are so accustomed to the common but sometimes confusing practice of talking about *symbols* for numbers—that is, numerals—as though they were the actual numbers that it will doubtless be many years before the traditional language forms are replaced by more precise terminology.

The authors have considered it unwise at the present time to use consistently the often strange verbal forms that are required to maintain the distinction. Nevertheless, teachers should remember that arithmetic deals primarily with numbers, but in writing arithmetical work and in talking about it, *numerals* are used to represent the numbers in symbolic form.

12-15 Looking back

Lesson Briefs for this lesson are on pages 27-30.

Overview

Children who have learned the principles of the number system through the use of the *Seeing Through Arithmetic* program will have a good background of understanding for their work in the fifth grade. The material in this lesson will help them review place value through the millions and the regroupings necessary for carrying in addition and multiplication and for borrowing in subtraction.

In reading large numbers, the child should understand that each group of three figures is read as hundreds, tens, and ones before the group name is used. He should also understand that in writing a three-figure numeral he uses places for the ones, tens, and hundreds; that in writing a six-figure numeral, the three figures that make up the thousands' part also represent ones, tens, and hundreds (the figure in ones' place stands for thousands, the figure in tens place stands for *ten* thousands, and the figure in hundreds' place stands for *hundred* thousands); that place value for ones, tens, and hundreds is maintained in the same way in writing a nine-figure numeral for millions (the figure in ones' place stands for millions, the figure in tens' place stands for *ten* millions, and the figure in hundreds' place stands for *hundred* millions). The work on page 12 begins with numbers in the ten thousands.

Teaching the whole class

Since some children will not have a thorough background in the number system, you may wish to precede this lesson with some basic work in grouping and regrouping objects and representing the numbers with numerals. Have on hand 10 single objects (such as small sticks), 10 small boxes labeled 10, 10 boxes labeled 100, and 10 boxes labeled 1000. These should

be in graduated sizes, if possible. Tell the children to assume that all the boxes contain the same kind of object. If you find it impossible to collect or make these boxes, drawings on the chalkboard may be substituted.

In a central place put 9 single objects and 2 boxes labeled 10. On the chalkboard draw a three-place tally space. Label the three places *hundreds, tens, ones.* Tell the pupils you want a record in this tally space of the number of boxes of 10 and the number of single objects. Ask a pupil to record the number of boxes of 10 in the space for tens. Then have him record the single objects in the space for ones. Tell him to write the numeral (29) beneath the tally space and read it.

Next place a box labeled 100 beside the boxes of 10. Let another pupil record this amount in the correct place in the same tally space; then have him write the new numeral (129) below it and read it.

To the left of the three-place tally space attach another section of three places, labeling each place (from the right) *ones, tens,* and *hundreds.* Under this section write the word *thousands.* Be sure to draw a longer or heavier line separating the places for the thousands from those at the right. Place two boxes labeled 1000 on the table and ask a pupil in which part of the tally space the numeral 2 should be put. If he has any difficulty, point out that there is a place for ones in the part of the tally space labeled *thousands.* After he has recorded the 2, have him write the complete numeral now represented in the tally space (2129). Direct him to relate each figure to the objects on the table.

Draw attention to the objects again. Remove all but the 9 single objects and put one more object with them. Remind the pupils that when there are 10 ones, we can think of them as 1 group of ten. (Remove the 10 single items and substitute one box of 10.) Tell the pupils that they can record this amount in tens' place in the tally space. Call on a pupil to write this figure in a new six-place tally space. Next place 9 more boxes of 10 on the table so there are 10 boxes in all. Ask how this number can be recorded. The children should remember that 1 can be placed in hundreds' place in the tally space. (Substitute one box of 100 for the ten boxes of 10.) Next, place with it 9 boxes labeled 100. Ask the pupils for another way to think of 10 hundreds. Have the 1 thousand recorded in thousands' place in the tally space. Have the numeral (1000) written below the tally space and read. Be sure the children understand why the zeros are written in the numeral but are not needed in the tally space.

Page 12: Explain to the children that the boxes and bags on this page contain wooden beads and sticks. Ask:

How many boxes of 1000 beads are stacked up in the pyramid in Picture A? Are 10 boxes of 1000 the same as 1 group of 10 thousand? How many other boxes of 1000 are there? How many bags of 100 are there?

Get the children to notice that the tally space has places for six figures. Point out that, starting at the right, there are places for ones, tens, and hundreds, then a longer line, and places again for ones, tens, and hundreds. Be sure the children notice the word *thousands* under the last section of three places. Explain that any figures written in these three places will mean thousands. Point out that to represent any amount in the thousands, at least four places are needed, and that five or six places can be used.

Next let someone explain which of the groups of objects the 1 in ten thousands' place stands for and why it belongs in this place. Use the same procedures with the remaining two figures. Then ask the children to explain why there are no figures in tens' and ones' places. Finally, call attention to the numeral below the tally space and to the number words beneath it that show how to read the numeral. Lead the children to see that the numeral occupies five places, that the two zeros show there are no groups of ten objects or single objects, and that the zeros correspond to the blank places in the tally space.

Draw attention to the comma in the numeral and explain that it makes the numeral easier to read. You can explain that in writing a numeral composed of more than three figures, it is customary to start at the right and set off each group of three figures with a comma. Remind the children again that each group of three figures is read just as if it stood alone and is followed by the name of the group (except for the first group on the right).

For Movie B, the first scene, have the children note where the 9, representing the boxes of sticks, has been put in the tally space. Let someone explain why no other figures are written in the tally space. Then have pupils note how this amount is written as a numeral and in words. Have the numeral read aloud.

Ask pupils to observe what is happening in the second scene. Be sure they realize that now there are 10 boxes of 10 thousand sticks. Let them observe how this amount is represented in the tally space. They should be able to explain that 10 ten thousands can be thought of as 1 group of 100 thousand. Have the numeral read aloud.

Page 13: Picture C can be taught in much the same way as the second scene of Picture B on page 12. Let the children discover that there are 10 boxes of 100 thousand. Then ask them to compare the tally space with the tally space for Picture B. They should discover that another section of three spaces has been provided and that once again the spaces are identified as *ones, tens,* and *hundreds.* They should note also the word *millions* written beneath this section. Ask the children if they know of another way to say 10 one hundred thousands. Some of the children will know that this is 1 million. Have them note where the 1 has been written. Ask them to observe how one million is written in number symbols and in words.

Draw on the board nine boxes, each labeled "1 million." Beside them draw a nine-place tally space with the proper labels above each place and the words *millions* and *thousands* under the two sections at the left. Let someone record the number of objects in the proper place, write the numeral below the tally space, and read it. Then draw one more box labeled "1 million." Ask:

Now how many boxes of 1 million each are there? When we have 10 groups, we think of them as 1 group of 10. In this case, because we are dealing with boxes of millions, we have 1 group of how many millions? Where should I put the figure 1 in the tally space? [*In 10 millions' place.*] *What if I had 12 boxes labeled millions?* [*Draw 2 more beside those you have.*] *We know that we can think of ten of these boxes as 1 group of ten million. I'll put 1 in tens' place of the millions' section. Now, how many other boxes of millions are there? Where should I put the figure 2?* [*In ones' place of the millions' section.*] *Will someone write this numeral on the board?* [*It may be necessary to help the child put commas in the correct places.*]

For the first block of exercises (A to F), remind the children that each group of three figures is read just as if it stood alone, except that it is followed by the group name, like *millions* or *thousands.* Remind them to say something for every figure except zero. Do not let the children say "and" when reading numerals. ("And" should be reserved for reading the decimal point later on, for distinguishing between dollars and cents, and for distinguishing between whole numbers and fractions.) Have the pupils take turns reading these exercises and the block of exercises (A to F) that follows.

Exercises A to R give more oral practice in reading large numbers. If the children ask why no comma is used in Exercise M, explain that four-figure numerals can be read easily without commas.

The children should now be ready to write numerals. Give them ample time to write Exercises A to K in numerals. Let them correct their work by taking turns in writing the numerals on the board.

Page 14: Get the pupils to describe what they see in Scene 1 of Movie A in terms of boxes of thousands, hundreds, and tens. Have them observe that the tally space has places for thousands, hundreds, tens, and ones. Ask if the numerals representing the boxes are in the right places. Help the pupils see that the 16 in the tally space for tens must be reorganized to express the number in the usual way.

Direct attention to the second scene. Get the pupils to tell how the boxes have been reorganized. If they do not understand where the extra box of 100 came from, explain that 10 of the 16 boxes of ten were put together and made into another box of 100. Ask how the figures in the tally space show this regrouping. Point out that each of the two scenes represents the same amount. Let someone read the numeral.

In the first scene of Movie B, get the pupils to explain the 5, 3, and 4 in the tally space in terms of the sticks. They must understand that no figure is entered in hundreds' place because there are no groups of 100 sticks. Have them compare the sticks in the second scene with those in the first scene. They should see that now there are 4 groups of 1000 sticks, 10 groups of 100 (1 group of 1000 has been broken up into hundreds), 3 groups of 10, and 4 single sticks. Have them explain why the 4, 10, 3, and 4 in the tally space accurately represent the sticks in Scene 2.

The tally spaces in Exercises A to E provide practice in writing numerals without help from pictures to show the regrouping of objects. Tell the pupils that the figures and question marks in blue indicate where some change has taken place or must take place. For A, have them compare the figures in the top row with those in the second row. The children should see that 10 of the 15 thousands have been changed to one group of ten thousand and then added to the 7 ten thousands. Help them see that when this has been done, only 5 one thousands are left.

In B, the children should observe that one of the groups of 6 ten thousands has been changed to 10 groups of one thousand. Thus we now have only 5 ten thousands but 12 one thousands.

Get the children to see that in C a change is being made from ten thousands' place to thousands' place. Have them interpret this change and explain what belongs where the question mark is. In D, ten of the 18 hundreds have been changed to one group of one thousand, which will make 6 thousands in

thousands' place. In E, children should note that there were enough thousands to make two more groups of ten thousand, leaving 3 one thousands.

Page 15: Exercises F to K should be handled in much the same way as the exercises on the preceding page. If Exercise J seems puzzling, help the children see that the blank spaces in the top row correspond to zeros in the numeral. Guide them to observe the change in ones' place. Since there are zeros in hundreds' and tens' places, the first numeral that can be changed is the 6 one thousands, which can be reorganized as 5 one thousands 10 one hundreds. One of these hundreds can be changed to 10 tens, and one of these tens to 10 ones, which are put with the 2 ones to make 12. In K, the children should see that 30 of the 31 one hundreds can be changed to 3 thousands. One hundred is kept in hundreds' place, and we add the 3 one thousands to 8, making 11 in thousands' place. But 10 of the 11 one thousands can be changed to 1 ten thousand, making 4 ten thousands and 1 thousand.

Exercises A to L may be answered orally first. Allow the children to draw and use tally spaces if they need to. If it seems desirable, you may also assign the entire block as written work.

Providing for the able pupil

These pupils may enjoy doing outside reading to find numerals in the number groups beyond millions (billions, trillions, etc.). Let them report to the class on their findings. They could illustrate sample numerals by writing them in labeled tally spaces on the chalkboard.

A group of pupils might like to work together in making large charts such as those pictured below. Such charts show how many zeros are used to express quantities from 10 to 100,000,000; what to say in reading numerals by groups; and the names of the places. They also demonstrate that each place (going from right to left) represents ten times as much as the preceding place.

Ask the pupils to look in newspapers, reference books, and magazines for numerals representing large numbers to read to the class. Each numeral should be in a sentence (e.g., "The population of Nevada was 176,322 in 1950."). Either direct the class to omit numerals beyond nine places or simply read them for the class. Satisfy any curiosity the children may have about larger numbers.

These pupils might also make sketches of large quantities (like those shown on page 14 of the textbook) to illustrate large numbers. If you wish, the children can exchange their papers and write numerals for the numbers illustrated.

MILLIONS			THOUSANDS					
hundreds	tens	ones	hundreds	tens	ones	hundreds	tens	ones
							1	0
						1	0	0
					1	0	0	0
				1	0	0	0	0
			1	0	0	0	0	0
		1	0	0	0	0	0	0
	1	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0

MILLIONS			THOUSANDS					
hundreds	tens	ones	hundreds	tens	ones	hundreds	tens	ones
	1	1	0	0	2	8	0	0
		5	0	0	7	3	1	0
			3	5	0	0	0	5
2	7	8	7	2	0	0	7	0
				4	9	0	0	0
					8	2	0	6
4	0	5	0	3	1	0	6	0

The game "Number spelldown" can be played by the able pupils while you are working with the slow learners. See Activity 5, described on page 359.

Helping the slow learner

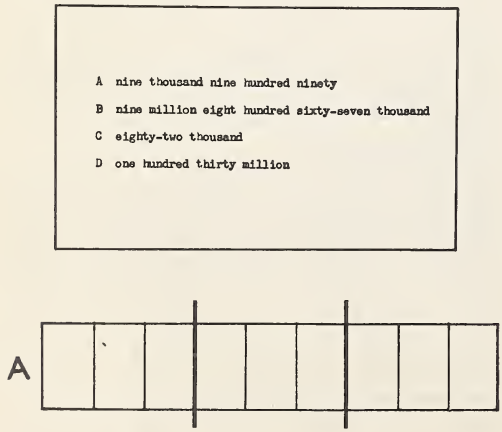
Give these children boxes (labeled 10, 100, and 1000) and single objects and have them use these boxes to show the meaning of numbers. Then prepare a few more boxes (in graduated sizes, if possible) to give practice with numbers in the 10 thousands, 100 thousands, and millions. You can limit the numerals above four places so that you will need only three boxes for each group. Label them *ten thousands, hundred thousands, millions, ten millions,* and *hundred millions.* Let the children work together in a group. (To illustrate the number 13,976, they would

need 1 box labeled 10,000, 3 boxes labeled 1000, 9 boxes labeled 100, 7 boxes labeled 10, and 6 single objects.)

To help the slow learners identify the various places correctly, work with a group of them in this way. Draw a nine-place tally space, labeling the three places in each section *ones, tens,* and *hundreds.* Beneath the appropriate sections write *millions* and *thousands.* First write the numeral 9 in ones' place and have a child read this as "9 ones." Then have him write it outside the tally space and read it. Erase the 9 in ones' place and write 9 in tens' place. Have it read as "9 tens," and written and read as "90." Continue writing 9 in each place, getting the children to tell what number is represented and having them write a numeral for it. You might ask questions like these, letting the children use a tally space to find answers, if necessary:

How many places do you always have for any number in the hundreds? What is the largest number that you can write with three figures? [*999*] *If you add 1 to 999, what number do you have? How many places do you need to write 1000? What is the largest number you can write with six figures?* [*999,999*] *What is one more than 999,999? How many places does it take to write this? What is the largest number you can write with nine figures?*

To give these children more experience in writing numerals for large numbers, prepare, or have the children prepare, sets of tally spaces with places through the millions (nine places) for them to fill in. Label each tally space with a letter. Also on the sheets prepare sets of number words (each labeled with a letter). The pupils are to translate the number words into number symbols and write the symbols in the tally spaces. A set of tally spaces and a set of number words may be placed on the same work sheet, if you wish. See the samples in the picture in Column 1.


A nine thousand nine hundred ninety
B nine million eight hundred sixty-seven thousand
C eighty-two thousand
D one hundred thirty million
A

A game called "Number along" can provide practice in regrouping. See Activity 6, page 359, for complete details.

16-17 Side trip

Lesson Briefs for this lesson are on pages 31-32.

Expanded Notes are not considered necessary for this lesson except for suggestions for the able pupil and the slow learner.

Providing for the able pupil

Let able pupils consult a children's encyclopedia and make a special report on Roman numerals. Ask them to find out why the letters I, V, X, etc., were used.

For another activity for the able pupils, have available a group of books with chapter numbers in Roman numerals. Also, have prepared a sheet (see the picture below) on which you have designated certain chapters in these books. Have the children fill in the number of the chapter in both Arabic and Roman numerals.

Book	Chapter	Roman numeral	Arabic numeral
Mystery in the Attic	The Trunk	V	5

From time to time let the able pupils translate the answers to exercises (such as those in the "Checking up" on page 18) into Roman numerals.

Helping the slow learner

These children might find the making of a Roman numeral chart helpful. See Activity 7, page 359.

18 Checking up

Lesson Briefs for this lesson are on pages 33-34.

Overview

In *Seeing Through Arithmetic 5* there are 69 tests that may be used to inventory what has been learned or to measure achievement. These tests will be found under the heading "Checking up." The first set is on page 18 of the child's book. Seven of these sets of tests are End-of-block tests that test the child's knowledge of topics just studied or abilities just acquired.

This enables the children and the teacher to use the tests *diagnostically*. If a child cannot respond, or if he makes errors that are not just the occasional careless slips characteristic of childlike behavior, he needs more learning experiences before going ahead with new work. These tests make it possible to know on which areas to focus further teaching.

It is true that children have difficulty in reading or studying typical arithmetic texts, with the result that both teacher and pupils come to regard the textbook chiefly as a source of practice exercises. In preparing *Seeing Through Arithmetic 5,* much attention was given to making a book that children could read, study, and understand. The pictures closely relate the learning to concrete situations. The complete, yet simple, explanations are written in a carefully controlled vocabulary. The best material for any needed reteaching, therefore, is found on the pages where the explanation was originally given. It should be a common experience for children to reëxamine materials they have studied earlier.

Some of the "Checking up" tests include mixed types of exercises (see, for example, page 111). These tests are more appropriate for securing achievement scores than for diagnosis. The End-of-block tests are primarily diagnostic.

The tests on page 18 require knowledge of the addition and subtraction processes and of the multiplication and division basic facts through the 36 group, all of which were taught in *Seeing Through Arithmetic 4*. For this reason, references for reteaching pertain to the Grade 4 book. (See "Comments" in the Lesson Briefs, page 33.) Reteaching references for later tests refer to those pages in Book 5 where the skills being tested were first taught. When test results indicate that a child has forgotten something or needs additional learning experience, help him to find and restudy the appropriate pages in his book. Your pupils should gradually become accustomed to looking back on their own to pages studied earlier when they are uncertain about how to get an answer to a problem or a computational exercise.

Providing for the able pupil

As a review of the multiplication and division basic facts, have these children arrange the facts according to number groups. At this point the multiplication facts should be limited to those with products of 36 or less and the division facts to those with dividends of 36 or less. See Activity 8, page 360.

After the able pupils have completed the review of addition on page 18, they might find it interesting to try adding numbers in a different way. For example, they might try adding the ones, tens, hundreds, etc., separately, and then adding these partial sums for the final answer. In using this method the children should see that they can add in any order. They can start by adding the ones, as shown in the first example, or by adding the hundreds first, as shown in the second example.

373	
628	741
470	895
11 (sum of the ones)	1500 (sum of the hundreds)
160 (sum of the tens)	130 (sum of the tens)
1300 (sum of the hundreds)	6 (sum of the ones)
1471 (final sum)	1636 (final sum)

The children can make up examples of their own for this activity, or they can use the examples in Test 3 on page 18. After they have worked a few examples in this way, they should be able to see that the way in which they ordinarily add is simply a short cut for this method.

In connection with the review and reteaching of the four processes, the able pupils should enjoy working with number sequences in which it is necessary to discover the relationship between the numbers in an arithmetical progression. In this activity the pupils are to find how each number in a given sequence was arrived at and to find several numbers in the sequence beyond those already given. They may even make up some number sequences of their own for their classmates to work with.

This activity may be used in connection with each "Checking up" section that reviews the four processes, including addition and subtraction of fractions. The number series used can be related to the process or processes covered by a particular "Checking up." If the test contains a review of subtraction of whole numbers, for example, the numbers in the series may be determined by subtracting whole numbers.

For a few examples of the types of number sequences to be used in connection with the various "Checking up" treatments, see Activity 9, page 360.

Helping the slow learner

If any of the children are having trouble with addition or subtraction basic facts, let them write the troublesome facts on cards and eliminate the cards as the facts are mastered. Encourage them to eliminate the cards as quickly as they can.

The game of "Target toss" can furnish practice on column addition of one-figure, two-figure, and three-figure numbers. See Activity 10, page 360, for instructions for playing the game and making necessary materials.

"Cork drop," Activity 11, page 361, is a game that can give practice in adding two-figure and three-figure numbers.

To help these pupils recall multiplication and division basic facts, let them start making a two-way basic fact chart. See Activity 12, page 361, for details.

A game that will give these children practice on basic facts is described in Activity 13, page 362.

CHARTING THE COURSE

Problem solving — the equation

The *Seeing Through Arithmetic* program uses distinctive methods to help pupils learn how to solve problems. For each type of problem the pupil is shown how to form an equation that tells what is happening in the problem situation. For example, suppose the problem is as follows: "The fifth-grade class had 24 arithmetic books. Then they got 8 new ones. How many did they have altogether?" In *Seeing Through Arithmetic 3,* pupils are taught how to write the equation 24+8=▭ to represent this situation. In this additive situation the number in each group (24 and 8) is known, and these numbers can be added immediately.

Sometimes the answer is not immediately obtainable by the process indicated in the equation. For example, suppose the problem reads: "The fifth-grade class had 24 new arithmetic books. There were 32 pupils in the class. How many more arithmetic books were needed?" In this example, the equation suggested by the situation is 24+▭=32. The answer is not obtainable by addition. In *Seeing Through Arithmetic 3,* pupils are shown a way of thinking about the situation that suggests that the indirect process of subtraction be used.

Other problems introduced in *Seeing Through Arithmetic 3* include those for straightforward subtraction situations that lead to equations like 8−5=▭, and comparisons by subtraction of two groups of known size that lead to similar equations. Still another situation arises when it is necessary to find how many there were to begin with when the number added and the resulting total are known. These additive situations lead to equations like ▭+3=12, and the indirect process of subtraction is used to find the answer. Problems of this type are also included in *Seeing Through Arithmetic 3.*

These types are all thoroughly retaught in *Seeing Through Arithmetic 4.* In conformity to the general plan of this program, materials are provided for a brief review of these same types in *Seeing Through Arithmetic 5* on pages 19-22. In this book, however, the symbol *n* replaces the use of the screen (▭) for "how many" *in equations.* The screen is still used for "how many" when the numeral that will replace it is not the final answer to a problem.

When a group is separated into equal subgroups, the situation is called *divisive.* In one kind of divisive situation, the number of objects in the original group is known and the number of objects in each of the equal groups is also known. For example, suppose the problem reads: "Mrs. Fisher has asked the girls to plant 12 flower bulbs in bowls. They are to plant 3 bulbs in each bowl. How many bowls will the girls need?" In this example, each of the equal groups contains 3 bulbs; so the bulbs can be picked up three at a time and put into bowls. The number of groups of 3 is to be found. This situation is expressed by the equation 12÷3=n. Situations of this kind are called *quotitive division* (or "measurement" division) situations.

In another kind of divisive situation, the number in each of the equal groups is not known. This number is what is wanted. The number of groups is, however, known. For example, suppose the problem reads: "Kathy and Jane Fisher are going to polish teaspoons for their mother. There are 16 teaspoons. If the girls share the work equally, how many teaspoons should each girl polish?" In this example, there are to be 2 equal groups, and the problem is to find how many are in each group. This situation can be represented by the equation 16÷n=2. Situations of this kind are called *partitive division* (or "sharing") situations.

The key to understanding problems of this kind consists in seeing that enough objects can be taken at one time (for example, 2 spoons) so that there is one for each group. This process may be repeated as many times as there are groups to be formed. This shows the

answer can be found by using quotitive division. In the example used here, $16 \div 2 = 8$ gives the number of times a group of 2 objects can be formed from a group of 16. Thus eventually a group of 8 objects can be formed by each girl, and this answers the original problem.

Simple situations of both quotitive and partitive types were introduced in *Seeing Through Arithmetic 3* by visual methods. They were carefully retaught in *Seeing Through Arithmetic 4*. In *Seeing Through Arithmetic 5* they are briefly reviewed on pages 24-26. In conformity to the plan followed throughout the program, these pages also provide for a review of simple multiplicative situations which lead to equations like $3 \times 8 = n$. These situations also were introduced in the third grade.

19-22 Exploring problems

Lesson Briefs for this lesson are on pages 34-37.

Overview

The "Comments" on pages 34-35 provide an extensive overview of this block of work.

Teaching the whole class

Remind the children that in the preceding lesson they reviewed addition and subtraction of numbers expressed with numerals of one, two, and three figures. If necessary, review briefly the use of the plus and minus signs in an equation. (They indicate the combining and the separating of groups.)

Page 19: Have the children read Problem A. Ask them why this problem requires an equation with a plus sign. Say:

Look at the first equation in the picture at the right. What does the square [or screen] stand for? Look at the next equation in the picture. Does this also describe the problem? Look at the last equation. Here the letter n *holds a place for the answer. From now on, in this book, you will find that* n *is used instead of a screen to hold a place for the answer in an equation. Now look at the equation under the problem. This equation tells the story of the problem.*

Be sure the children understand what each of the numerals in this equation stands for. Ask someone to go to the chalkboard, write the numerals in computational form, and add. The rest of the pupils may do the work at their desks. If necessary, remind the children to line up the points under each other and write the dollar sign with the first numeral and the answer. When they have found the answer, proceed somewhat as follows:

Now imagine the $3.89 put with the $.98 instead of the way you just wrote the numerals. Would the answer be the same? Bill, write the equation on the board that goes with this way of doing it. Now put the numerals in computational form and add. Is your answer the same as when $.98 was added to $3.89? Whenever you imagine groups combining, it does not matter which numeral you write first in the equation or in the computation.

Call attention next to Problem B. Help the pupils see that this problem calls for a subtraction equation. Ask the pupils to be ready to explain the equation. Then call on a pupil to compute at the chalkboard to find the answer. The other children can verify his answer by doing the same work at their desks.

Continue in the same way with the rest of the problems. Help the pupils see that in Problem D, they can imagine the number of girls as gone from the total. Then the number left is the number of boys.

Page 20: Ask the pupils to read Problem A. Help them see that the first scene of Movie A shows the 24 books the class had. Let them tell what else the picture shows. [More books are being brought in.] Make certain they realize that they cannot tell how many more books are being brought in. Have them look next at the equation at the right and ask them to explain it in terms of Scene 1. Be sure the pupils understand that they cannot add to find the answer because the number being added to 24 is to be found.

Direct attention to the second scene. Help the pupils see that this scene shows the 32 books needed. Remind the pupils that they already know how many books the class had. Point out that 24 of the 32 books are dimmed and, if necessary, explain that this suggests the removal of part of the group. The books in strong color show the number of books brought in. Ask if subtracting will show us how many books were brought in. Have someone go to the chalkboard and subtract 24 from 32.

If any pupils have trouble understanding this, dramatize the action with objects. Collect 32 books, and have two children demonstrate why an addition equation must be used and why removing the 24 books, shown by subtraction, produces the correct answer.

Next direct attention to Problem B. Be sure the pupils understand from reading the problem that Nancy had an unknown number of crayons to begin with and that Scene 1 of Movie B shows that she bought a new box of 16 crayons. Then have the pupils look next at the equation at the right and ask them what the numerals and the letter *n* stand for. Then ask them why they cannot find the number that *n*

represents by adding. Tell them to look at the second scene of Movie B. They should see that this scene shows the number of crayons that Nancy now has. Then suggest that to find out how many crayons Nancy had at first, the new box of 16 crayons can be thought of as gone. The dimming of 16 crayons suggests this, and the action suggests the process of subtraction. Let the pupils count the crayons remaining in strong color. Then ask a pupil to go to the board and subtract 16 from 23.

If necessary, repeat the entire problem-solving procedure with objects (preferably crayons), so that the slower-learning children can see why, even though the situation is additive, the process to use in computing is subtraction.

Page 21: Have the pupils read Problem C. In the first scene of the movie get them to identify the two groups in terms of the problem. Then call attention to the first statement at the right of the second scene. They should see that 5 of the 16 pictures have been matched with the 5 scrapbooks. Suggest that removing the 5 matched pictures from the total number of 16 pictures will show how many more pictures there are than scrapbooks. Get them to explain the equation. Then have them look at the final scene and see that the 5 matched pictures are dimmed and can be imagined as gone. Consequently, they should subtract 5 from 16. Let a pupil write on the chalkboard the equation with 5 replacing *n*.

Finish the lesson by using objects to compare groups. By removing the matched objects in the larger group, you can show how to find how many are unmatched.

Page 22: Discuss the first three problems. Be sure the pupils can explain the numerals, sign, and letter *n* in each equation. For each problem, have a pupil put the computation on the chalkboard and find the number that *n* represents.

Point out that in Problem D the question asked is, "How many *fewer* fifth graders went on trips?" The answer would be the same if the question had been "How many *more* fifth graders stayed at home?"

Assign Problems G to N as written work. For each problem have the pupils (1) write an equation that tells the story, (2) compute to find the answer, and (3) rewrite the equation with the answer in place of *n*.

Supply answers so that each pupil can verify his own work. When all the pupils have verified their work, find out which problems caused the most trouble and center class discussion on them. For problems that only a few children missed, let those who got the correct answers work with the other children and explain the solutions.

For any problems missed by most of the children, have the solutions put on the board, if possible. The solutions may then be discussed.

Providing for the able pupil

Problems involving the five types just studied may be cut from old textbooks and pasted on 3″ x 5″ cards. From a file of such cards the children may select problems to solve. Be sure the problems used do not exceed the range of the processes taught. See Activity 14, page 362, for complete details.

A group of able children may play a game entitled "Make up a problem." A child must make up a problem for an equation written on the board by another child. See details in Activity 15, page 363.

Helping the slow learner

Find out which types of problem situations give these children the most trouble. Then adapt Activity 14 (see page 362) as follows: Prepare simple problems that involve only basic facts. Then ask the pupils to write an equation for each and use the letter *n* to hold a place in the equation for the answer. (If necessary, let the pupils use objects to make the groups indicated by the problems.) Have them demonstrate the action (real or imagined) to find the answer. Finally, ask them to rewrite each equation with the answer in place of the letter *n*. An able pupil may supervise the work of a group of slow learners.

The slow learners who have trouble with problems that involve how many more or fewer there are in one group than another may use the following activity. Put several sets of two kinds of objects on a center table (7 crayons, 16 pencils, for example). Then let the children take turns making up problems about how many more or how many fewer there are in one group than in another. The child who makes up a problem may call on someone else to solve the problem with objects and then write the equation and do the computation necessary to find the answer.

Children who have difficulty with the "How many more were added" type of problem should be given opportunities to dramatize situations of this type. Small objects may be given them to work with. The children can work in pairs, a slow learner with an abler pupil. The slow learner can select a number of the objects, and his partner can give him some more, without telling him how many. Then the first child should perform the actions necessary to find how many were added. As a result of such dramatization, the child should write the equation and do the computation.

Children who have difficulty in finding how many there were to begin with may try the following ac-

tivity. Let them work in pairs. (An able pupil might be paired with a slow learner.) Each pair needs a box of objects (shell macaroni, stones, jacks, corn, corks, etc.) and a cloth about 12 inches square. One child should take a handful of the objects and, without counting, put them under the cloth. The other child, then, should take a group of objects out of the box and count them. He puts these on top of the cloth. Then one of the children removes the cloth so that all of the objects fall into one group. The other child counts all of the objects. Then they are to find how many objects were put under the cloth. This they can do, of course, by removing the number that were in the known group from the total group and counting the remainder. The children should write an equation to tell what happened, using n to hold a place for the answer, and then compute to find the answer.

23 Thinking straight; Keeping skillful

Lesson Briefs for these lessons are on page 38.

Expanded Notes are not considered necessary for these lessons.

24-26 Exploring problems

Lesson Briefs for this lesson are on pages 39-41.

Overview

Three types of problem situations are reviewed in this lesson: (1) finding the size of the equal groups when the total group and the number of equal groups are known (partitive division); (2) finding the number of equal groups when the total group and the size of the equal groups are known (quotitive division); and (3) finding the total when the size and number of equal groups to be combined are known. The lesson ends with a problem set that tests the ability of the child to identify problem situations, make equations to describe them, and find the answers.

Children who have used the *Seeing Through Arithmetic* program in earlier grades should have no trouble in analyzing and solving these three problem types. For other children, the partitive division type should receive the greatest emphasis. Note that, although the computational procedure is the same in this type as in the quotitive division type, the equation is different. It is important that the children understand what each part of a division equation stands for. They should understand that the first numeral is the total group, the numeral after the division sign is the size of the equal groups into which the total group is to be divided, and the numeral after the equals sign is the number of equal groups.

Teaching the whole class

Page 24: Have the pupils read Problem A and observe the first scene of Movie A. Ask the pupils how they know that the teaspoons are going to be *divided* equally. When they are certain that a division equation is necessary, direct attention to the equation. Ask them to explain why the letter n is placed after the division sign. [Because the size of the equal groups is unknown, and the numeral representing the size of the group must always come after the division sign.] Be sure they know what the 2 after the equals sign stands for.

Tell the pupils that the next scene and its explanation suggest a way to find the missing number. Have them see that, since each girl has removed 1 spoon, a group of 2 has been removed from the total group of 16 spoons. Tell the children that the girls will continue taking a spoon apiece (or a group of 2 spoons) away from the 16 spoons until all are gone. Make clear that the third scene shows the final result.

If possible, have a group of 16 objects on hand to represent the spoons. Let two pupils remove groups of 2 spoons from the whole pile, each pupil putting 1 spoon in front of him each time. When the spoons have been divided into two equal groups, have the spoons in each group counted.

Now tell the pupils that, to get the answer for $16 \div n = 2$ by arithmetic, they can think of the problem as one of finding how many groups of 2 there are in 16.

Now ask a pupil to divide 16 by 2 and compare the answer with the spoons in each group in the last picture. Have someone read the last statement on the page and supply the missing number. Then tell all the pupils to put this number in place of n and divide to see if they get 2.

To conclude the work on this page, you may ask the children to make up other "sharing" type problems. They can call on their classmates to solve the problems, first with objects and then by arithmetic.

Page 25: After the children have read Problem B, have them look at the first scene of Movie B to verify the statements made in the problem. Ask how they know that this problem is about the combining of groups. Help them see that the groups to be combined are *equal* and that it is, therefore, a multiplication problem.

Have them look next at the equation. Let someone read aloud the explanation for each numeral and the letter. Ask him to give the answer and verify it by

the second scene of Movie B. You may wish to point out that the numeral standing for the number of equal groups always comes before the times sign and the numeral standing for the size of the equal groups comes after the times sign. You might let the children take turns making up other simple multiplication problems that involve basic facts and call on someone to write the equation and give the answer for each.

Turn their attention next to Problem C. Let the pupils read the problem and decide whether groups are to be combined or separated and whether they are equal groups or not. They should decide that this problem requires a division equation. Let them discuss the first scene of Movie C to determine that, while it shows the total number of flower bulbs, they cannot be sure how many of the bowls pictured will be needed.

Discuss the equation and the explanatory phrases. Call on a pupil to give the answer and explain what it means. Explain that a bowl can be matched to each of the 4 groups, as shown in the second scene of Movie C. Let someone read the last sentence and complete it.

Page 26: Tell the pupils that on this page they will find problems of the three kinds just studied. As they read each problem, they are to decide which kind of problem it is and write the equation that describes the situation (using n to hold a place for the answer). Then they are to find the answer and rewrite the equation, putting the answer they found in place of the letter n.

If you think it necessary, discuss Problem D with the children and help them formulate the equation. Discuss why the equation is made as it is.

When all the children have completed the work, supply answers and let them verify their own work. Discuss any problems that caused trouble. Use objects, if necessary.

Providing for the able pupil

Problems involving the three problem types just studied may be cut from old textbooks and pasted on 3″ x 5″ cards. From a file of such cards the children may select problems for which they can make equations and find the answers. Be sure the problems used include only the multiplication and division basic facts reviewed up to this time. See Activity 14, page 362, for complete details.

A group of able children may play at the chalkboard a game entitled "Make up a problem." A child must make up a problem for an equation written on the board by another child. See complete details in Activity 15, page 363.

Helping the slow learner

If these children have difficulty with problems involving partitive division, give them many opportunities to act out sharing situations with objects. Then have them write the problem situations in equation form and divide to get the answers.

Find out also if any other type of problem situation gives these children trouble. Then adapt Activity 14, page 362, as follows: Let the children use objects to make the groups and separate or combine them as indicated by the problem. Direct them to make an equation. Then have them verify their answer by arithmetic.

CHARTING THE COURSE

Multiplication and division basic facts

In the *Seeing Through Arithmetic* program, the multiplication basic facts and the division basic facts are organized for teaching purposes into two major groups. Products of 10 or smaller (and the corresponding quotients for dividends of 10 or smaller) are introduced informally in *Numbers in Action*. These products and quotients are retaught in *Seeing Through Arithmetic 3*, and the facts whose products (and dividends) are 36 or smaller are taught by visual methods. In *Seeing Through Arithmetic 4* these basic facts are all retaught, and the basic facts with products and dividends from 40 to 81 are introduced.

To complete the teaching of basic facts, there remains the task of systematically reteaching the basic facts which have products and dividends from 40 to 81. Materials for this reteaching are provided on pages 27-36 of *Seeing Through Arithmetic 5*. The methods used involve the grouping of objects in various ways and make use of the pupil's knowledge of the number system. For example, 40 objects can be arranged so that there are 5 rows of 8 and, at the same time, 8 rows of 5. The

same 40 objects can also be seen as 4 groups of 10 objects each, and hence the total is known to be 40. Thus the pupil can see that $5\times8=40$ and $8\times5=40$. Similar methods are used for reteaching the division basic facts.

Pages 37 and 38 provide materials for a review of the remainder in division. This topic was also taught earlier in both Grades 3 and 4 of the *Seeing Through Arithmetic* program.

27-30 Looking back

Lesson Briefs for this lesson are on pages 41-44.

Teaching the whole class

Page 27: To begin this lesson, have a pupil read the introductory statement at the top of the page. If you wish to see how much the children remember from the previous grade, have them use the first block of equations on page 30 as a written inventory test. By making such an inventory of what the children know, you will be aided in determining how detailed your teaching needs to be. Remember, however, that the children must know these basic facts thoroughly before they go on to the processes of multiplication and division. Taking time with this lesson will pay dividends later on.

Ask the children to count the masks in the first horizontal row in Picture A. Then ask how many rows of 8 masks there are. Get the children to think of the 5 rows of 8 masks as 5 groups of 8 which are combined. Discuss what the 5 and 8 in the first statement stand for in terms of the picture. Point out that when the 5 groups of 8 masks are thought of as combining into one large group, the multiplicative action can be expressed as it is in the second and third lines of the text. In discussing the third line, point out that the number of groups comes first in the equation and the number in each group comes next; thus, 5×8 means 5 groups of 8 masks each, or a total of 40 masks. Make sure the children understand that each circled group contains 10 masks and that there are 4 groups of 10.

Now have someone read the line of text describing the reverse basic fact for 40. Let the class study the 8 vertical rows, or groups, of 5 masks. Then, as before, discuss the last two lines of text and make sure that the children understand what each numeral in the equation stands for.

Picture B can be handled in much the same way as Picture A. You may ask questions like these:

How many horns are in the first row across the top? Are there 7 horns in each of the other horizontal rows? How many rows are there? Can we say that there are 6 groups of 7 horns? Now think of the 6 groups of 7 horns as one large group. In the first two lines of text for Picture B, what numeral should be read where the screen is? In the third line, what does n *stand for? How many tens are there? How many single horns are there? There are how many horns altogether? In the equation* $6\times7=n$, *what does the 6 stand for? The 7? The* n?

In like manner discuss the reverse multiplication fact for the 42 group. Have the children note the number of horns in each vertical row and the number of vertical rows, and then find the total number of horns by counting the groups of 10 and the single horns. Let several children read the lines of text and supply the missing numerals.

Discuss Picture C in a way similar to the procedures outlined for Pictures A and B.

Page 28: Adapt the discussion and questions for Pictures A and B to Pictures D, E, and F. Note that for Picture E there is only one multiplication basic fact. Be sure the pupils understand why. The first block of equations at the bottom of the page (A to K) contains the multiplication basic facts just reviewed. In the second block (A to L) the facts just reviewed are mixed with multiplication basic facts studied previously. Let the children take turns reading the equations in both blocks and supplying the answers. Then have them write the complete equations.

Page 29: The multiplication facts on this page may be handled in somewhat the same way as those on pages 27 and 28. Note that Picture I has only one multiplication basic fact.

Page 30: Use the method suggested for the previous pages to reteach the multiplication basic facts for the 72 and 81 groups. If you used the equations at the bottom of this page as a diagnostic test, you may use them again now as a retest. The second block of equations will serve to check how well the children know the multiplication basic facts in general.

It is important at this time to make sure individual and group difficulties are cleared up before new work is introduced. If necessary, form special groups for eliminating difficulties that still exist.

To give your class extra practice with the multiplication facts, choose from the activities mentioned below those which you feel will give your pupils the kind of review they need most.

As one review of multiplication basic facts, have the children write on a sheet of paper a number group—for example, 40—and beneath it write all the multiplication basic facts that have a product of

40. In this way have them review all the multiplication facts studied up to this time. (See Activity 8, page 360.)

The children may also make multiplication practice cards that have the basic facts for any given group in equation form on one side and the products on the other. (See Activity 16, page 363.) Working individually or in pairs, the children should turn the equation side of the card up and then write or say the product. They can verify their work by using the reverse side of the card.

A child who has difficulty with some of the basic facts may make another kind of multiplication practice cards. On one side of a card have him write, with the answer, a basic fact he finds troublesome and, on the other side, the basic fact with *n* where the answer belongs. Have him practice by first reading or writing the facts with the answers and then saying or writing them without looking at the answers. As he learns each basic fact, let him eliminate that card from his set. (See Activity 17, page 363.) The desire to eliminate cards should motivate the child to learn the facts.

The children may also make "tables" of the multiplication basic facts. They may make tables for all the groups studied or for just the groups that are most difficult. (See Activity 18, page 364.)

Multiplication tables may be written on practice cards with the size of the group indicated on one side of the card and the tables in equation form on the other side. (See Activity 19, page 364.)

A multiplication basic fact chart may be made for reviewing all the basic facts. If the class started making such a chart in connection with the work on page 18, it can be completed now. (See Activity 12, page 361.) Be sure everyone in the room knows how to use this chart.

The whole class may participate in a "football game" that provides practice on the multiplication basic facts. If you wish to give your class special practice with the larger groups, cut out four cardboard footballs and number each football 6, 7, 8, or 9. (See Activity 20, page 364.)

"All-in-a-row" may be played by several pupils or by the whole class. Divide 5″ x 5″ cards into 16 squares and write in the squares numerals for the number groups you want your class to review. (For this lesson, you will probably use the 40 to 81 groups.) Arrange the numerals differently on each card. Follow the directions given for Activity 21, page 365.

Providing for the able pupil

During the special activities let the able children work with the slow learners. In this way you can have more groups working at the same time, and you will be free to supervise all the groups and to give special attention to the slow pupils.

The able pupils may also help make the practice cards and charts that the slow learners need for extra practice.

Of the various activities described above, those most useful to the able pupils are: writing multiplication equations for the number groups just reviewed, Activity 8, page 360; making multiplication tables, Activity 18, page 364; completing the multiplication basic fact chart, Activity 12, page 361.

"Which one," a game that is illustrated for division in Activity 22 on page 366, may be adapted for multiplication and used here.

Helping the slow learner

To give the slow learners more opportunity to visualize the multiplication basic facts, have them arrange objects in groups to represent multiplication equations. Then, by imagining the objects as arranged in groups of 10, they can determine the answer (product).

Working with objects may be followed by arranging small paper squares in groups representing multiplication equations. (See Activity 23, page 367.)

Other activities especially useful to the slow learner are: making multiplication practice cards for the difficult basic facts, Activity 17, page 363; using the multiplication chart for all difficult basic facts, Activity 12, page 361; putting the multiplication tables on practice cards, Activity 19, page 364; participating in the game "All-in-a-row," Activity 21, page 365; and "Number football," Activity 20, page 364.

31-36 Looking back

Lesson Briefs for this lesson are on pages 44-47.

Teaching the whole class

You may use the first block of equations (A to S) on page 36 to inventory the pupils' knowledge of the division basic facts reviewed in this lesson. From the results you will be able to tell what you must reteach in this lesson. If there is a substantial class weakness, you will want to take plenty of time to clear up any imperfect learning. Use as many of the following procedures and activities as you feel are necessary. Explain to the children that since this lesson reviews some of the division basic facts they learned in Grade 4, it is wise to see how well they remember these facts.

Page 31: Ask a pupil to read the statement beside the heading.

Let the children examine Picture A. Have them observe the number of birds in the first horizontal row. When the children find that there are 10 birds, ask them how many rows of 10 birds there are and how many birds there are in all. Then have the first line of text read. Explain that this lesson shows how a large group of objects can be divided into smaller equal groups. Ask how many birds are in each group and into how many equal groups the 40 birds have been divided. Summarize by saying that the 40 birds have been divided into 5 groups, each containing 8. Have a pupil read the rest of the text and supply the missing numerals.

Go on to Picture B and say:

This picture shows the same birds, arranged in the same rows. But here the 40 birds are divided into equal groups in another way. How many birds are in each group? How many equal groups are there? Picture B shows us, then, that 40 birds can be divided into 8 groups of 5 birds in each.

Call on several children to read the text for Picture B and supply the missing numerals.

To visualize the divisive action, first let the children divide 40 objects (beans, paper clips, kernels of corn, etc.) into groups of 8 and determine the number of equal groups. Then have them separate the 40 objects into groups of 5 and see how many equal groups there are now. The abler children may find other ways to separate their objects into equal groups (4 groups of 10, 10 groups of 4, etc.), but do not stress these, since they do not represent *basic* facts.

Pictures C and D may be taught in the same way as Pictures A and B.

Page 32: You may use the procedures given for page 31 to reteach the division basic facts on this page.

Page 33: For Pictures I, J, and K, adapt the procedures used for pages 31 and 32.

Use the equations at the bottom of the page to test the children's knowledge of the division basic facts reviewed so far in this lesson. Before doing this, however, you may wish to use one or more other activities to give the children extra practice.

Provide sheets of paper on which are nine rows of X's arranged like the objects shown in the text. Give each child two sheets for each of the number groups 40, 42, 45, and 48 and one sheet for 49, which has only a single basic fact. Label each sheet with the number group it represents. Direct the children to cross off all X's not needed for the group. Then have them write the two basic fact equations for each number group, one on each of the two sheets, and then show the divisive action for each by drawing circles around the equal groups. The abler children may wish to use more sheets to show ways other than those used in the book for circling equal groups. Also, the abler children may prepare rows of X's for the less able children to use in showing division facts.

When the children are competent in working with this exercise, call on them to read and supply the answers for the equations at the bottom of page 33. Note that the first block is made up of the division basic facts just reviewed, while the second block contains other basic facts studied previously. After oral practice, the children may write the complete equation and the answer, or just the identifying letter with the answer beside it. As soon as you feel the children have learned this group of division basic facts, go on to pages 34 and 35, where the remaining division facts (quotients of 56, 63, 64, 72, and 81) are retaught.

Page 34: Adapt the procedures used for pages 31-33 to this page.

Page 35: The procedures used for teaching pages 31-34 may be adapted to this page. Again have the children separate objects into equal groups. Then pass out duplicated sheets of X's for the 56, 63, 64, 72, and 81 groups. Only one sheet is needed for 64 and 81, since each has only a single basic fact. Again encourage the abler children to circle equal groups in as many ways as they can.

Before assigning the equations on page 36, you may wish to use one or more other activities. Use only those activities that will give your pupils the practice they need most.

For general review, let the class use the multiplication and division chart for all the division basic facts through the 81 group. (See Activity 12, page 361.) The slow learner should use this chart to verify the basic facts he finds difficult.

All the children should practice writing "tables" for several number groups (see Activity 18, page 364). If you wish, have each child make tables for the groups that give him the most difficulty. Such tables will help the pupils to see relations among the facts.

Any child who is having difficulty may write on special practice cards the facts he does not yet know. He should eliminate the cards, one by one, as he learns the facts. (See Activity 17, page 363.)

Master cards and division basic fact cards may be made for "All-in-a-row," and used by the whole class or small groups of children. (See Activity 21, page 365.)

Choose from the activities given below for the able pupil and the slow learner any that will benefit the whole class.

Page 36: Have the class use the first block of equations (A to S) as a retest.

The class may take turns giving the answers to the second block of equations orally. They then may write either the whole equation and the answer or just the answer beside its identifying letter.

Do not leave this lesson until the children know the division basic facts well. If necessary, repeat some of the activities and provide extra work on the most difficult basic facts. Perhaps some of the children who have successfully eliminated difficulties might tell the others how they did it. You may find it profitable to set up special groups of children who should spend a few minutes each day working with the basic facts.

Providing for the able pupil

The abler pupils may write the division basic facts for the eleven number groups they have reviewed in this lesson. (See Activity 8, page 360.)

Making the basic fact lists may be followed by making division practice cards. The group number is written on one side of the card and the corresponding equations on the other. (See Activity 16, page 363.)

Also, have the children write on a sheet of paper the division "tables" as described in Activity 18 on page 364.

The division tables may also be made on practice cards. Put on one side of the card the numeral for the group and on the other the equations. Arrange these tables to show increasingly larger numbers divided by a given number. (See Activity 19, page 364.)

"Which one," a game that you may have adapted for use with pages 27-30, will be enjoyed by these pupils. (See Activity 22, page 366.)

The abler pupils can also supervise the work of the slow learners.

Helping the slow learner

The slow learner will profit by making practice cards for the division basic facts that are most difficult for him. (See Activity 17, page 363.)

Two other kinds of practice cards will also help the slow learner: the cards containing all the division basic facts for each number group from 40 to 81 (see Activity 16, page 363) and those containing the division tables (see Activity 19, page 364).

Another activity these pupils will enjoy is "Number football." (See Activity 20, page 364.) For this lesson use the number groups from 40 to 81. Let one of the abler pupils supervise the game and make sure that correct answers are given.

If your class has made a division basic fact chart (see Activity 12, page 361), let the slow learner refer to it whenever he is unsure of any division fact.

Use any other activity described for the able pupils that you feel would have value for the slow learners.

36 Thinking straight

Lesson Briefs for this lesson are on page 48.

Expanded Notes are not considered necessary for this lesson.

37-38 Looking back

Lesson Briefs for this lesson are on pages 49-50.

Teaching the whole class

Division with remainders was taught in *Seeing Through Arithmetic 4,* pages 118-119 and 176-177, and for this reason this lesson will be a review for most of the children in your class. Therefore, you may wish to have your class start the lesson by using the third block of exercises (A to N) on page 38 as an inventory test. The results of this inventory will help you decide how much time you should spend on pages 37 and 38.

Page 37: Give each pupil 15 small objects (paper clips, buttons, kernels of corn, beans, etc.). Ask the pupils to divide the 15 objects into equal groups without a remainder in as many ways as they can. Let them try any size group, but, of course, they will find that only groups of 5 and groups of 3 divide it without a remainder. Ask if all numbers can be divided into equal groups without remainders. Then have them take away one object from the total group of 15 so that 14 remain. Let them divide 14 into equal groups without a remainder. Then have them take away another object so that a group of 13 is left. Let them try to divide the 13 objects into equal groups without a remainder. Repeat this process twice more to let the children see that 12 can be divided into equal groups in three different ways without a remainder, but that 11 cannot be divided into equal groups without a remainder. Emphasize that there often is a remainder when you divide.

Point out also that it is important to know which numbers can be divided into equal groups without remainders. Knowing the numbers that can be divided equally enables the children to recognize immediately the numbers that have remainders, and

then they are able to find out how much is left over. For example, have the children lay out 10 objects and try to divide them into groups of 3. They can see that the next number smaller than 10 that can be divided into groups of 3 without a remainder is 9. It follows that 10 can be divided into 3 groups of 3 with a remainder of 1. If necessary, let the children work in a similar way with groups other than 10.

It would be advisable also to direct questions to help the children discover that the *number in the equal groups* makes a difference as to whether or not there is a remainder. For example, $14 \div 2$ and $14 \div 7$ do not have remainders; $14 \div 3$, $14 \div 4$, and $14 \div 5$ do. Then have them look at Picture A on page 37. Ask:

How do you know there are 40 arrowheads altogether? [*4 rows of 10 each*] *There are how many arrowheads in each ringed group? How many groups are there? Are any arrowheads left over? Then if 40 arrowheads are divided into groups of 8, how many equal groups are there?*

Let one or more children read the text accompanying Picture A and supply the missing numbers.

Discuss Picture B in a similar way. After the children have determined the total number of arrowheads, have them determine how many arrowheads there are in each group and how many groups of 8 there are. Ask how many arrowheads are left over when 43 is divided into equal groups of 8. Get the pupils to explain that since 5 groups of 8 are 40, they can tell that 3 arrowheads will be left over.

Let several children read the text for Picture B and supply the missing numbers.

Adapt the procedures used for Picture B to Picture C.

The work on this page may be followed by the use of duplicated sheets on which groups of X's are arranged in rows of 10, with some extra X's in another row. Text beside the X's may follow this pattern: $26 \div 4 = n$ and ▒ remainder. Let the children draw circles around the X's for the designated groups and write the answers for the accompanying text.

Page 38: Have the children do Exercises A to I orally first and then write on their papers the identifying letters and the answers. Another way to use this exercise is to have them say the quotient and the remainder (if any) for each number. Call on the children at random and have some of them give answers for one or two numbers and others the answers to a whole set of numbers. After oral practice, you may have the children write the quotients and remainders for all of the numbers or for just those that seem most difficult.

Call on children at random to answer Questions A to H with *yes* or *no*. If you wish, they may also write their answers on paper. If the answer is *yes*, have them tell what the remainder is. For example, the answer to A would be *Yes—1* or *Yes—remainder 1*.

The last two blocks of exercises (each block A to N) may be used to test how well the children can divide with remainders. You may wish to use the first one after Questions A to H have been finished, and the second one after the children have had further practice with Activity 24, page 367. If you used the first practice block at the beginning of the lesson as an inventory test, use the second block now as a retest to check on the pupils' ability to divide with remainders.

Providing for the able pupil

Each of the abler pupils may make up one or more sheets similar to the duplicated sheet described for use at the end of page 37. Let the child choose a number between 12 and 89 and decide on the size of the group into which he will divide it. Let him draw X's or objects to represent the total number and then circle the groups into which the number is to be divided. Then he should write the equation with the answer and the remainder (if there is one). Some pupils may wish to make a series of sheets for display showing what happens to the answer and the remainder when a number such as 37 is divided into groups that are different in size.

Pairs of the abler children may play the game "Remainders." See Activity 24, page 367.

Helping the slow learner

Let the slow learners work with objects to visualize answers and remainders. They, too, may use duplicated sheets that contain the groups that seem most troublesome for them.

The game "Remainders" will give them additional practice and may be played as often as their attention is held by it. See Activity 24, page 367.

39 Checking up; Keeping skillful

Lesson Briefs for this lesson are on pages 50-51.

Overview

End-of-block tests are intended to be diagnostic and should reveal the items of the preceding lessons that the child failed to master. Test 1 includes the retaught multiplication facts, Test 2 includes the retaught division facts, Test 3 includes division with remainders through 40, and Test 4 includes division with remainders through 82.

CHARTING THE COURSE

Problem solving — subtractive types

One of the problem types to be retaught in this grade arises when we want to find how many items there were at first when the number gone and the number left are known. For example: John had some white mice. He sold 6 of them. Then he had 4 mice left. How many mice did he have before he sold the 6 mice? This situation is subtractive. When expressed in arithmetical symbols, the equation $n-6=4$ is obtained. However, subtraction cannot be used because the original number (or minuend) is not known. The way to solve this equation is suggested if one imagines the 6 mice that were sold are put back with the 4 that remain. So the addition process is used to find the answer. In *Seeing Through Arithmetic 5* the pupil is shown on pages 40-41 how to analyze the situation.

Another type of problem situation involves finding how many are gone when the number to begin with and the number remaining are known. For example: Patsy made 23 potholders. She sold some of them one Saturday. Then she had 11 potholders left. How many potholders had she sold? Traditionally, this has been called the "how many are gone" type of problem.

The action in this situation is subtractive, but if an effort is made to dramatize the situation with objects, a difficulty arises. The number of potholders sold, which is the number to be taken away from 23, is unknown. In symbols, the situation is represented by the equation $23-n=11$. The child who sees this situation as it really is, either on his own or as the result of a realistic dramatization, needs to know why, in computing, the answer can be found by subtracting 11 from 23.

In *Seeing Through Arithmetic 5*, this problem is analyzed for the pupils on pages 43-44. It should be noted that in this situation, in which the unknown number is involved in the action, it is not enough to imagine the situation restored or put back as it was originally. That is, we must do more than just imagine that the potholders that were sold have been returned. This act of returning does restore the original group, of course; so then, starting again from the original group, those that were *not* sold (or a group equal to them) can be set aside or taken from the original group. This action suggests the subtraction of the number not sold (the number left) from the original number, and this process solves the problem.

40-42 Exploring problems

Lesson Briefs for this lesson are on pages 52-53.

Teaching the whole class

Page 40: Let one of the pupils begin this lesson by reading Problem A. Then have another child read just the first sentence in Problem A. Explain that Picture A illustrates this first sentence. Ask the class if it is possible to tell how many mice John had by looking at Picture A. Then explain:

The problem tells us how many mice John sold and how many he had left, but it does not tell us how many he had at first. So we do not know what numeral to use in the equation for the mice he had to begin with. We'll use the letter n *for this numeral.*

Explain that in Picture B we see John putting the mice he has sold into a smaller box. Tell the children to count the mice in the box to verify the statement, "He sold 6 mice." Discuss what has been written in the equation to show this (-6). Point out that -6 shows the action of John selling 6 mice. Then say to the children:

In Picture C we are looking at the big box again. In it are the mice that were left after John had sold 6 mice. Counting the mice, we find there are 4 left. The equation tells us that if we subtract the number of mice John sold [6] *from the total number there were to begin with* [n], *then there are 4 mice left. We put* n *first in the equation because we do not know what numeral to use for the number of mice that he started with.*

Page 41: Have the children look at Picture D and count the mice John had left. Then have them look at Picture E and count the mice John sold. State again that what they want to know is how many mice John had before he sold any of them. Let the children do some thinking on their own about how the original number of mice can be found.

After several children have told how they think it is possible to find how many mice there were at first, have them look at Picture F. Tell them to imagine that if the 6 mice John sold were put back with the 4 mice he had left, we would have all the mice he started with. Tell the children that the dotted background in the picture indicates this imagined combining of the groups. Explain that this means that we should add the numbers 4 and 6. Draw attention to the computation below Picture F.

If a pupil has difficulty in understanding the method of working this kind of problem, let him use objects to demonstrate the steps shown on pages 40 and 41. Tell him to take out a sheet of paper and a small handful of objects (paper clips, cardboard cut in small pieces, beans, kernels of corn, etc.). Direct him to conceal the objects under the sheet of paper before he counts them. Ask him if he knows how many objects he has under the sheet of paper. Discuss what he would write for the number he is to find in an equation. Of course, you are working for n as a response.

Next, instruct him to take a few of the objects away from the group under the paper and to count the objects he has removed. Direct him to write on the paper the part of the equation that shows what has happened so far. Be sure he understands that a minus sign must be used to show that he has taken away some objects from the original number he had to begin with. The pupil might write "n − 10." Discuss with the pupil the action that so far has been symbolized.

When the child understands this much of the problem, have him uncover the objects that are still under the paper. Tell him to count the objects and decide where in the equation he will write the numeral telling how many objects are left.

Tell the pupil that now he has two groups of objects on his desk—the group he took away from the objects he had to begin with, and the group that was left. Explain that he will be able to find out how many objects he had to begin with by combining the group he took away and the group that was left. Be sure he understands that the only way to find the sum of the two groups is to add. Therefore, he must think: "Add the number of objects in the group I took away to the number of objects in the group that was left." Have the pupil verify his work by pushing together both groups of objects on his desk and counting the total number.

The slow learners may do better if you let two of them work together, one drawing a handful of objects and taking several away from the original group, and the other writing the equation to illustrate the action that took place.

Page 42: Ask one of the pupils to read Problem A. Discuss with the class the two facts that are known in the problem—Dick spent $.87 and had $2.36 left. Ask the children what it is that is not known [how much money Dick had to begin with]. Then study together the equation and its accompanying text. Make sure the class understands the way the equation is built—that since the original quantity of money is not known, the n must come first. Have the pupils visualize the action of finding the amount of money Dick had in the beginning by combining what he spent and what he had left.

Problems B and C may be handled the same way as Problem A, or you may give the children some time to work independently on the two problems and then discuss their work.

The children should work Problems D to I independently without any prior discussion. Tell them that these problems include types that require equations different from the one just studied (Problems D, F, G, H). After the children have completed their work, discuss the problems. Concentrate the discussion on the problems that most of the pupils had wrong.

Providing for the able pupil

Cards containing problems for which the children must find how many items there were to begin with when the number taken away and the number left are known may be prepared for use as independent work. Some other problem types should be included in the group of cards given to the children. See Activity 14, page 362.

Helping the slow learner

Give these children additional time for using objects to visualize the steps used in building equations for the problems studied in this lesson. Use the plan for working with objects that was described in the notes for page 41.

After the slow learners have worked with objects, let them use the cards mentioned above including problems for which they must find how many items there were to begin with when the number taken away and the number left are known. See Activity 14, page 362.

43-45 Exploring problems

Lesson Briefs for this lesson are on pages 54-56.

Teaching the whole class

Page 43: Begin this lesson by having one of the children read Problem A. Then have another child read

only the first sentence in Problem A. Explain that Picture A illustrates this first sentence. Note that the potholders are arranged in two groups of 10 with 3 extra ones. Discuss the two facts known in the problem—that Patsy made 23 potholders and that she had 11 left after she sold some of them. Ask the class whether or not the problem states how many potholders Patsy sold.

Go on to Picture B and have the class look at the picture while one of the children reads the first statement. Proceed somewhat in this manner:

The story in this problem tells us that Patsy sold some potholders. But we do not know how many she sold. So we must use the letter n *in the equation to hold a place for the number of potholders she removed. The problem also tells us that the number of potholders Patsy sold was taken away from the 23 she had to begin with. So we will use a minus sign.*

Have the children count the number of potholders in Picture C and tell which statement in the problem the picture illustrates. Then ask several children to read the equation and restate the meaning of 23, the minus sign, *n*, and 11. When the children understand how the equation has been built, ask them if they have any ideas about how to find the number of potholders Patsy sold. Give the children an opportunity to express their own ideas and resolve their disagreements, if they have any.

The potholders in Picture D have been laid out so that each one can be seen and counted. Let the children verify for themselves that the number shown is the same as the quantity Patsy had to begin with.

Page 44: When the children look at Picture E, have someone in the class recall that Patsy had 11 potholders left after she had sold some of them. Point out the ring around the 11 she had left. Be sure pupils understand any 11 holders can be indicated.

In Picture F they are to imagine that 11 potholders are gone from the total number Patsy had to begin with. Explain that dimming 11 potholders in Picture F is to help them imagine that 11 are gone from the original 23. Say to the class:

When we take away the number of potholders Patsy had left from the total number she had to begin with, the group of potholders that remains is equal to the number she sold. When we say we take away 11 of the 23 potholders, it suggests that we must subtract 11 from 23.

Direct attention to the computation at the right of Picture F. Have someone compute and tell the answer that is to be written in place of the screen. Then let the class verify the work by counting the 4 rows of 3 potholders in the picture. Let them insert the answer 12 in place of *n* in the equation at the right to show that the process they followed and the answer they got are correct.

Although this type of problem and equation were taught in *Seeing Through Arithmetic 4,* pages 130-133, some of your children may not understand the steps for building the equation and computing the answer that have been described in Pictures A to F. Let these children have objects to use while they again go through the steps shown in the pictures.

Let these slower children have several concrete experiences with this type of problem. Direct them to take a handful of small objects, count them, and then remove part of them without counting. Tell them to hide the removed objects under a piece of paper and to count those remaining. Have them write an equation to show how many they had to begin with, how many were left, and the unknown quantity they hid. Then have them think: "The number I hid is one part of the original number. The number I have left is the other part of the original number. If I imagine that I take away the number of objects I have left, I will then have the *other part,* or the objects I hid."

After working with objects, turn to the problems at the bottom of page 44 and ask one of the children to read Problem A. Discuss with the class the two facts known in the problem—that Jane had 30 bookmarks to begin with and that she had 16 left after she gave some away. Study the equation and its accompanying text. Be sure the children understand that, since they do not know the number of bookmarks that were given away, *n* must follow the minus sign. Have them show the action of separating the 30 bookmarks into two groups—one group of 16, which is the quantity she had left, and the other group, the quantity she gave away. Therefore, they can find the number she gave away by subtracting 16 from 30, as the statement below the equation reads. Let one of the children write the computation on the board, find the answer, and put it in the equation in place of *n*.

Problems B and C may be handled in the same way. Perhaps you will want the pupils to work through the problems independently and discuss the process and the answers with them after they have finished.

Page 45: The children may work Problems D to M independently without any prior discussion. It would be well to tell them that the problems include types that require equations different from the one just studied (Problems G, H, J, M). After the children have completed the set of problems, discuss their work with them.

Providing for the able pupil

Cards containing problems for which the children must find how many items were taken away when the original number and the number left are known may be prepared for the children to use independently. Some other problem types should be included in the group of cards given to the children. See Activity 14, page 362.

The able children may help you prepare problems for the game "Solve it" (Activity 25, page 368). Inspect the problems before including them.

Helping the slow learner

Give these children additional time for using objects to act out the steps used in building equations for the problems. Use the plan for working with objects described in the notes for page 44.

After the slow learners have worked with objects, let them use the cards containing problems for which they must find how many items were taken away when the original number and the number left are known. (See Activity 14, page 362.)

Although these children are probably not able to help you write problems for the game "Solve it," some of them will be able to play the game. (See Activity 25, page 368.)

46-47 Using arithmetic

Lesson Briefs for this lesson are on poges 56-57.

Expanded Notes are not considered necessary for this lesson.

47 Looking back

Lesson Briefs for this lesson are on poge 58.

Expanded Notes are not considered necessary for this lesson except for suggestions for the able pupil and the slow learner.

Providing for the able pupil

The able pupils can extend the exercises on page 47 by writing and answering such exercises as the following: 7×200, 7×2000, $7 \times 200{,}000$; 3×800, 3×8000, $3 \times 80{,}000$; etc. This activity will help them to extend their understanding of the use of zero as a placeholder.

Helping the slow learner

Provide packages or small boxes, each labeled with such numbers as 20, 30, 400, 500, etc. Each child should pick up (at one time) two or more of the packages or boxes and tell what number (40, 180, 120, etc.) is represented.

CHARTING THE COURSE

Multiplication process

In the *Seeing Through Arithmetic* program, the multiplication process is first taught in Grade 4. *Seeing Through Arithmetic 5* provides for a complete reteaching of the process. This material is organized in two sections. The first (pages 47-59) shows pupils how to multiply by a number that is represented by a one-figure numeral (hereafter referred to briefly as a "one-figure number"). The second (pages 60-64) shows them how to multiply by a two-figure number.

When pupils understand the number system, the process of multiplication can be viewed simply as a more or less elaborate regrouping operation. In this process groups of tens, of hundreds, of thousands, and so on, play a prominent rôle. Multiples of these basic numbers (for example, 20, 30, 200, 4000) are, of course, also used. Page 47 of *Seeing Through Arithmetic 5* provides material for helping pupils recall the effect of multiplying 10 and multiples of 10 by a one-figure number. These ideas are then used (pages 48-59) to help the pupils understand how to multiply any number by a one-figure number.

When the multiplier is a number of two or more figures, multiplying by 10 and by multiples of ten is involved. The grouping in this situation is quite different from that involved in multiplying 10 or a multiple of ten by a number of one figure. This can easily be demonstrated with objects and pictures. Nevertheless, the product of 30×2, for example, is the same as the product of 2×30. The latter is easily understood as 2 groups of 3 tens each, which makes 6 groups of tens, or 60. Then 30×2, or 30 groups of 2, will also be 60. In *Seeing Through Arithmetic 4*, examples of this kind were used to lead pupils to the generalization that the prod-

uct of a one-figure number and any number that ends in zero also ends in zero. Similar generalizations hold if the larger number ends in two or more zeros. In *Seeing Through Arithmetic 5*, these ideas are reviewed on page 60. They are then used to help pupils understand how to multiply by two-figure numbers. The materials for this purpose are found on pages 61-64. Pupils see, for example, that multiplying by 36 involves first multiplying by 6, and then by 30, and finally combining these two results by addition.

The multiplication process involves a variation in the carrying procedure that requires careful teaching Whenever a partial product is 10 or a number larger than 10, the number of tens must be *remembered* until the next multiplication has been done, and then it is added to the partial product. In *Seeing Through Arithmetic*, this is taught visually by showing how one or more groups of ten in the partial product are put to one side until the next partial product has been found.

Similarly, in multiplying a number of three figures, any hundreds obtained from multiplying and grouping the tens must be remembered until after the hundreds' figure in the original number has been multiplied. This procedure can, of course, be extended to multiplying numbers of more than three figures. As numbers become larger, demonstrations with objects become unwieldy because of the large number of objects required. It is important to help pupils understand the nature of the process, and then increasingly to make use of the number symbols. The tremendous power of arithmetic comes from the fact that people can learn to use the symbols instead of the objects themselves, and so achieve great savings in time and energy.

48-52 Learning how

Lesson Briefs for this lesson ore on poges 59-62.

Overview

Work in this lesson is confined to the multiplication of numbers involving two figures by numbers involving one figure. The four-step teaching method, described in detail on page 59 of the Lesson Briefs, is used to present this material.

During the work with the SEE step on pages 48-49, you will, of course, work closely with the children explaining each step carefully. For the THINK step on pages 50-51, help the children answer the questions, but encourage them to think for themselves. In the TRY and DO steps on page 52, the children are expected to work independently. You may need to remind them how to use the dollar sign and point when multiplying numbers that represent amounts of money.

Teaching the whole class

Since children differ in their needs, depending in part on the work accomplished the previous year, you may find it helpful to use Exercises E to L in the first block of the DO step on page 52 as an inventory test. The results should disclose how much reteaching will be necessary and enable you to determine which pupils should be given other work to do at this time. (See "Comments," page 59.)

If the results indicate that thorough reteaching is advisable, you may want to review with the children the idea that multiplication is used when equal groups are put together into one big group. Remind the children that they already have had practice in multiplying two-figure numbers (those ending in zero). Their experiences with regrouping have shown them that when they have more than 9 ones they can make a new ten, and that when they have more than 9 tens they can make a new hundred. In this lesson all they have to do is to use these ideas in one problem.

Page 48: Have the children read the statement that tells them what they are going to learn in this lesson and then read Problem A. Have them examine Picture A and relate it to the problem. (Be sure they understand that there are 17 envelopes in each group and that there are 4 groups altogether.) Direct attention to the equation, and discuss what each numeral means in relation to the picture.

Have the children next tell what is happening in Picture B. Make sure they understand that all the single envelopes are being put together. If necessary, explain that the dimmed-off envelopes represent those groups that are not being considered at this time. Direct attention to the computation and explain what the large black 4 and 7 represent (4 stands for 4 groups, and 7 for the number of ones in each group). Let someone tell what the answer is when 7 ones are multiplied by 4. Direct attention to Picture C and get someone to explain that 2 new piles of 10 envelopes each have been made. Discuss the computation and let someone relate the large black 8 to the picture.

Page 49: Discuss Picture D, the text, and the computation. Let a child relate the text to the picture. Make sure everyone understands that the 2 new piles of 10 envelopes are being pushed aside until after the next multiplication has been done, but that they must be remembered.

Direct attention to Picture E, and let a child explain what is happening. Call on another child to relate the undimmed objects in the picture to the black 1 and 4 in the computation. Make sure everyone understands that the large black 1 means 1 ten. In the same way, continue to discuss and relate Pictures F and G to the text and computation.

Page 50: Ask the children to read Problem A. By means of questions, help the children relate the problem to the picture. Let someone tell what the numerals in the equation stand for. Proceed to Picture B, and relate the text and computation to the picture. The children should answer the questions in the text. Then direct attention to Picture C, and let the children answer the questions and explain their answers. Have them relate what is happening in the picture to the questions and the computation.

Page 51: Pictures D, E, F, and G should be handled in the same way.

Page 52: In the work with the TRY step, individual differences among pupils should be recognized. Pupils should work only as many of the examples as they need to before proceeding to the DO step, which offers no assistance but requires each pupil to work entirely on his own. Observe the work that is being done to make sure that pupils who consider themselves ready to proceed to the DO step really are able to do so. Slower pupils may need considerable help. Exercises A to P are to be used only by those pupils who require additional work.

Answers to the problems and exercises in the DO step should be available so that the children may verify their answers.

Providing for the able pupil

"Try for the Number," Activity 26, page 369, may be used by able students with the following adaptations: Number sets of pink cards consecutively with the decade numbers from 20 to 90. The winning score would then be within 100 points of 1000.

Activity 10, "Target Toss" (page 360), may be adapted for multiplication in this manner. Give each player six jar rubbers—either two each marked 1, 2, and 3, or two each of black, red, and white. When a player rings a numeral with one of his black rings (or one marked with the figure 1), he scores just that number and no more. If he succeeds in ringing a numeral with one of his red rings (or one marked with the figure 2), his score is twice the number he rings. If he succeeds in ringing a numeral with one of the white rings (or one marked with the figure 3), his score is three times the number he rings. Let several of the able children take turns being scorekeeper for this game. Each player should figure his own score for his white or red ringers; the scorekeeper should total all the ringers for each player to see who has the highest score. The difficulty of this game may be controlled by the size of the numerals you hang on the target hooks and also by the value you give the rings. Some students can work with multipliers such as 7, 8, and 9.

Helping the slow learner

"Target Toss," Activity 10, page 360, may be adapted for slow learners by keeping the numbers small. It may be advisable, also, to give each of the children only four rings—two black (or labeled 1) and two red (or labeled 2).

53-59 Learning how

Lesson Briefs for this lesson are on pages 62-66.

Teaching the whole class

Pupils who successfully completed the inventory test given at the beginning of the previous lesson may be expected to make an equally good showing in a similar test involving three-figure multiplicands. You might use Exercises C to L in the first block of the DO step on page 59 for this purpose. Pupils who complete this test satisfactorily may be excused from participating in this lesson. A suggestion for keeping such pupils busy at this time will be found in the Lesson Briefs, page 62.

Page 53: Have the children read the statement at the top of the page. Then direct attention to Problem A and to the picture. When the problem has been read, ask a child to point out the 4 groups of 135 pencils. Call attention to the equation. Let a child tell what the 4 in the equation stands for and relate it to the picture. Let someone else tell what the 135 in the equation stands for and relate it to the picture.

Now ask what Picture B shows and why part of it is dimmed off. Let a child relate the text and computation to the action in the picture. Have him relate the undimmed part of the picture to the black figures in the computation.

Page 54: Treat Picture C and its text in a similar way. You might also ask such questions as:

Where did the pencils that are being made into 2 new bundles of 10 come from? Are there any single pencils left? Look at the computation. What tells that no single pencils are left? What should we do next?

Have the children look at Picture D and relate the picture to the text and computation.

Page 55: Have the children look at Pictures E and F. Get them to relate each picture to the text and

computation as they go along. When you discuss the large black figures in the computation opposite Picture E, make sure the children understand that the 3 represents 3 *tens*. In discussing Picture F, let a child explain where the 2 new bundles of ten came from, and let another child explain that 14 tens equal one hundred forty.

Page 56: Continue discussing pictures, text, and computation, as you did for the three preceding pages. Bring out the relationship between each picture and its text, between the text and the computation, and between the computation and the picture. When you have worked through this page to the last sentence and the equation, have the children turn back to page 53, read the problem, and answer the question.

Page 57: Since this page begins the THINK step, lead the children into finding the right answers by asking any additional questions that seem necessary. Work with each picture and its text carefully, but avoid direct explanation. Instead, encourage the children to do the explaining. In Pictures B and C, make sure that everyone understands how the black numerals in the computation are related to the action in the picture.

Page 58: Continue working as before until the THINK step has been completed.

Page 59: Work through Problem A in the TRY step with the class, and then let the children go ahead at their own speed. Explain that if anyone is quite sure he understands how to do all the problems in this step, he may go on to the DO step. You will need to observe the work that children do by themselves in this TRY step, to make sure that only those who really do know what they are doing continue to the next step, and also to find out which children need further help from you. Very slow learners may need considerable individual help.

In the DO step the children should work entirely on their own. Provide a set of answers that they may use to verify their work when they have finished Problems A and B and Exercises C to V.

The final block of exercises (A to T) is intended for extra work, to be assigned if you think more practice is needed.

Providing for the able pupil

Activity 26, described on page 369, may be adapted for use with this lesson by making a new pack of pink cards that have three-figure numerals such as 254, 121, and 309 written on them. The game is then played as described.

Activity 10, "Target Toss" (page 360), may also be adapted for use with this lesson. Hang three-figure numerals, such as those suggested in the above adaptation of Activity 26, on the hooks and put one-figure numerals on the rings. Players should compute the score for each "ringer" on paper.

Helping the slow learner

For the slow children Activity 26 (page 369) may be adapted for use with this lesson by using on the white cards such numerals as 101, 120, 208, etc.

Activity 10 (page 360) can be adapted for the slow learners by using numerals like 101, 110, and 204. Use two three-figure numerals like these and three two-figure numerals (80, 71, 12, etc.) on the hooks. Use one-figure numerals on the rings.

60 Looking back

Lesson Briefs for this lesson are on pages 66-67.

Expanded Notes are not considered necessary for this lesson.

61-64 Learning how

Lesson Briefs for this lesson are on pages 67-70.

Teaching the whole class

The work in this lesson completes the teaching of the multiplication of whole numbers. The work is confined to two-figure multipliers, but when the procedure has been established, the children may be allowed to try multiplication with multipliers of any size.

Before proceeding with this lesson, be sure that all of the children can multiply two-figure numbers by tens and hundreds (see page 60). This understanding becomes basic when the child is required, for example, to multiply 36 by 14 and must understand why one of his partial products is 420 and not 42.

If you wish to inventory your pupils' knowledge of the work taught in this lesson, you may use Exercises C to K in the DO step (page 64) for this purpose.

The pictures in the SEE step (pages 61 and 62) suggest a meaningful way to use objects to develop the ideas of the lesson, if this seems necessary. Provide 45 boxes (matchboxes, for example), each labeled "10," 4 larger boxes, each labeled "100," and 76 objects small enough so that 10 of them will fit into one of the "10" boxes (corn, sunflower seeds, etc.). The lesson may be interrupted at any point to let those who are having difficulty work out the step with objects, perhaps with the assistance of some of the able students. Or you may want to let the

slow pupils work through the lesson with objects after you have completed the teaching of the lesson.

Multiplication of the ones in the SEE step may be illustrated with objects as follows: Have 12 children each set up a group of 38 on a large table. (Each child should pick out 3 boxes marked "10" and 8 single objects.) Then have two children combine their groups and let everyone discover that in the 2 groups of 38 there are 6 tens and 16 ones. Get a child to regroup the 16 ones by filling a new "10" box with 10 objects. Let the children check the tens and ones they now have with the computation in the book.

Page 61: Have someone read the problem aloud and then direct attention to the picture. Ask how many groups of 38 beads there are. Let a child explain the arrangement of the beads (each bag contains 10). Call attention to the equation and let the children explain it in terms of the problem and the picture. (You may need to explain that the 12 in the equation represents the 12 *groups of beads* the 12 girls will make.) Be sure the children understand that they are to think of the 12 as 1 ten and 2 ones.

When discussing Picture B and its text, draw attention to the fact that 10 of the 12 groups have been dimmed off because we are now concerned with only 2 of these groups. Explain that the dotted background indicates that the 2 groups of 38 are to be thought of as one group. Then have someone relate the computation to the picture.

If any child has difficulty understanding this multiplication of the ones, let him use objects, as explained under "Teaching the whole class."

Page 62: Let one of the children explain what the undimmed part of Picture C represents (the beads 10 of the 12 girls will make). Then have everyone read the text. Let the pupils take turns relating the text to Picture C. Let someone explain the meaning of the dotted background. (The 10 groups of 38 are to be thought of as put together.) Have a child relate the computation to the text and let another child relate the computation to the picture. Be sure everyone understands that the 10 represents the 10 *groups of beads* 10 girls will make. Direct attention to the rearrangement of the 10 groups of 38 beads (Picture D) as 3 hundreds and 8 tens.

Again, slow pupils may need to work out this step with objects. Explain that when they have more than 9 tens, they are to make hundreds by putting 10 small boxes into one big box.

Be sure the children understand that Picture D shows the 76 beads to be made by 2 girls (groups on the right) and the 380 beads to be made by 10 girls (groups on the left). Now relate the computation opposite Picture D to the picture. If necessary, refer back to the computation opposite Picture B (page 61) and opposite Picture C to explain how the numbers 76 and 380 have been found. Discuss the dotted background and what it means.

Next tell the children that usually all the computation is written together in one place. Direct attention to the way the numerals in the computation opposite Picture E are arranged—in columns representing ones, tens, and hundreds—and discuss why such an arrangement is important.

Page 63: After the problem has been read, draw attention to the 6 gumdrop animals at the top of the picture and relate them to the 6 in 36 (in the problem, equation, and computation). Then relate the 30 gumdrop animals in the rest of the picture to the 3 in the 36. Ask how many groups of 14 gumdrops will be needed to make 6 animals and how many groups of 14 gumdrops will be needed to make 30 animals. Then ask:

What are we asked to find? How can we find this? What numeral stands for the size of the group? What numeral stands for the number of groups? How are these numerals written in computational form? What do we multiply by first? What do we multiply by next?

Then let the children take turns reading the text, supplying the missing numerals, and relating the numerals in the computation to the text as it is developed.

It is impractical to work examples such as this with objects, since too many objects are required. However, children who still do not grasp the process of multiplying by a two-figure number may use the objects assembled for the SEE step, as described in "Teaching the whole class," to do such simple exercises as 12×25, 14×17, etc.

Page 64: Discuss Problem A (in the TRY step) and its solution with the class, and then assign Exercises B to D as written work. Make sure everyone understands why we conventionally use the smaller number as the multiplier. [It seems easier because there will be fewer partial products.]

Point out, also, that when a number representing an amount of money is multiplied, it is important to place the point and dollar sign correctly in the number multiplied and in the answer. Warn the children to keep the numerals in ones', tens', and hundreds' places directly under each other. The children should not look at the computation in the book until they have completed their own work on Exercises B to D.

If any pupils fail to do the TRY examples successfully, analyze their work carefully to find the nature of their errors and then provide appropriate help. Some children may need to review the preceding lesson on multiplying by tens and hundreds (page 60).

Pupils should complete Exercises A to T of the DO step without assistance. Provide answers for these exercises so that pupils may verify their work when they have finished the assignment.

Exercises in the second block (A to R) may be assigned to those pupils who need extra practice.

Providing for the able pupil

Able pupils enjoy working with "magic numbers." Tell them to multiply 37 first by 3, and then by multiples of 3 (for example, 3×37, 6×37, 9×37, etc.). They should proceed as far as 27×37 and note what happens. (The products will be 111, 222, 333, etc., up to 999.)

When they have done this, tell them to double 37 and to work with the result (74) in the same way. They should use 3, 6, 9, and 12 as multipliers ($3\times74=222$; $6\times74=444$; $9\times74=666$; $12\times74=888$). Then tell them to double 74 and find out how many times it can be multiplied by 3 or by a multiple of 3 before the figures in the product become dissimilar. ($3\times148=444$; $6\times148=888$. Here the sequence ends, since $9\times148=1332$.)

Able pupils also like to experiment with different ways of multiplying large numbers. Out of this should come the realization that our traditional method of multiplying has become most common mainly because it is the most efficient. The method shown below, while cumbersome, may give the pupils a deeper understanding of what is really involved in multiplication, and it does make the checking of errors in computation easier.

With this method, the children can multiply in any order they choose, but they must be aware of the real value of each partial product, and they must add the partial products properly. In the example on the left, the ones are multiplied first, the tens by the ones next, and so on, with the hundreds multiplied by the tens last. The example on the right reverses this procedure. Point out the danger of omitting one of the partial products. It would be easy to forget to multiply the tens by the hundreds, for example, when using a three-figure multiplier.

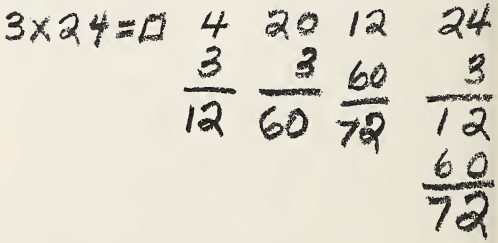
449		416	
86		92	
54	(6×9)	36000	(90×400)
240	(6×40)	900	(90×10)
2400	(6×400)	540	(90×6)
720	(80×9)	800	(2×400)
3200	(80×40)	20	(2×10)
32000	(80×400)	12	(2×6)
38614		38272	

Encourage the pupils to find a principle that tells how many partial products there should be in the method described above. (The number of partial products is equal to the product of the number of *figures* in the multiplicand and the number of *figures* in the multiplier.)

Permit pupils to explain any unusual multiplication situations they discover through reading.

Helping the slow learner

You may give these pupils cards containing examples in which two-figure numbers are multiplied by decade numbers (10, 20, 30, etc.). Direct the children to estimate each answer in hundreds. For example, for 20×37 a child might write "700" if he thinks the answer will be in the 700's, or he might write "800" if he thinks the answer will be in the 800's. The slow pupil may profit from further work with the multiplication of two-figure numbers by a one-figure number, simplified in the following way:



Later he can remember (or carry) the ten until he has completed the multiplication of tens.

65-66 Using arithmetic; Checking up

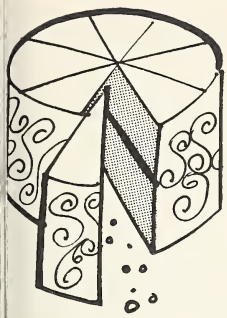
Lesson Briefs for these lessons are on pages 70-72.

Expanded Notes are not considered necessary for these lessons.

67 Thinking straight

Lesson Briefs for this lesson are on pages 72-74.

Expanded Notes are not considered necessary for this lesson.



CHARTING THE COURSE

Division process

In the *Seeing Through Arithmetic* program, the learning of the division basic facts is based upon the quotitive, or "measurement," type of divisive action. For example, the fact $40 \div 8 = 5$ is learned by observing (as on page 31) that when a group of 40 is separated into groups of 8, there are 5 such groups. In general, division is taught as a process for finding how many equal groups of a certain size can be found in a given group. In *Seeing Through Arithmetic 4,* pupils are shown how to use this process with one-figure divisors and two-figure quotients. Materials are also provided for teaching division by a two-figure divisor leading to two-figure quotients, but this block of work is considered optional in the fourth grade. *Seeing Through Arithmetic 5* provides fully detailed materials on pages 68-74 for reteaching division with one-figure divisors. Pages 78-84 provide materials for teaching (or reteaching) division with two-figure divisors. Applications to problems involving money are given on pages 86-88.

Division is generally regarded as the most difficult basic process in arithmetic. One reason for this is that it is not a direct process, like addition or multiplication, and estimating and adjusting are involved in all but the simplest of examples. The quotient figure must be estimated, step by step; and if at any step it is not "correct," a correction or new estimate is made, and the process continues.

The computational process of division is based upon subtracting equal groups until the original number (represented by the dividend) has been used up. This process is shown both pictorially and by symbols in *Seeing Through Arithmetic 5,* but at first no effort is made to subtract as many equal groups as possible. In an example like $864 \div 50$, the usual procedure consists of first subtracting 10 fifties, or 500, and then subtracting 7 fifties, or 350, to obtain a complete quotient of $10+7$, or 17, and a remainder of 14. However, one can subtract the fifties in groups of other sizes. This is shown on page 79, where in the second step only 5 fifties, instead of 7, are taken. Then 2 more fifties are subtracted in a third step, to obtain the quotient as $10+5+2=17$.

Much of the difficulty in division has arisen because it has been customary to insist upon getting the correct quotient figure in each step. It is not possible for even the most mature and skilled computers always to get the right quotient figure on the first trial. When the nature of the process is understood, however, they can proceed and correct their work. One procedure is to erase the work that was done with the incorrect estimate, and start over. Another and simpler procedure is to continue the work and adjust the computation as one goes along. This procedure is used to introduce the process in this book. As children grow more mature and skilled in their division work, they are taught gradually how to improve their estimates and are encouraged to do so. At the outset, getting the most efficient estimate is regarded as less important than understanding the process.

In this book, the partial quotients are written at the right of the dividend, and a vertical line is drawn to separate them from the rest of the work. This arrangement makes it easy to add them to find the complete quotient, which is written beside the remainder, if there is one. Thus, the complete answer is in one spot.

For some years past in the United States, the conventional manner of writing division has placed the quotient above the dividend and used a horizontal line to separate them. This scheme has not always been used in this country, and at the present time is not used in most other countries. The practice of placing the quotient at the right is not an innovation. It is a return to a practice that was formerly almost universal.

Some of the "difficulties" in division involving the placement of numerals, "bringing down" figures from the dividend, and typical "zero difficulties" are largely avoided when pupils understand the division process. These difficulties arise when division is taught by reliance upon a set of rules (divide, multiply, compare, subtract, bring down, etc.) which are learned without understanding. The pictorial development of long division in *Seeing Through Arithmetic 5* is designed to develop understanding as well as skill in the use of the process.

Various refinements of the process are suggested in a "Thinking straight" section on page 77 and a "Keeping skillful" section on page 91. Teachers should also

Expanded notes 65-67

look ahead to pages 136-137, where the pupils are taught that dividing or multiplying a divisor and dividend by the same number does not change the quotient. This leads to another step toward more mature methods of estimating the quotient (see pages 204-206). Instead of merely being given rules for estimating the quotient, pupils are introduced to the principles involved. The basic idea involves rounding both the dividend and the divisor to convenient multiples of ten. Thus in the example 11,468÷23, the divisor 23 may be rounded up to 30 and the dividend rounded down to 9000. Then both 30 and 9000 may be divided by 10, and the quotient is estimated by dividing 900 by 3 to obtain 300.

Most methods of estimating the quotient that are in common use require that the dividend be rounded down to a multiple of 10, and in the example above, the dividend is usually treated as though it were 11,000. It should be noted, however, that when the divisor is rounded *up* to a multiple of 10, the estimate obtained will never be too large. It is, therefore, always possible to continue the division by subtracting more multiples of the divisor until the remainder is less than the divisor.

Still another step forward is taken on pages 210-212, where the pupil is shown that in some cases the remainder may be used to form a fraction and to write a fraction numeral as part of the result of a division problem. Pages 213-214 are designed to develop judgment as to when it is and is not appropriate to express a result with a fraction numeral. This decision always depends upon the nature of the situation in which the problem arises. In the past, as soon as pupils had learned something about fractions, they were often taught to express the answer in a division problem as a mixed number. Many times such a result in a concrete situation is absurd. Judgment is always necessary, and definite efforts should be made to help pupils understand this and to develop the ability to make these judgments.

68-74 Learning how

Lesson Briefs for this lesson are on pages 74-79.

Overview

If you are not already familiar with the division method taught in this book, be sure to read "Charting the course" on pages 279-280 and study pages 68-74 of the pupils' book. The explanation provided on these pages is simple, complete, and illustrated at every step. The "long division" method taught here provides a procedure that is the same for divisors of any size. It is introduced in this lesson with divisors of only one figure, but the method is general. Moreover, the first example (on pages 68-70) is one with a remainder, which is the general case. The lesson explains division as a process of repeated subtraction, and the reasons for each step are made clear by pictures and text. With this method the difficulties of estimating the quotient figure are avoided, because the way the work is written does not require that the exact figure be obtained before the work can proceed.

While examining pages 68-74, you should observe that: (1) the process makes use of repeated subtractions; (2) the complete quotient can be obtained in various ways, some of which are more efficient than others; (3) it is not necessary to learn and use the most efficient procedure at the beginning—efficiency can be built up gradually by the pupils; (4) the computation is written in a way that contributes to understanding.

Note, too, how this method avoids the frustration that results because of failure to estimate the correct quotient figure, which in other methods is followed by erasing and a fresh start that may also turn out to be "wrong." Note, finally, that at each step the entire partial quotient is written (for example, 10—not just 1—in the problem on page 68). Thus the question of where to place the "1" in the quotient does not arise. The same applies to divisors with two figures (see pages 78-80).

Observe that in the TRY step (pages 73-74), where the child begins to work independently, each example has been worked for him in four different ways, and in each case the way that demonstrates the highest level of skill is shown in red.

The DO step (page 74) provides practice. Most groups will need to use only the first block of exercises (A to V). The second block of exercises (A to T) may be used if additional practice is needed.

Since many of the children will have studied division with one-figure divisors earlier, the amount of time to be spent on this lesson and the lesson steps selected for presentation should be determined by the abilities of the children. With a group that shows knowledge and understanding, it is unnecessary to do more than review the process. However, a group that studied division with one-figure divisors in the fourth grade should be retaught the process from the beginning if you feel they do not understand it.

Teaching the whole class

If you think it is necessary, have the children review the division basic facts and division with remainders.

Tell them that now they are going to learn how to divide larger numbers.

Page 68: Have a child read the problem. Then let the class look at Picture A and discuss it in terms of the problem. Be sure the children accept the fact that each group of toys in the picture contains 10 toys. Have a child read the equation and explain what each numeral and *n* stand for.

When the children look at Picture B, explain that these toys are the same as those in Picture A, but that they are dimmed off because something else in the picture is more important now. Ask how many boxes they think Kathy can fill with the 50 toys if she puts 3 toys in each. Get them to make quick guesses. Then guide the discussion somewhat as follows:

Let's find out exactly how many boxes she can fill. How many boxes are there in the picture? Let's start with those. Look at the computation at the right. The black 10 stands for the 10 boxes that Kathy will fill first. The red up-and-down line separates the 10 from the other figures.

Now look at Picture C. In this picture a group of 3 toys is in front of each box. Are some toys left over? To find out how many toys are left over, we must first find how many toys have been used.

Direct attention to the text at the right of Picture C. Have a child read the first statement; then have all the children look at the computation. Be sure they understand that the black 30 stands for the number of toys that have been used, and that it is written under the 50 so that we can subtract to find how many toys are left to be put in groups of 3.

Page 69: Explain that, in Picture D, the 10 boxes of toys are dimmed off because now we are interested in how many toys are left. Have the children note how the computation shows that 20 toys are left. To summarize the work to this point, say that there were 50 toys to be put in groups of 3 to a box, that 10 boxes have been filled and 30 toys were used to fill them, and that now 20 toys are left to be put into boxes.

Now explain that some of the 20 toys left are going to be put into boxes. Draw attention to the 6 new boxes in Picture E, and ask a child to read the statement beside the arrow. Point out that the black 6 in the computation stands for the 6 boxes that are to be filled, and that it is written under the 10 that stands for the 10 boxes that are already filled.

Ask the children to tell what Picture F shows. They should observe that each of the 6 new boxes has a group of 3 toys in front of it. Have a child read the statement beside the first arrow. Direct attention to the computation and have a child read the text beside it. Ask what the black 18 stands for. Explain that 18 of the 20 toys that were left have been used, and that the 18 in the computation is written under the 20 so that we can subtract to find how many toys are left now.

Page 70: Have the children explain what Picture G shows (the 10 boxes filled first, the 6 boxes filled next, and the 2 toys left over). Let one of them read the statement beside the first arrow. Discuss the computation. Remind the children that the 20 represents the toys that were left after the first 10 boxes were filled and that the 18 stands for the 6 groups of 3 toys that were just put into boxes. Let a child read the statement beside the computation and relate the computation to the statement and to the picture. Be sure all the children understand that the black 2 stands for the 2 toys that are left over. Ask if enough toys are left to fill another box. Explain that since too few toys are left to fill another box, we must now find out how many boxes of 3 toys each have been filled.

Direct attention to Picture H. Point out that the picture shows the 10 boxes that were filled first and the 6 boxes that were filled next. The dotted background is to help the children see all of the boxes as one group. Have a child read the statements above the computation and relate them to the picture. Let the children count the boxes, if they wish. Next direct attention to the computation. Ask what the black 10 and the black 6 stand for. Be sure the children think of them as representing 10 groups of 3 and 6 groups of 3. Relate the addition of the 10 and the 6 to the picture. Be sure the children understand that the total stands for 16 groups of 3 toys, and that the remainder stands for 2 toys. Let a child read the completed equation.

The procedures described for the problem in the SEE step may be adapted for the problem in the THINK step. For the work in this step the children must answer questions and supply the missing numbers.

Page 71: Continue the discussion begun on the preceding page, having the children answer the questions and explain the pictures and computation. Be sure they know which numerals stand for the number of lollipops in each group and which stand for the number of groups of lollipops.

Page 72: The THINK step is concluded in Picture G on this page. Note that there is no remainder in this problem. Have the children explain why there is no remainder and bring out that this problem was worked in the same way as the one in the SEE step, in which there was a remainder.

Page 73: As always in the TRY step, the children should work the problems independently and then compare their work with that shown in the book. Four solutions are given for each example on this page, at four successively more efficient levels. The one that is most efficient, then, is the one printed in red at the extreme right of each series. Tell the children to write each example in computational form, find the answer, and then compare their answer with that in the book. Tell them that although each *answer* must be the same as that given in the book, the numbers they use in finding it may be different from those in the book, since each example can be worked in several different ways. The children should realize that the example shown in red is the one that saves the most work in computation.

Page 74: The last example in the TRY step is at the top of this page. When the children have finished Examples A to D in the TRY step, discuss their work with them. Have them put on the board a few of their solutions that are different from those in the book. Get the children to see that it is quicker and easier to use one or two larger partial quotients than it is to use many smaller ones.

Finally, let the children work the exercises in the DO step independently. Average groups should work only the first block (Exercises A to V). If more practice is needed, use Exercises A to T.

Providing for the able pupil

Direct the abler pupils to examine Examples A to D on pages 73-74, and get them to note that many of the partial quotients can be combined to get larger partial quotients. (Have them cover the red examples at the right when they do this.) In Example B the three 20's can be combined to make 60, or the 50 and the 10 can be combined to make 60.

When the children have examined the partial quotients for Examples A to D, tell them to examine their work for Exercises A to V of the DO step in the same way. For each exercise they should decide which of their partial quotients can be combined into a larger partial quotient.

This activity will help the children develop judgment in their work with division.

Helping the slow learner

The slower children may have several kinds of difficulties. One is in understanding the reasons for each step of the division process. Another is in doing the necessary multiplication. If lack of ability to multiply is causing the trouble, have the children review the multiplication basic facts, especially those for the higher number groups, and also review multiplication with multipliers in the decades (10, 20, 50, etc.).

A good technique for helping the slower children understand the division process is to let them work out problems with objects and write down what they do for each step in computational form. With the very slow children, it may be necessary to let them take out one group at a time until they understand exactly what they are doing. Have them start with the problem in the SEE step on page 68, so that they can proceed from the manipulation of objects to the more advanced treatment of the same problem given in the book.

Put 50 small objects to represent toys on a table, and let one child take out successive groups of 3 while the other children look on and write the computation for each step. This should show them clearly the subtractive nature of the division process. Then suggest that two or three groups of 3 be taken out at a time, and again have the children write the computation. Let them do several problems in this way, if necessary. Most children will not need to start on such a primitive level, but they will need to manipulate objects involving just a few groups before they can solve division problems at even the simplest level of skill shown in the book.

75-76 Using arithmetic; Keeping skillful

Lesson Briefs for these lessons are on pages 79-81.

Expanded Notes are not considered necessary for these lessons.

77 Thinking straight; Keeping skillful

Lesson Briefs for these lessons are on pages 81-82.

Expanded Notes are not considered necessary for these lessons.

78-84 Learning how

Lesson Briefs for this lesson are on pages 82-86.

Overview

The best time for children to begin work with two-figure divisors is immediately after they have studied one-figure divisors. Such a procedure rounds out and completes their knowledge of the division process. It also gives the children an opportunity to enrich and generalize their understanding of the process. Division with a two-figure divisor, as it is taught here, is not a new and much more difficult form of division. The division process is the same whether the divisor is a two-figure number or a one-figure number.

Teaching the whole class

Page 78: Before presenting the work in this lesson, you may wish to review multiplication with numbers whose numerals end in zero.

Let the children discuss the picture on page 78 in terms of the problem. The children should understand that there are 864 pennies in the pile, and that they are to find out how many rolls of 50 pennies each can be made from the 864 pennies. When they are interpreting the equation, be sure they understand that n stands for the number of groups of 50 that can be made from the 864 pennies.

Remind the children that they have learned to divide using a one-figure divisor, and tell them that all division can be done in the same way, no matter how large the divisor is.

Direct attention to the first statement beside Picture B. Ask why 10 is a reasonable number of rolls to start with, and how we know that 10 rolls will not take more than 864 pennies. Have the children look at the computation, and ask someone to read aloud the statements beside it. Let the pupils discuss whether or not this step is in any way different from the first step when they divide by a one-figure number. Be sure the children understand that the black 10 stands for 10 groups of 50 pennies each, and that the black 500 stands for the total number of pennies in the 10 groups of 50.

Let the children discuss Picture C and explain what it shows. They should observe the 10 rolls of pennies, and know that each roll contains 50 pennies. They should also observe that some pennies are left, but that it is impossible to tell how many. Have them refer to the computation to find out how many pennies are left.

Page 79: Remind the children that there are still 864 pennies to be put into rolls. Have them look at Picture D and observe that five more rolls have been made. Ask why 5 is a reasonable number of rolls to try next. Have the children explain the black numerals in the computation.

Next have the children look at the computation beside Picture E, observe the subtraction, and note that now 114 pennies are left. Ask if any more rolls of pennies can be made, and let a child explain why.

Have the children examine and discuss Picture F. They should observe that two more rolls of pennies have been made. Ask why 2 is a reasonable number of rolls to make this time. Let them explain what the black numerals in the computation stand for, and why each is written where it is.

Page 80: The children should look at the computation beside Picture G and note that there is a remainder of 14. They should explain what the 14 stands for. Let them relate this numeral to the picture, which shows 14 pennies that still have not been put into a roll. Ask if another roll of pennies can be made, and let a child explain why not.

Have the children examine and discuss Picture H. Remind them that they have been finding out how many rolls of 50 pennies can be made from 864 pennies. Let them point out in the picture the 10 rolls that were made first, the 5 rolls that were made next, and the 2 rolls that were made last. Tell them that these three groups of rolls must be combined to find out how many rolls in all were made. Call attention to the dotted background, which helps them see the three groups as one. Direct attention to the computation and let various children explain what each black numeral stands for. They should relate each numeral to the picture.

Page 81: Ask the children to read the problem at the top of the page and think about what the numerals and n stand for in the equation. Have a child tell what they stand for. As you work through Steps A to E with the children, have them answer the questions in the text, verify each step of the computation, explain what the black numerals stand for, and tell what numerals should be where the screens are. They should be able to explain what must be done with the partial quotients in Step E to find the answer. Then have them read the equation at the bottom of the page, supply the name of the missing number, and tell what each numeral in the equation stands for.

Page 82: The TRY step gives the children a chance to solve division examples and compare their solutions with solutions given in the book. Tell them to write the computational form for Equations A and B and find the answers, and then to compare their work with that shown in the book. Make it clear that their work may differ step by step from that in the book, but the final answer must be that shown in the book. Point out that the shortest way of finding the answer is printed in blue at the right of the page.

Page 83: Tell the children to work Examples C, D, and E as they did Examples A and B on page 82. Remind them that when they finish each example, they are to compare their work with that shown in the book. Encourage them to work each example in as short a way as they can. When the children have finished, discuss with them the various ways in which they worked the examples.

Page 84: Have the children work Problems A and B and Exercises C to X independently. When they

have finished, let different children show their solutions on the chalkboard. Each child should be able to explain and justify his work. Exercises A to V may be used as additional practice.

Providing for the able pupil

The activities suggested for the able pupil in the Expanded Notes for pages 68-74 can be adapted for this lesson. (Use the examples and exercises on pages 82-84.) The objective again is to acquire skill in estimating the largest possible partial quotient. Encourage the children to use decade numbers in estimating as often as possible.

Helping the slow learner

The activities suggested for the slow learner in the Expanded Notes for pages 68-74 can be adapted for use with this lesson. The objective is to help the child understand the subtractive nature of division and to understand what each numeral in his computation stands for. Be sure that these children know the multiplication and division facts and know how to multiply with numbers whose numerals end in zero.

84-85 Using arithmetic; Keeping skillful

Lesson Briefs for these lessons are on pages 87-88.

Expanded Notes are not considered necessary for these lessons.

86-88 Exploring problems

Lesson Briefs for this lesson are on pages 88-90.

Teaching the whole class

You will find it helpful to have on hand cards or other objects to represent tickets and 200 pennies (real or play money).

The most important thing in this lesson is to develop a thorough understanding of the problem situation, so that the child may interpret his answer correctly after computing. The only way the child can work division problems that involve dollars and cents is to ignore or eliminate the point and dollar sign while he is computing. Consequently, when he arrives at the answer, he is faced with the problem of what to call it. Is it cents, or is it dollars and cents? Does it represent objects or people? If there is a remainder, what does the remainder represent?

No child can answer such questions sensibly unless he knows from the beginning exactly what he is trying to find out. Frequently, however, what he is trying to discover and does discover, through the computation, is the answer to a question that has not been directly asked. For example, the problem may put the question this way: "How many tickets at $.22 each can Sally buy with $2.00?" In order to understand this situation, the child must be able to relate the known and unknown quantities in it to the three parts of a division equation: the total amount, the size of the equal groups, and the number of equal groups. Before he can make the equation, he must decide whether the quantity to be found represents the size of each group or the number of equal groups. The form of the equation will depend upon whether the unknown quantity represents the number of equal groups (quotitive or "measurement" division) or the size of the equal groups (partitive division). When he computes to solve a problem in quotitive division he must also be able to associate his answer (the number of equal groups he found) with the concrete aspects of the problem situation. (For example, the number of groups of $.22 contained in $2.00 is associated with, or corresponds to, the number of tickets Sally can buy.)

If the child knows that the question "How many tickets at $.22 each can Sally buy with $2.00?" may be restated as "How many groups of 22 pennies are contained in 200 pennies?" he will be able to make the correct equation, compute, and interpret his answer. In thinking out this problem, he will see that 22 pennies is the size of the group, 200 pennies the total amount, and that the unknown number (to be discovered through computation) is the number of groups of 22 pennies contained in 200 pennies. He can then formulate the equation $200 \div 22 = n$. When he finishes his computation, he will know that in 200 pennies there are 9 groups of 22 pennies with 2 pennies left over. Because he knows that 22 pennies (or $.22) is the amount needed to buy 1 ticket, he should have no difficulty in interpreting his answer correctly as 9 tickets (the number of tickets that can be bought with 9 groups of 22 pennies), and the remainder will be seen as 2 pennies and not as tickets.

In *Seeing Through Arithmetic 3*, pages 92 and 93 children are taught to make a correspondence between groups of money and the objects such groups of money would buy, between groups of objects and the containers needed to hold groups of this size, etc. This work is reviewed on pages 66 and 67 of *Seeing Through Arithmetic 4*. If some children fail to make this correspondence easily in the problem on page 86 in this book, even after seeing it demonstrated with objects matched against groups of pennies, you may find it necessary to review these two earlier lessons. Have the pupils match objects with pennies

in very simple divisive situations until they understand the correspondence (page 86) between the number of groups of 22 pennies and the number of tickets Sally can buy.

In this program, the division basic facts are taught by means of the "measurement," or quotitive, type of situation. In this situation, the number of equal groups is to be found. The teaching of the division process also is based upon the "measurement" type of divisive action—that is, groups of known size (the divisor) are taken away, or subtracted, from the original group (the dividend).

The problem on page 86 involves the "measurement" divisive idea. When the amounts are expressed in pennies, the equation is $200 \div 22 = n$, and the divisor, 22, represents the number of pennies each ticket costs.

In problems of the "sharing" type (partitive division), the situation is different, as is shown by the problem on page 87. In this type of problem the size of the equal groups (the divisor) is unknown. When the amounts are expressed in pennies, the equation is $200 \div n = 22$. The 22 in this equation represents the *number of groups* (of pennies), of equal but unknown size, that each of the 22 boys will pay for his share. There will be one group of pennies for each boy. In the computation, however, the divisor, 22, represents *the size of the group removed from the total supply* in each round of sharing. (Each time the 22 boys gave a penny apiece, a group of 22 pennies was removed from the total amount to be shared.) As in the problem on page 86, the computation is done to discover how many groups of 22 are contained in 200.

The difference between these two problems lies in the meanings of the concrete aspects of the situations. This difference can be shown in a meaningful way by demonstrating these problem situations and their solutions with objects. In the first problem, the groups of 22 that are removed from 200 remain intact, since the correspondence is 22 to 1 (22 pennies to 1 ticket). But in the second problem, the groups of 22 that are removed from the 200 are immediately partitioned among 22 boys, since the correspondence is 22 to 22 (22 pennies to 22 boys, or 22 pennies shared by 22 boys). In this problem the answer (9 groups of 22 with a remainder of 2) represents 9 rounds of sharing. Since the children can see, in a demonstration with pennies, that in one round of sharing each boy paid 1 penny, they should recognize without difficulty that in 9 rounds of sharing each boy will pay 9 pennies and that 2 pennies will be left over.

Page 86: When the children have read the problem and have studied Picture A, let someone count the money in the picture and relate it to the problem. Then discuss the meaning of the numerals in the equation. Ask which numeral represents the total amount, and have a child relate this numeral to the problem and to the picture. Ask which numeral represents the size of the equal groups. (You may remind the children, if you like, that the numeral following the division sign in an equation always represents the size of the equal groups.) Let a child relate this numeral to the problem. Ask what *n* represents, and bring out the fact that since the numerals in a division equation always stand for the total amount, the size of the equal groups, and the number of equal groups (in that order), the unknown numeral in this equation must represent the number of groups. Ask why finding the number of groups (of pennies) will tell how many tickets Sally can buy. If necessary, you might say something like this:

We know that $2.00 is the total amount. We know, too, that 22 cents is the size of the equal groups. Do we know how many groups of 22 cents there are in $2.00? Isn't that what we have to find out? Is 22 cents the same as 22 pennies? How many tickets can Sally buy with one *group of 22 pennies? Can Sally buy as many tickets as there are groups of 22 pennies in $2.00? Why can you think of $2.00 as 200?*

Ask a child how many pennies there are in one dollar and how many there are in two dollars. Direct attention to Picture B and explain that it shows all the money in Picture A changed into pennies. Tell the children that when they compute with dollars and cents, they can omit the dollar sign and point from the computation if they think of all the money as changed into pennies.

Then direct attention to Picture C and ask why the pennies in the picture are being put into groups of 22. Be sure the pupils observe that the first group of 22 is being made. Let a child relate the 22 in the computation to the picture. Ask again how many pennies are needed to buy 1 ticket. Let another child relate the 200 in the computation to all the pictures, to the text in Problem A, and to the equation. Let another child relate the 22 in the computation to the problem and the equation. Have the pupils verify the computation and explain the significance of the answer. Be sure they understand that the 9 represents the number of groups of 22 pennies that are contained in 200 pennies, and that 2 pennies are left over. Let a child explain how we know that these

9 groups of pennies will buy 9 tickets. Let another child explain why the remainder, 2, stands for pennies and not tickets.

Page 87: Have the children read the problem and tell which numeral represents the total amount. Help them see that Picture A shows this amount. Ask where the numeral that represents the size of the equal groups is written in a division equation. Have them tell what *n* represents in this equation and how we know that this numeral must stand for money and not boys or books. Ask the children if any of the numbers they know to start with in this problem are the same as any of the numbers they knew to start with in the preceding problem. Help the children discover the difference between these two problems. You may ask such questions as the following:

What were you trying to find in the problem on page 86? What did you find when you computed? What did the result of the computation tell you? Did you know how many equal groups there were? What did the 22 stand for? Is that what the 22 stands for in this new problem? What do you want to find in this problem? Do you know how many equal groups there are? Do you know the size of each of these groups? What does the 22 stand for in this problem? What will it stand for in the equation?

Lead the children to see why the number of equal groups in the equation is the same as the number of boys. (If 22 boys share the cost of the $2.00 book equally, each will contribute an equal number of pennies. Since there are 22 boys, there will be 22 equal groups of pennies.) Let the children discover that since we do not know how many pennies are going to be in each of these 22 groups, we cannot make any complete group immediately with objects. Show the children how the groups are built up gradually. The best way to show this is by a demonstration with 200 pennies (real or play money). You may, however, use markers instead.

Put 200 pennies, piled in groups of 5, on a table where everyone can see what is happening. Touch each group of five pennies and let the children count aloud to verify the amount. Then put all the pennies in a pile. Tell the children that this pile contains the same number of pennies that the boys need to buy the book. Ask how the 22 boys can find out how many pennies each must give if they are to share the cost of the book equally. Someone will probably suggest that, to start with, one penny should be taken from the 200 pennies for each boy. Let a child do this, counting aloud so that the children know that the correct number of pennies is being taken from the total supply. Have the child put the pennies in a row on the table. Be sure the children understand that a group of 22 pennies, one penny for each of the 22 boys, was taken from the 200 pennies. Put the problem on the board in computational form and explain that the divisor represents the group of 22 pennies that will be taken away from the 200 pennies in one round of sharing.

Let the children discover that they could go on taking out groups of 22 until all the pennies have been equally divided, but that it would take a long time. Ask for suggestions as to how the sharing process might be speeded up. Then put back the pennies that have been taken out, and ask another child to start over, this time taking out a larger group for each of the 22 boys. If you wish to have this demonstration follow the computation in the book step by step, tell the child who is taking out the pennies to take out a group of 9 for each of the 22 boys. Ask the class how many pennies in all have been taken from the total supply. Ask how many rounds of sharing each group of 9 represents. Write these numerals on the board, and later on let the children compare them with those in the book. Ask how many pennies are left. Ask if there are enough pennies left for each of the 22 boys to give another penny. Get someone in the class to explain that the 2 pennies left are the remainder. Write the numeral 2 on the board.

Next, direct attention to Pictures B, C, and D and to the computation. Let the children take turns relating each picture to the computation. Copy the computation on the board and let someone explain each numeral in it. Then go back to the equation and let a child read it, supplying the name of the missing number and explaining what it means.

Slow pupils may need to go through a number of similar examples, taking out groups one at a time, then two at a time, etc., before going on to the problems on page 88.

Page 88: Discuss Problem A with the class. Let someone explain what would happen if Patsy decided to take out one penny at a time for each of the presents. (A group of 6 pennies would be removed from the total amount.) Let the children tell what information they have been given in this problem as well as what they must find out. Let someone explain why 430 can be divided by 6 in the computation. Let a child show how to do this computation on the board, and have another child explain what the answer and the remainder represent. If you think it is necessary, let one of the children use pennies to show what has happened.

Discuss Problem B in detail also. Have the children do the computation on their papers; then discuss the answer.

For Problem C, let the children tell what must be discovered by the computation, what the numerals in the equation represent, and why the number of groups of 35 pennies can be translated into packages of paper. Let someone explain why the remainder stands for money and not packages of paper.

Let the children work Problems D to L by themselves. Tell them to think about the problem, decide what must be discovered, write the proper equation, do the computation, and then rewrite the equation, filling in the missing numeral. Provide answers so that pupils may verify their work when it has been completed. Then note which problems proved to be most troublesome, and discuss them with the whole class.

Providing for the able pupil

Activity 15, the "Make up a problem" game described on page 363, may be adapted to provide experience with partitive division involving money. The child who is "It" writes on the chalkboard an equation with the first numeral expressing dollars and cents, *n* following the division sign, and a numeral for the number of groups. He calls on another child, who makes up a problem to fit the equation and finds the answer.

Helping the slow learner

Let slow learners make up division problems involving dolars and cents in which the size of groups must be found. Amounts used in the problems should be small enough so that the sharing action can be easily demonstrated with real or play money.

89-90 Using arithmetic; Checking up

Lesson Briefs for these lessons are on poges 91-92.

Expanded Notes are not considered necessary for these lessons.

91 Keeping skillful

Lesson Briefs for this lesson are on poge 93.

Expanded Notes other than provision for the able pupil are not considered necessary for this lesson.

Providing for the able pupil

Able pupils should be encouraged to improve their solutions of division problems by using larger partial quotients and fewer steps. Any of the following activities will contribute toward this end.

Make a set of 8″ x 10″ cards. On one side of each card put a division example written in computational form and on the other side put the largest possible partial quotient ending in zero. Arrange the cards so that the examples become progressively more difficult. Hold up the cards one by one, and ask the children to look at each, decide what the answer will be, and write their estimates. Explain that, when estimating, it is better to give an answer that is a little too small than one that is a little too large. When you have held up five or six examples, show the answers. Let the children compare and discuss their estimates.

Then show another series of more difficult examples. Let the pupils estimate and compare their estimates with the partial quotients on the back of the cards. Ask them whether they think their estimates have improved.

After the children have had some practice in estimating quotients, you may let them have a spelldown. Choose two teams of children of about the same achievement level. Prepare a set of 8″ x 10″ cards on which division examples in computational form are written on one side and the largest possible partial quotients on the other. The examples should all require approximately the same level of skill. For example, none should have a divisor as small as 2, with a two-figure quotient; none should be more difficult than the children have already worked. There must be at least as many cards with different examples on them as there are children, because no example should be used twice in the same game.

Show a card to the first child on a team, and let him give an estimated partial quotient. This estimate must be either the largest possible partial quotient or less. If the estimate is larger than the correct quotient, the child is out of the game. If the first child gives an acceptable estimate (one that is the same or less than the computed quotient), show the same card to the first child on the second team and ask him if he can give a better estimate. If his estimate is closer to the largest possible partial quotient, the first child is out. If it is not, the child on the second team is out. If he accepts the first child's estimate, let him estimate the quotient for another example; then give the second child on the first team a chance to accept or correct it. Continue the game, using the above procedures, until each pupil has had a turn, or until all the children but one are out. If a child is out because he overestimates a quotient, do not use that card for the next child on the other team, since he has a clue to the answer and has had longer to think about it.

Be sure the children understand that they are not trying to give the complete answers in this game, but are just trying to estimate the largest first partial quotient. Give each child the same amount of time to give his answer. If a child does not answer in the allotted time, he is out, and the next card in the set is shown to the next child on the other team.

CHARTING THE COURSE

Changing measures

By the time children have reached the fifth grade, most of them have had considerable experience with measurement. They have measured lengths, weight, capacity, time, and temperature. They know some of the commonest equivalents, such as 3 feet equal 1 yard. In an instructional program in arithmetic it is desirable to check up on this knowledge from time to time, to reteach facts or principles that have been forgotten, and to extend the pupils' knowledge. *Seeing Through Arithmetic 5* provides materials for these purposes on pages 92-95.

In *Seeing Through Arithmetic 4*, pupils are shown how to analyze situations that involve changing measures to larger or smaller units. Suppose a quantity measured in quarts (for example, 7 quarts) is to be expressed in pints. Each quart may be thought of as replaced by a group of 2 pints. This suggests multiplying 2 by 7. On the other hand, suppose a quantity measured in pints (for example, 18 pints) is to be expressed in quarts. Since it takes 2 pints to make one quart, the 18 pints may be thought of in groups of 2. This suggests finding how many groups of 2 are in 18, and this is quotitive division. These ideas are retaught on page 94.

Up to this point, in problems requiring conversion from one unit to another, the data have been so restricted that only multiplication basic facts or division basic facts were needed. Now, however, the pupils can apply the multiplication and division processes to conversion problems. Thus to change 1440 seconds to minutes, they can divide 1440 by 60. Pages 94 and 95, therefore, provide problems for thus extending the abilities of the pupils to deal with measurement situations.

92-95 Looking back; Keeping skillful

Lesson Briefs for these lessons are on pages 94-97.

Expanded Notes other than suggestions for the able pupil and help for the slow learner are not considered necessary for these lessons.

Providing for the able pupil

The able pupils will probably have no difficulty in working with the lesson on measurement equivalents and abbreviations. As soon as they finish, you might let them work the first crossword puzzle in Activity 27, page 370, and the cross number puzzle suggested below for the slow learners (Activity 28, page 371), and then let them try making a crossword puzzle. If acceptable, these crossword puzzles could be used by the slow learners.

The abler pupils should enjoy doing independent reading on measurement to discover how our measures evolved. Children's encyclopedias and the book *How Much and How Many** may be used as sources. The findings can be presented to the class in any one of several ways—dramatizations, quiz programs, mural designs, etc.

Helping the slow learner

Crossword and cross number puzzles can be used to give children a review of measurement equivalents and abbreviations in a way that they will enjoy. See Activity 27 (page 370) and Activity 28 (page 371) for sample puzzles.

Scrambled sentences also can help children become familiar with measurement ideas such as abbreviations and equivalents. A set of scrambled sentences with answers is provided below. Each item consists of two scrambled sentences which the children are to unscramble. Write sets like these (without the answers) on 4" x 6" cards, and let the children work them in their spare time.

A. equals three yard feet one things one 12 equals dozen

(One yard equals three feet. One dozen equals 12 things.)

**How Much and How Many* (The Story of Weights and Measures) by Jeanne Bendick. Whittlesey House, McGraw-Hill Book Co., Inc., New York, New York.

B. equal a one of pints tbs. quart is is short writing two way to tablespoonful
(One quart is equal to two pints. Tbs. is a short way of writing tablespoonful.)

C. used 0° time boils is above to 212° tell water a at clock
(A clock is used to tell time. Water boils at 212° above 0°.)

D. cup pints 8 equals 8 equal gallon ounces 1 fluid 1
(8 fluid ounces equal 1 cup. 1 gallon equals 8 pints; *or* 1 cup equals 8 fluid ounces. 8 pints equal 1 gallon.)

E. the dry for 32 c. bushel abbreviation equal cup quarts is 1
(The abbreviation for cup is c. 32 dry quarts equal 1 bushel.)

CHARTING THE COURSE

Multiple-step problems

Many problems involve two or more computational processes to obtain the answer. For example: "Tony bought 2 cards of fish hooks with 10 hooks on each card. Jim bought 2 cards of fish hooks with 12 hooks on each card. How many fish hooks did the boys buy in all?" This is a multiple-step problem that may be solved by multiplying twice and then adding. "Multiple-step," as used here, refers not to the details of the processes, but to the number of successive complete basic operations (addition, subtraction, multiplication, division) required to obtain the answer.

In the past teachers have very often introduced multiple-step problems without first encouraging the children to "see where they are going." In the problem quoted above, for example, they would multiply 10 by 2 to find how many fish hooks Tony bought. Then they would multiply 12 by 2 to find how many fish hooks Jim bought. They might recognize dimly that in the end these two products would have to be combined by addition, but in a very real sense, they failed to see the problem clearly as a whole *from the beginning*. It is true that abler pupils gradually gained this foresightedness through experience with many similar problems, but for many pupils the solving of multiple-step problems has been a more or less blind procedure.

If pupils are to succeed with multiple-step problems, they must learn from the beginning how to analyze the problem situation as a whole before rushing into computation. One way to focus attention on the total situation is to consider the equation that expresses it in symbols. In the example above, the form of this equation is indicated by writing ▭+▭=n. Here the first "screen" (▭) represents the number of fish hooks Tony bought, and the second screen represents the number Jim bought. The form of the equation shows that these two numbers are to be added to find the number of fish hooks bought by both boys. (See page 96 of *Seeing Through Arithmetic 5.*) This form makes clear that two numbers, both as yet unknown, must be found before the important final step of addition can be carried out. Finding the number of fish hooks Tony bought is, of course, a simple one-step multiplication problem. Finding the number of Jim's fish hooks is a similar problem. Usually, these separate one-step problems are not the source of the difficulty in multiple-step problems. What puzzles the pupils is how the situation as a whole is to be put together.

It is important to observe that many multiple-step problems can be seen and solved in more than one way. For example: "Tony fished for 35 minutes in the morning and 45 minutes in the afternoon. He did this every day for 3 days. For how many minutes in all did he fish during the 3 days?" One way of viewing this situation is to observe that all 3 days are alike; so multiplying the combined morning and afternoon periods by 3 will give the answer. This analysis may be shown by writing 3×▭=n. This leads to adding 45 and 35, and then multiplying the sum, or 80, by 3. Another way of viewing the situation is to see that the 3 mornings are alike, and the 3 afternoons are alike, and that the final answer can be found by adding all of the morning time to all of the afternoon time. This analysis may be shown by writing ▭+▭=n, and leads to multiplying 35 by 3, then multiplying 45 by 3, and finally adding these products (105+135=n). Either method of solution is acceptable.

Among two-step problems, those that involve finding averages are of special interest. The idea of average is very widely used, and children need an understanding of averages for use in work and play, both in school and out. Too often they are merely given a rule: Add the numbers, and then divide the sum by the number of numbers that were added.

Fundamentally, an average is a single number that is used in place of a whole set of numbers. As a simple example, consider what single number might be used to replace 8, 12, 19, and 21 and yet not change the total (8+12+19+21). It is not immediately obvious that 15 is a good answer. If, however, the original numbers are thought of as measures of length (for example, the length of 4 sticks), the total length would be 60. If this length were divided equally (that is, partitioned) among the 4 lengths, each would be 15 units long. Actually, the length 8 is 7 units under the average, and 12 is 3 units under it. Together, they are 10 units under the average. The length 19 is 4 units over the average, and 21 is 6 units over it; so together these are 10 units over the average. Thus the 10 units of shortage below the average are balanced by the 10 units of extra length above it. This will be true only when 15 is used as "the average."

When the meaning of average is understood as suggested above, finding the average may be seen as a two-step problem. In the first step, all of the measures are combined—this is an additive situation. The sum is then partitioned, or separated, into as many equal parts as there are measures originally. In some situations the sum is already known, and only the partitioning remains to be done. In *Seeing Through Arithmetic 5,* pages 101-104 may be used to develop the idea of average so that children will be able to understand as well as compute averages.

In solving problems by division, a question often arises about what to do with the remainder. There is no single way to handle this situation, and no single "rule" can be applied in all cases. It is necessary to think about the meaning of the remainder in each particular problem situation, and to use judgment in using the remainder in the solution. Pages 105-106 of *Seeing Through Arithmetic 5* focus attention upon the need for judgment in using the remainder. Later, after fractions have been studied, other ways of handling the remainder can also be considered. Using the remainder to make a fraction is not taught until fractions have been taught.

96-100 Exploring problems

Lesson Briefs for this lesson are on pages 98-101.

Overview

This lesson presents the multiple-step problem. Solution of such a problem requires a grasp of the master situation and the writing of a "master" equation necessitated by this overall view. This equation must express the principal action—additive, subtractive, multiplicative, or divisive—that solution of the problem requires. Additive action is expressed by the overall equation ▭+▭=n on page 96. The screens stand for the numbers that must be found before the equation can be solved and the value of *n* (the answer required by the problem) determined.

The master equation 21 − ▭ = n, on page 98, expresses the subtractive action of the question asked in the problem. The presence of one screen indicates that only one number has to be found before the equation can be solved. Multiplicative and divisive actions are expressed by the two master equations, 3 × ▭ = n and $4.20 ÷ ▭ = n, on page 99.

It should be noted that the so-called "steps" in a multiple-step problem are the separate procedures the child must go through to find the numeral he will use in the equation for the problem he is solving.

Teaching the whole class

Page 96: Tell the children to read Problem A. Then have them look at Picture A and identify the cards of fish hooks. They should see that the hooks on the red cards are Tony's (10 hooks on each card) and that those on the yellow cards are Jim's (12 hooks on each card).

Point out to the class that the question asked in the problem indicates a combining action, and get the children to see that to find the answer, the unknown number of Tony's fish hooks and the unknown number of Jim's fish hooks must be combined.

Turn attention next to Picture B and the reading material opposite. The dotted background in the picture suggests that Jim's fish hooks and Tony's fish hooks are to be thought of as combined. The equation at the right represents the combining situation in symbols. A red screen is used to stand for the unknown number of Tony's fish hooks, and a blue screen is used to stand for the unknown number of Jim's fish hooks. The *n*, of course, stands for the answer, the total that results from the combining of the two groups.

Continue the discussion of the equation until you are sure the children see how this equation represents the additive action required by the question in the problem.

Now have someone read the last statement and explain why the number of hooks Tony bought and also the number Jim bought must be found. Ask the children if enough information is given in the problem to enable them to find the numbers that are represented by the screens in the equation. When they conclude that there is enough information, tell them that on the next page they will see how to use it.

Page 97: Discuss Picture C, the equation, and the text. The children should observe that now they are dealing with Tony's fish hooks, that 2 groups are to be combined, and that, since the groups are equal, the process is multiplication.

Use the same procedure for Picture D and the text opposite. The pupils should recognize these fish hooks as those Jim bought. When they have found how many fish hooks each boy bought, have them look at the equation opposite Picture B on page 96. Ask if they now know what numerals belong where the red and blue screens are. Have someone substitute the numerals in the equation. Then have the class look at the equation opposite Picture E and read the text. Tell the pupils to add to get the answer.

Some of the abler pupils may wish to suggest a solution that is different from that shown in the book. They may feel that adding is preferable to multiplying when finding the numerals that belong where the red and blue screens are. Such thinking is permissible, but do not let the children forget that the basic, or master, equation must show the action (additive in this case) required by the question in the problem.

Page 98: Ask the pupils to read Problem A. Then bring out in discussion that the central situation is that of removing some fish from the original group. Go on to an examination of the master equation, which shows the number in the original group, the screen standing for the number removed, and the remainder, represented by *n*. The pupils should see that before they can find the number represented by the screen, they must find the total number of fish removed from the original group of 21 fish. Refer to Picture B, which shows the fish each boy sold.

The dotted background in Picture C implies that the two groups of removed fish are to be combined.

Page 99: In Picture D the dimming of the 9 fish shows that they are to be thought of as gone. Let the children discuss why the combined groups should now be thought of as gone. Discuss the equation opposite Picture D, and let the children decide why the numeral 9 can be used to replace the screen in the first equation. Tell the pupils to subtract to find the answer. Then they should write the equation with the answer inserted in place of *n*.

For Problem A on page 99, help the children see why the situation is basically multiplicative. Point out that Tony fished for the same amount of time each day for three days. In the master equation, then, 3 stands for the number of days (or the number of equal groups). The size of the equal groups (or the number of minutes Tony fished each day) is as yet unknown; so the screen is used to represent this number.

Discuss how to find the total number of minutes Tony fished each day. Lead the children to see that the number of minutes he fished each morning must be combined with the number of minutes he fished each afternoon. Have someone put the numerals in computational form on the board and add. Then have the pupils substitute this total of 80 minutes in the basic equation. Now let someone do the multiplication indicated by the equation and find what number is represented by *n*.

Problem B may be handled in much the same way. Here the pupils must see that the situation involves division in which the number of equal groups is to be found.

First discuss the basic equation in terms of the problem situation. The pupils should then see that to find the number the screen stands for, they should find the total amount earned each day. The sum of $40¢$ and $20¢$ gives them the size of each group. When this total has been found, ask someone to write the equation on the board, replacing the screen with the correct numeral. He should then divide to find the number that *n* stands for.

Page 100: Assign the problems on this page as written work. Instruct the children to read each problem carefully and study the help given. The correct basic equation is provided for each problem, but the children must decide how to find the numeral to replace the screen. Also they must be able to explain why the basic equation is correct.

Tell the pupils first to write the basic, or master, equation for each problem on their papers. They then should show the necessary computation for finding the numeral to replace the screen. Next they are to rewrite the equation with this numeral inserted. If they need to compute on paper to get the answer (*n*), tell them to show this work, too.

For ways of correcting the children's work and ways of handling class discussion, see the suggestions given for the problems on page 19 of the pupils' book (page 262 of this *Teaching Guide*).

Providing for the able pupil

"Problem relay" is a game that these children will enjoy. See Activity 29, page 372.

Helping the slow learner

The slow learners can also participate in the "Problem relay" game. However, for these children you may want to use the adaptation suggested in the last paragraph of the description. See Activity 29, page 372.

101-104 Exploring problems

Lesson Briefs for this lesson are on pages 101-103.

Overview

On these pages the finding of averages is taught as an application of the method used to solve two-step problems, which the children studied on pages 96-100. The pupils are taught that the average is the size of the group that results if a certain number of unequal groups are put together and this new group is then divided into the same number of equal groups.

The basic equation the pupils use in symbolizing this situation is that used for partitive division ($24 \div n = \dot{3}$, for example). In most cases a screen will have to be used in the basic equation in place of the dividend, since the number that is the dividend must be found by adding the unequal groups. The pupils should see that finding an average is a kind of equalizing process in which unequal groups become, in imagination, equal.

Two types of problems involving averages are developed fully. One type (page 101) involves objects that are counted, not measured (other examples—fruit sold by the dozen, toys owned by somebody), and the other type, beginning on page 102, involves objects and materials that are measured, not counted (other examples—apples bought by the pound, fish measured in inches). The lesson concludes with a set of problems for the children to solve.

Teaching the whole class

Page 101: Before beginning this lesson, you may want to introduce the idea of "average." Statements and questions such as the following may be used:

Suppose you bought 7 toys at different prices and someone asked you about how much each toy cost. Why would the price of the cheapest toy not be a good answer? Why would the price of the most expensive toy not be a good answer? How would you answer this question? Or suppose we wanted to compare the height of the boys in this class with the height of the girls to see which group is taller. How could we do this? Would it be a good idea to compare the heights of the tallest boy and girl? Let us see if there is an easy way to answer these questions.

Now have the problem on page 101 read. Let someone count the money in each of the four groups of coins in Picture A and relate each group to a boy mentioned in the problem. Point out that if each of the boys had loaned the same amount, each group of coins would have the same value. The pupils may observe that in this case some of the boys would have loaned less than they actually did, while others would have loaned more.

Next direct attention to Picture B. Establish first that the coins pictured here are the same coins as those in Picture A. Tell the children that now they are to think of the four groups as one group and imagine that it is to be divided into four *equal* groups. Lead the children to see that they have already studied this situation (finding the size of equal groups when they know the number of equal groups). Then point out that so far they do not know how much money in all is to be divided. Therefore, in making a master equation, we use a screen to represent this unknown amount. Have the children examine the equation in the book and explain each of its parts.

The children should now see that first they must find the numeral that replaces the screen in the equation and that this numeral can be obtained by finding the total amount the boys loaned. See page 98 for a suggestion for a method of indicating the work.

Let them add to find this total and have them verify it by counting the money shown in Picture C. Be sure the pupils note the rearrangement of the coins in Picture C to make counting easy (two groups of one dollar each and a third group of $.60).

Page 102: Now that the total ($2.60) has been found, it can be used in the basic equation, and the equation can be solved. Have the pupils study the equation. Ask them to divide $2.60 by 4. Tell them that the answer ($.65) means that if each boy had loaned $.65, the club would have received the same total amount and that $.65 is the *average* amount loaned.

Now have the pupils study Picture D. They should see that each of the four groups has a value of $.65. They should see, further, that these coins are the same coins that were used in the other pictures and that each picture shows the same total amount of money.

Next have the pupils turn to the new problem under the heading "See." Call on someone to read it aloud and relate the picture to it. Discuss with the children what the expression "average weight" means. Ask how many boxes of groceries Paul delivered.

Page 103: In connection with the work labeled B, you may first ask questions like these:

Why should we think of all the groceries as one large amount? Do we know how much the groceries weigh in all? Can we find out? Why should we think of them as divided equally into 7 boxes?

Tell the pupils to examine the equation and have someone explain each part of it in relation to the picture.

Next go on to the "Compute" step. As the pupils read the text material, have them do the computation. Be sure they understand every step. You might ask someone what numeral could be put on the large box in Picture B. Let another pupil explain why the 7 boxes of groceries in Picture C are each marked "9 lb."

Page 104: Discuss Problems A and B. In Problem A call attention to the word *per*. Have the pupils note that *per* has the same meaning as the word *an* in the preceding sentence. Point out that *per* is used again in Problem B. Bring out that in Problem B the average can be found in one step because the total amount is given in the problem and this numeral can be written immediately. Then assign Problems C to I as written work. Tell the children to write the basic equations and compute to find the missing numbers. Tell them that in some of these problems the total is given.

Have answers available so that the children may verify their work. Then discuss any problems that caused trouble.

Providing for the able pupil

The able pupils will enjoy finding the average of many things that are of interest to young people their age. For example, working either individually or in committees, these pupils might find the following averages:

(1) The average height of the boys (or girls) in the class. Measure each child to the nearest inch. (Pupils should disregard remainders if they occur in finding the average.)

(2) The average weight of the boys or of the girls. Weigh each child to the nearest pound.

(3) The average weight or height of the class. When doing this, the individual weights or heights must be totaled and divided by the number of pupils. Do *not* allow the pupils to total the averages found in (1) and (2) above and then divide by 2.

(4) The average daily attendance of the class, the grade, or the school, based on the record for a week or a month.

(5) The average time members of the class spent in watching TV during a week.

(6) The average amount members of the class spent on movies during a month.

Perhaps the pupils will think of other projects involving amounts for which they would like to know the average. When such averages have been determined, they may be used as a basis for posters to be displayed on the bulletin boards.

In connection with the lesson on finding the average, the able pupils might be interested in seeing how the concept of average can be applied to numbers in a sequence. Suppose, for example, the numbers that are to be averaged are 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, and 17. The children have learned that, to average these numbers, they should add them and then divide the sum by 11, since there are 11 numbers in all. A short-cut method for averaging these numbers is simply to find the average of the first and last numbers (that is, $7+17=24$. $24\div2=12$). 12 is the average for these two numbers and is also the average for all the numbers in the sequence above. This method can be used to find the average for certain other types of number sequences. If, for example, the numbers to be averaged are all of the even numbers from 2 to 24, again simply average the first and last numbers. Thus the average for these numbers is 13.

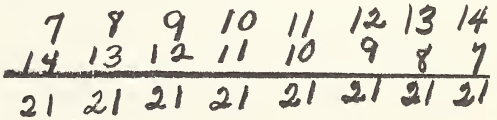
After you have demonstrated this method to the children, let them practice it by using the examples below and others they make up for themselves:

Find the average of all the odd numbers from 7 to 31.

Find the average of all the numbers from 1 to 99.

Find the average of every third number from 6 to 117 (6, 9, 12, 15, . . ., 117).

Another application of this principle can be used to find the sum of the numbers in a series. Suppose you want to find the sum of all the numbers from 7 to 14. First add 7 to 14. Then, since there are 8 numbers in the series, multiply the sum, 21, by 8. Finally, divide the product, 168, by 2. The sum of the numbers from 7 to 14 is 84.



The work above demonstrates why this method works. First, numerals for the numbers to be added are written in order. Then they are written in reverse order, and the sum for each pair of numbers is found. In each case the sum is 21. There are eight 21's, and $8\times21=168$. But since each number has been

added twice, this product must be divided by 2. $168 \div 2 = 84$. 84 is the sum of the numbers from 7 to 14.

After you have demonstrated this example for the pupils, you might proceed with a second example in this way:

Now let's find the sum of all the even numbers from 12 to 36. What is the sum of the first and last numbers? [*48*] *What must this sum be multiplied by?* [*There are thirteen 48's.*] *Why must this product be divided by 2? Is 312 the sum of all the even numbers from 12 to 36? Add these numbers in the usual way to verify this answer. Which way takes less time?*

Have the children use this method to find the sum of other number series (for example, all the numbers from 1 to 100, all the odd numbers from 1 to 99, etc.).

The game of "Call the average," Activity 30, page 372, will furnish groups of children with practice in finding averages and in estimating. It is suggested, however, that this game be postponed until the children have studied the next lesson, which deals with remainders in division.

Helping the slow learner

The activity of finding average weights, heights, etc., described above for the able pupils, may be adapted for the slower learners. Instead of finding averages for the whole class, ask these children to find the average for a group of five or so, or for a row or table of children.

The game of "Call the average," Activity 30, described above and in fuller detail on page 372, may be adapted for the slower children. As stated above, this game may be withheld until the children have studied the following lesson, which is concerned with remainders in division.

105-106 Thinking straight

Lesson Briefs for this lesson are on pages 104-105.

Expanded Notes are not considered necessary for this lesson except for suggestions for the slow learner and the able pupil.

Providing for the able pupil

The game of "Call the average" will give children not only practice in finding averages, but also in working with remainders. A complete description of how to prepare for this game and how to play it is provided in Activity 30, page 372.

Helping the slow learner

The game of "Call the average" can be adapted to the needs of the slow learner. See Activity 30, page 372.

CHARTING THE COURSE

Perimeters

Arithmetic is frequently used to find "the distance around" an object. Among problems of this kind are those that involve finding "the distance around" triangles, rectangles (including squares), and other polygons. The technical term for "the distance around" a polygon is *perimeter,* and this word is now commonly introduced in the fifth grade. In *Seeing Through Arithmetic 5* the meaning of the term *perimeter* is made clear by pictures (on pages 107-108) that show that a perimeter can be thought of as "straightened out," thus becoming the total of the lengths of the sides. Problems about perimeters on page 109 are kept simple and quite concrete.

107-109 Moving forward

Lesson Briefs for this lesson are on pages 106-108.

Teaching the whole class

To introduce this lesson, you might first discuss some everyday situations that involve finding the distance around some polygon, like a city block, a building, or some polygonal figure. Begin somewhat like this:

Today we are going to study how to measure the distance around a field, a building, or a piece of cloth. Suppose Mary is making a handkerchief for her mother. She plans to put lace around the edge of the handkerchief, and she does not want to buy more than she needs. How can she find about how much lace to buy?

Have a piece of paper ready to represent the handkerchief. Bring out in the discussion that Mary must find the distance around the handkerchief.

Some children may see that she can measure each side of the handkerchief and then add the four numbers resulting from the measurement.

As another example, say that Mr. and Mrs. Brown are going to put wallpaper on the walls in a bedroom. They plan to put a border at the top and need to know about how much border to buy. Again the children should see that the Browns must find the distance around the room.

Another example could be to find the distance a boy ran in making a home run in baseball. Draw a picture on the chalkboard to show the standard baseball diamond. To determine how far the boy ran, the total length of the sides must be found. (Each side is 90 ft.)

Page 107: Tell the children first to read the text beside the heading "Moving forward." If you wish to refresh their knowledge of the term *polygon,* you might proceed somewhat like this:

What was the shape of the handkerchief Mary was making? [Either a rectangle or a square] What might have been the shape of the bedroom Mr. and Mrs. Brown were going to paper? [Again, either a rectangle or a square] What is the shape of a baseball diamond? [A square] Look at the pictures on page 109. Do these pictures show only squares and rectangles? What else do they show? [Polygons with 3 sides, 4 sides, 5 sides, 6 sides] We are going to learn how to find the distance around many kinds of polygons.

What kinds of polygons are shown in Pictures A, B, and C on page 107? Polygons A and B have how many sides? Polygon C has how many sides? Remember that a polygon always has three or more sides. Some polygons have sides that are equal. Some have sides that are parallel. Some polygons have sides that are neither equal nor parallel. What do we mean when we say sides are equal? When we say they are parallel?

Some children in your class may not remember what the term *parallel* means, even though it was used on page 36 of their book. If so, draw two parallel lines on the board and explain that parallel lines are an equal distance from each other at all points and never meet or come together. Now refer to Polygons A and B and discuss whether the opposite sides are equal and parallel. Then discuss the sides of Polygon C.

Call on someone to read the text under Polygon A. Review what is meant by square corners and right angles. Let the children find the polygons with right angles and the polygon that has no right angles. Call on someone to read Exercise A and answer it. Be sure the children understand that the square is a special case of the rectangle, which, in turn, is a special case of the parallelogram. (The rectangle is a parallelogram with all angles right angles, and the square is a rectangle with all sides equal.) Ask another pupil to read Exercise B, supplying the missing numeral.

If you wish, the children may measure the sides of Polygon D with their rulers to verify the length of each side. Get them to decide that this polygon is a square and to justify their decision. Then explain that to find the distance around the square, they can imagine the four sides straightened out in a straight line (see Picture E). Have them note Picture F and the equations below. Get the children to explain what the first equation shows (the lengths of the four sides added, to give the total length of 4 inches). Then discuss why the second equation can also be used to find the distance around the square. Let one or more children read Exercises C and D. Note the introduction of the term *perimeter* in Exercise D. Use the term orally as much as possible so that the children will become familiar with it. You may want to let them trace the perimeters of Polygons A, B, and C and demonstrate how each perimeter can be shown as a straight line. Refer again to the handkerchief, the wallpaper border, and the baseball diamond, and let several children explain how the perimeter of each can be found.

Page 108: The sides of the polygons shown in the balance of this lesson are labeled with the lengths they represent. The children, therefore, must imagine the true lengths of the sides. That is, the pictures on pages 108 and 109 are scale drawings. Do not get into a discussion of scale drawing at this point, however.

Ask how many sides Polygon G has, how long each side is, what kind of polygon it is, and what has been done in Picture H. Let someone explain the equations for Picture H and supply the answers. Discuss Exercises E to G.

Pictures I, J, and K show how to find the perimeter of a rectangle that is not a square. Have the children note the lengths of the four sides. Ask why this polygon is not a square. Discuss the terms *length* and *width.* Then ask someone to explain what Picture K shows and read the equation, supplying a numeral to replace *n.* Another pupil may summarize the discussion by reading and answering Exercises H, I, and J.

The use of letters to identify the sides of a polygon was introduced on pages 6-7. In Picture L the sides of the rectangle are thus identified. Use Exercises

K to O as a guide when you discuss finding the perimeter of this polygon.

One approach to the discussion of Polygon M might be this:

Picture M shows what kind of polygon? How many sides does this polygon have? Are the sides equal in length? Are any of them parallel? What is shown in Picture N? Read the equation and then answer Exercise P.

So far we have learned how to find the perimeters of squares, rectangles, and triangles. Page 109 shows still other polygons.

Page 109: First of all, let the children find on this page the three kinds of polygons that they have just worked with (Polygons A, B, D, E, and G). Then have them examine Polygon F and tell how many sides it has and if they are equal. Have them study Polygons C and H, count the number of sides each has, and tell whether or not the sides are equal.

After the children have become familiar with the polygons at the top of the page, let them work independently to find the perimeters (Exercise A). When they have completed this work, discuss the answers with the class. Then discuss Exercises B, C, and D.

After the two methods of finding the perimeter of a square (Exercise C) have been discussed, you might ask the children if two methods can be used to find the perimeters of any other polygons at the top of the page. If necessary, point out the equality of the sides of Polygons C and H and explain that the length of one side of each polygon may be multiplied by the number of sides the polygon has.

When discussing Exercise D, a few pupils in the class may notice that Polygons A and F each have two equal sides. Thus the length of one such side can, in each case, be multiplied by two and the product added to the lengths of the other sides. Do not emphasize this short cut, since some children may not understand it.

When you are sure the children can find the perimeters for all the polygons at the top of the page, let them work Problems E to H independently. Tell them that for each problem they should first draw a rough diagram that illustrates the polygon described in the problem. On the diagram they are to write the lengths of its sides, and then they are to find the perimeter. Examine each child's diagrams and computation when he has finished, to make sure he understands the ideas involved in the concept of perimeter.

Providing for the able pupil

Encourage able children to draw as many different kinds of polygons as they can, then label the lengths of the sides, and give the perimeter of each. To add interest, they also may make up a problem about each polygon they draw.

These children might also measure the boundaries of selected states on a large map and find their perimeters. These, too, may be drawn, colored, and labeled for display. Similar projects to be carried out may include finding the perimeters of large objects in the classroom or school building, or of a football field or tennis court, etc.

An activity on judging lengths (Activity 31, page 373) may be a challenge to these children.

Helping the slow learner

Give the slow learners much experience in measuring objects and then finding the perimeters. (Use a small box, a sheet of paper, a ruler, a postcard, a book, etc.) Since most of these objects will be rectangles or squares, cut out polygons that are not rectangles from construction paper and let the children find their perimeters.

Activity 31, page 373, may be used by the slow learners, but for them it may be a trial-and-error problem rather than one of judgment. Do not expect them to work longer than the activity holds their interest. It is not wise to have children of unequal ability work together on an activity of this kind.

110-111 Checking up

Lesson Briefs for this lesson are on pages 108-110.

Expanded Notes are not considered necessary for this lesson.

CHARTING THE COURSE

Number pairs — fractions and ratios

The idea of *rate* and the idea of *comparison* are both of great importance in the applications of arithmetic. Problems that involve one or the other of these ideas occur very frequently in everyday living. Tradi-

tionally there has been little or no definite teaching of the idea of rate or the idea of comparison in the fifth grade, even though use has been made of them in problems in that grade or earlier. If, however, understanding is to be achieved, it is essential that these ideas be carefully developed. *Seeing Through Arithmetic 5*, therefore, provides activities for this purpose on pages 112-123.

The idea of rate is involved whenever one measure is paired with another. Suppose, for example, that 12 pieces of candy were sold for 15 cents. (See page 112 of the pupils' book.) This statement expresses a rate. To write this rate, two numerals are required. The numeral 12 shows the number of pieces that were sold. The numeral 15 shows the number of cents needed to buy them. There is a pairing, or correspondence, of 12 pieces to 15 cents. This same rate can be stated in other ways. For example, the rate can be expressed as 4 pieces for 5 cents, or 8 pieces for 10 cents.

In stating the rate as 12 pieces per 15 cents, the numerals 12 and 15 are used to express an *abstract* numerical relationship. A pair of numerals used in this way is called a *ratio*. In this example the ratio is 12 per 15. The same relationship is also expressed by the ratio 4 per 5 and by the ratio 8 per 10. The words *pieces* (of candy) and *cents* are used to express concrete aspects of the problem situation from which this ratio came.

In general, every rate also involves a ratio. As another example, suppose a train travels 120 miles in 2 hours. A rate of 120 miles in 2 hours is the same as 60 miles in 1 hour, or 60 miles per hour. The rate, in this example, is expressed by the ratio 120 per 2 or by the ratio 60 per 1.

Ratios are often written with one numeral above the other as in the example shown at the right. $\frac{12}{15}$ This symbol does not represent a fraction.

In the example above, the "pieces of candy" and the "cents" are two entirely different kinds of objects. One cannot say that "the candy is a fraction of the money."

(In this Guidebook, for mechanical reasons, ratios will sometimes be printed with a diagonal line, as 12/15.)

In general, every rate can be stated by using any one of many different ratios—that is, pairs of numerals. For example, in *Seeing Through Arithmetic 5*, pupils learn that 4/5, 8/10, 12/15 and other ratios all express the same rate in the candy-buying situation discussed above. Consequently, it is correct to write 4/5=8/10=12/15=16/20=20/25. This statement is obtained by developing an understanding of rate through using various ways of grouping pieces of candy and cents. Although computational methods exist for showing that two different expressions such as 4/5 and 8/10 actually represent the same rate, they are not introduced at this point. The focus of attention is upon the concepts, rather than upon computational details.

Comparison situations involve two groups of objects, and two numerals are required to express the comparison. For example, suppose Bob has 10 model planes and Paul has 15 model planes. (See page 119 of the pupils' book.) If Paul says, "Bob, I have 15 planes to your 10," he states a comparison. The ratio 15 to 10 expresses the abstract numerical relationship in this situation. The same relation is also expressed by other ratios, such as 3 to 2 or 9 to 6. The same situation also gives rise to a different ratio, namely, 10 to 15, if the other boy's planes are made the basis of the comparison. The comparison idea grows out of *group to group* relationships. These ideas are developed on pages 119-123 of *Seeing Through Arithmetic 5*.

Fractions play a very important rôle in arithmetic. Pupils have been introduced to fractions in earlier grades, but in Grade 5 the study of fractions becomes much more intensive. A discussion of some of the central features of the way fractions are taught in *Seeing Through Arithmetic 5* may be found in the section "Guiding Principles" on pages 6-18.

The fraction idea is used in situations where only a part of something is being considered. A fraction numeral tells *how much* of it is being considered if the whole thing is represented by 1. For example, suppose something is divided into three equal parts and then two of these parts are taken. The child learns to symbolize the relation between these two parts and the original whole by using a new kind of numeral, namely, $\frac{2}{3}$. In this numeral, the 3 shows the number of equal parts into which the whole was divided. The 2 shows the number of these parts that are being considered. The whole numeral tells how much of the original whole is being considered. The part (represented by $\frac{2}{3}$) has the same relation to the whole (represented by 1) as 2 has to 3. This relation idea is a fundamental aspect of the fraction concept, but the fraction concept also involves other ideas. The fraction idea grows out of *part to whole* relationships.

The same ideas are used when the whole is a *whole* group of things. In this case, suppose the original whole

group is first separated into 3 equal parts, or subgroups. Next suppose only 2 of these subgroups are considered. The relation of the number of objects in these 2 parts to the number in the original group may also be symbolized by the numeral $\frac{2}{3}$. This numeral does not, by itself, tell how many objects there are in the two parts of the original group that are being considered. It does indicate that there will be 2 objects for each 3 that were in the original group.

These ideas are the basic meanings for situations that involve fractions. Here, as elsewhere, the child can best acquire these meanings by carrying out the actions with objects, and then using the number symbols to show what he has done. Pictures can help him not only to acquire these ideas, but can guide him to the actions that are basic to the meaning. The pictures and text on pages 126-131 of *Seeing Through Arithmetic* 5 provide materials for a careful review and extension of the fraction idea and the way fraction numerals are used to express it.

There are three types of problem situations that are symbolized by using number pairs. One of these types involves rates, another involves comparisons, and the third involves fractions. It is not possible to tell from the symbol alone which of these situations the symbol is intended to represent. The meanings are based on the concrete situations and are not inherent in the symbols. If children are to deal successfully with each of these types of problem situations, they must learn to distinguish among them regardless of how they are symbolized by numerals. Pages 132 and 133 are of special significance because they help pupils to do this.

112-117 Moving forward

Lesson Briefs for this lesson are on pages 111-114.

Teaching the whole class

One way to begin this lesson on expressing rates is to ask the children to give prices they pay for common school articles such as pencils, erasers, pen points, leads, crayons, etc. Point out that sometimes there is a set price for each item, but that sometimes items can be bought at a smaller price for larger quantities. They may, for example, buy 12 leads for 5 cents and 3 pencils for 5 cents. Explain that the price they pay for these items is called a rate, and two numbers are used to express a rate. Have one of the pupils go to the chalkboard and write in his own way the price, or rate, he paid for several items he got in quantity. Bring in other rates, such as "25 miles in 3 hours," to avoid having children get the idea that rates apply only to prices.

Page 112: Have two boys read as a dialogue the speeches of Don and Jim, and have a third child read the first paragraph of text. Have someone give the two numerals that express the rate at which Jim bought candy. Have someone read the statement that says the rate can be expressed in different ways. Continue somewhat as follows:

Picture B and the text show us one way to express the rate at which the candy was sold. Count the pieces of candy. Now count the pennies. As Jim said, he bought 12 pieces for 15 cents. We can say that he bought the candy at the rate *of 12 pieces for 15 cents. We may write the rate as "12 for 15," but there is also another way to use the two numerals to write the rate. The numerals in red show us that 12 may be placed over 15. Remember that the upper numeral represents the number of pieces that were bought and the lower numeral represents the number of cents needed.*

Have several children take turns reading the text that explains this way of writing the rate. Draw attention to the red text below the numerals. Explain that instead of the word *for*, the word *per* is used to read a rate. In other words, the rate at which Jim bought the candy is read "12 per 15."

Have a pupil read the last three lines of text. The term *ratio* will not bother the children if you explain that it is a word used for the way a rate is written. You may say that the word *ratio* is used as the name of two numerals when they are placed one over the other to express a rate. Using the word often as you discuss the way a rate is expressed and asking questions that require the children to use it meaningfully will be helpful.

Before going on to page 113, have the children look again at Picture B. Ask them if they can see any way to find out how many pennies they would need if they wanted to buy only 4 pieces of candy. Let them think about this for a few minutes. Then go on to page 113.

Page 113: Tell the children to study Picture C. Tell them to count all the pieces of candy and all the pennies so that they will see that there are still 12 pieces of candy and 15 pennies. Ask them whether they can explain what has been done. After some discussion, call on a child to read the text for Picture C. Make sure all the children see that there are 4 pieces of candy and 5 pennies in each group. Then go on to the first two lines of text for Picture D. Let someone explain why some of the

groups are dimmed off in the picture. Then continue somewhat in this manner:

If you can buy 4 pieces of candy for 5 cents, what are the two numerals you would use to write the rate? How would this rate be written? Look at the ratio shown in red type. What does each of the numerals stand for? How do you read the ratio? Does "4 per 5" express the rate illustrated by Picture D? Does this ratio express the same rate shown in Picture B?

After the children understand that, at the rate of 12 per 15, they can buy 4 pieces of candy for 5 cents, ask them how the rate could be expressed if they bought 8 pieces of candy. Let them think about this a little, then direct their attention to Picture E and the accompanying text. Again let the children explain the dimming off in the picture and decide upon the two numerals they will use to express the rate as it is illustrated by Picture E. Have several children read the text accompanying Picture E and explain the meanings of the two numerals in the ratio.

Before going on to page 114, you can suggest other situations by asking questions similar to these:

Now imagine that you wanted to buy 20 pieces of candy. How much would you pay for them? Or perhaps you wanted to buy 16 pieces. How much would they cost?

Page 114: By this time several of the children may see that the rate at which the candy is bought can be expressed as 4 per 5, no matter how much is bought. Let these children help explain Picture F to the class.

Tell the pupils to count all the pieces of candy in Picture F and then all the pennies. When they have agreed that there are 20 pieces of candy and 25 pennies, have someone read the text and explain the ratio given in red.

Ask one of the children to read Question A and use Picture F to find his answer. Stress again that the candy and pennies in Picture F have been divided into groups, each containing 4 candies and 5 pennies, and that the amount of money paid for the candy depends on how many groups are bought. The rate of 4 per 5 is basic. The rate for 2 groups is 4 per 5 taken twice, or 8 per 10; the rate for 3 groups is 4 per 5 taken 3 times, or 12 per 15; the rate for 4 groups is 4 per 5 taken 4 times, or 16 per 20; and the rate for 5 groups is 4 per 5 taken 5 times, or 20 per 25. Therefore, a different ratio can be written to express each way of saying the rate 4 per 5, and all the ratios listed in Exercise B are illustrated in Picture F. Let several children supply the numerals to replace the screens in the ratios in Exercise B.

Then tell the class that another set of ratios is illustrated in Picture F. Read to the children the text at the bottom of the first column. Emphasize that the ratios they have just studied were written to express the number of pieces of candy first. The numerals representing the candy were placed over the numerals representing the money. Explain that if they now think of the amount of money first, they should place the numeral that represents the money over the numeral that represents candy. Let a pupil read the text accompanying the ratio 25/20 and explain what each numeral stands for.

Then have someone read Exercise C and, with the help of Picture F, fill in the screen as well as explain the rate 5 per 4 and the ratio 5/4.

Let another child read Question D and give the answer.

When you discuss Exercise E, again emphasize that all the ratios express the same rate. Use the groups in Picture F to help clarify this for the children. If 5 cents is paid, 4 candies are bought; if 2 groups of 5 pennies, or 10 cents, are paid, then 2 groups of 4 candies, or 8 candies, are bought. If 3 groups of 5 pennies, or 15 cents, are paid, then 3 groups of 4 candies, or 12 candies, are bought, etc. Be sure the children see that all the ratios express the rate 5 per 4.

Page 115: A new rate situation is introduced on this page, but the same steps are followed as for the rate just studied. Therefore, you may adapt the procedures for page 112 and for Picture C on page 113 to this page.

Page 116: Procedures suggested for Pictures D and E on page 113 and for page 114 may be adapted to this page.

Page 117: Give the children a little time to study the pictures at the top of the page. Tell them to decide what rates each picture illustrates, and then let them discuss the rates. When you feel they are ready, have them do the first block of exercises (A to T) by writing each ratio and beside it the letter of the picture that shows the rate. When the children have finished the exercises, discuss each ratio with them. Be sure the pupils understand how to correct any errors they have made.

You may want to use the second block of exercises (A to K) with your class in different ways. The slow learners may be able to do little more than write the rates as ratios. The able pupils may be told that there are different ways to express some of the rates given in the exercises. These children may use circles, X's, or other simple marks to illustrate the rates, and they also may be encouraged to separate the items

into equal groups whenever possible and write as many ratios as they can to express each rate. Check the work of each child and make sure he understands how to correct any errors he has made. Let several children put their work for one or more exercises on the board for the whole class to discuss.

Providing for the able pupil

The able pupils might enjoy playing "Ratio." See Activity 33, page 374. The game is played in groups of four, and picture cards are provided to aid the child in seeing the formation of rates and ratios. You may wish to arrange the children so that one or two who understand rates and ratios are grouped with a couple of children who are having difficulty in understanding them. In this way, the abler pupils will help to verify the work done by the others.

You might also have several able pupils make up a set of statements that express rates at which items may be bought similar to the second block of exercises on page 117. The children can write the statements on 3″ x 5″ cards, five on a card. You can then make use of these cards by having the slow learners draw pictures to illustrate the rates suggested by the statements and write the rates by using ratios.

Helping the slow learner

The child who does not see through this work very readily will be helped if he is given extra practice in using materials to illustrate rates and ratios. Objects and coins may be used to illustrate the rates expressed in the pictures on page 117.

You might write on a sheet of paper a group of rates expressed in the form 3 per 2, 4 per 2, 5 per 7, etc., then have the children make circles, X's, etc., to illustrate them and write the rates by writing the ratios in the form 3/2, etc. (Be sure the pupils use a horizontal line, not a slanting line, when they write ratios.) In this case also the children may understand the meaning more readily if they use objects.

When they are able to do the above exercise, give them the cards that were mentioned earlier and which may have been made by the able pupils (or by you). Have them write the ratios for the rates expressed in the statements. As before, the children may make circles or other simple marks, to help them understand what they are doing.

The game "Ratio" (Activity 33, page 374) may be played by all the children. Since it is played by children in small groups, the slow learner may be put with children who understand rates and ratios better than he does and who, therefore, can help him. This game will give the slow learner good practice in matching rates illustrated by pictures with ratios that express the rates.

118 Thinking straight

Lesson Briefs for this lesson ore on poge 115.

Expanded Notes are not considered necessary for this lesson.

119-123 Moving forward

Lesson Briefs for this lesson ore on poges 116-119.

Teaching the whole class

Page 119: To introduce this lesson you can give two girls and two boys a short selection to read. Time them with a stop watch and write on the chalkboard the number of minutes it took each child to read the selection. Then make a comparison between the times for the two boys and another comparison between the times for the two girls. (Simplify the times so that comparisons are easily made.)

Other comparisons can be made between objects or experiences familiar to the children. Such comparisons might include the number of windows in the classroom compared with the number of windows in the library, the number of hours one child sleeps at night with the number of hours another child sleeps, the number of pencils one child has in his desk with the number another child has, etc. Be sure to use the word *compare* in these exercises.

Then tell the children that they are now going to learn the proper way to write, read, and interpret comparisons. Have them open their books to page 119, and let two children read the dialogue of the two boys in Picture A. Emphasize the fact stated in the text that *two* numerals are used to express a comparison—in this case, 15 and 10.

Ask the children to count the red planes and the black planes in Picture B and explain how Picture B and Picture A are alike. Then have a pupil read the text accompanying Picture B and explain the meaning of each numeral in the ratio. Point out that since the ratios in this lesson are expressing comparisons, the word *to* instead of *per* is used when reading the ratio.

Call on someone to explain what has happened in Picture C. Let the pupil tell the class how many of each boy's planes have been put into each group in the picture. You may point out that numerals other than 15 and 10 can be used to compare Paul's and Bob's planes. Let someone read the text to summarize the counting done by the children and the explanation you have given.

The way in which the groups of five are formed from the total group will be clearer to the children i

you demonstrate with a group of 15 objects and a group of 10 other objects how all of the objects can be separated into 5 groups, 3 of one kind, and 2 of another kind. Perhaps you will want each child to have experience in grouping. This can be done by letting the children cut 15 pieces of construction paper of one color and 10 pieces of another color to represent the planes in the pictures. Then have them separate the pieces of paper into the groups shown in Picture C.

Page 120: Have the children look at Picture D and tell why 4 of the groups have been dimmed off. Then ask someone to read the text accompanying Picture D. Be sure the children understand that the comparison is based on only *one* combined group of 3 red planes and 2 black planes, and that the proper way to read the ratio showing the comparison is "3 *to* 2."

If the children are using pieces of paper to represent the planes, have them move to one side all the groups except one and verify the ratio with the pieces of paper they have.

Handle Picture E in the same way as Picture D. You will want to point out to the children that the smallest groups that the sets of the planes can be separated into will consist of 3 of Paul's planes and 2 of Bob's. Let them discover that the relationship in Picture E is the same as that in Picture D, and that it also can be expressed as "3 to 2."

Page 121: Tell the class that Picture F uses groups of 3 and 2 to show what happens when Paul buys 3 more planes and Bob buys 2 more planes. Now they have 18 and 12. If the children are using pieces of construction paper for illustrating the work, have them cut out another set of 3 and 2. Then let them make the correct number of groups (representing 3 of Paul's planes and 2 of Bob's) to help them write and explain the ratios for Exercises A, B, and C. You might discuss what would have happened in the comparison if Bob had bought 3 more planes instead of 2 more. [The relationship between the number of Paul's planes and the number of Bob's planes would have been spoiled.]

Then you may continue somewhat in the following manner.

So far in this lesson you have thought of Paul's planes first when making the comparisons. Since Paul had more planes than Bob, the numeral for the larger number was placed above the line in the ratio. But there is another way to compare the boys' planes, and that is to think of Bob's planes first. Then, to write a ratio comparing the planes shown in Picture F, the 12 will be written above the line and the 18 below the line. In other words, you think of Bob's 12 planes first and then Paul's 18. You would say, "Bob has 12 planes to Paul's 18," and you would read the ratio as "12 to 18."

When we look at only one *group of planes and think of Bob's planes first, we describe the comparison by writing the 2 over the 3 in the ratio and read it "2 to 3." Since this is the comparison based on the smallest possible number of planes in each group, this group is repeated for other comparisons we may make. Thus, if we take one group and double it, we have 2 groups, and the comparison is between 4 of Bob's planes and 6 of Paul's. The ratio is written 4/6 and read "4 to 6."*

Discuss Exercises D and E, and have the children take turns supplying numerals for the screens.

Page 122: Ask the children to study the pictures and then tell whether the ratios that can be made for the three pictures are the same or not. Note that Exercises A to E all apply to Picture A and that they outline steps that can be adapted when discussing Pictures B and C. In each case, you will want to compare first the objects in the top row with the objects in the bottom row and write all the ratios possible for that comparison, and then reverse the process so that the children think first of the objects in the bottom row and compare them with the objects in the top row. Let the children take turns discussing Exercises A, B, and C and supplying numerals for the screens. Then give them time to think on their own about Exercises D and E before discussing them.

Exercises F and G apply to Picture B, and you may discuss the picture in terms of these two questions only, or you may apply the steps outlined for Exercises A to E also.

You will probably want the children to do Exercises H and I independently. They should first study Picture C and then relate the ratios to the groups in the picture. The use of pieces of construction paper to represent the groups in Picture C will aid the pupils, especially the slow learners, to understand which ratios to choose.

Page 123: Before having the children write out the exercises in Block 1, give them time to study the six pictures at the top of the page. Take one picture at a time and discuss with the children the different ratios that can be written for it. This will help them find the groups for the ratios in Block 1.

When the children are familiar with the pictures, read the text for Block 1 and make sure all the children understand the example that is given. Then have them write on paper each ratio and the letter for the picture that shows the relationship the ratio

expresses. Verify the answers and, if necessary, give further explanation to help the children understand how the pictures illustrate the ratios.

For Block 2, have the slow learners make circles, X's, etc., to illustrate each statement and then write out the one ratio that expresses the comparison by using only the information in the statement. The able pupils will be able to write two ratios expressing the comparison expressed in Statements C to L, thinking first of the objects mentioned first as being compared to the objects mentioned second. The other ratio is based on taking the objects in the reverse order. The able pupils may also be encouraged to use pieces of construction paper to illustrate the two groups of objects in each statement and then show how these objects can be separated into smaller equal groups to illustrate other ratios that can be written to express the result of the comparison.

Providing for the able pupil

Activity 33, described on page 374, can be adapted to the material in this lesson by having the pictures on the cards show comparisons between two or more groups of two kinds of objects.

The able pupils can be encouraged to make up statements expressing comparisons similar to the statements in Block 2 on page 123. Have several of the children write their statements on 3″ x 5″ cards, five on a card. These can later be used by the slow learners for extra practice in writing ratios. You can write a number or letter the cards for easy identification.

Helping the slow learner

Duplicate a sheet containing several pictures of objects separated into equal groups, similar to the pictures on page 123. Indicate under each picture the number of ratios the pupils are to write for the picture (2 ratios, 3 ratios, etc.). Label each picture with a letter. Have the children write on paper the letter for each picture and then opposite the labelling letter write the ratios for the picture.

For further practice you may wish to have these children use the 3″ x 5″ cards containing statements of comparison made by the abler pupils. Tell the children to draw pictures to illustrate the comparison statements and then write a ratio for each statement. After the extra practice, perhaps some of these children will be able to see that more than one ratio can be written for each statement.

Use the adaptation (see notes for the able pupil above) of Activity 33, page 374, with these children. You may wish to pair a couple of slow learners with a couple of children who are not having difficulty in writing ratios for comparisons.

124-125 Using arithmetic; Keeping skillful

Lesson Briefs for these lessons ore on poges 119-121.

Expanded Notes are not considered necessary for these lessons.

126-131 Moving forward

Lesson Briefs for this lesson ore on poges 121-125.

Overview

The work in this lesson reviews the meaning of proper fractions and the reading and writing of the numerals that symbolize such fractions. Since this is not new work at the fifth-grade level, a relatively mature approach is possible.

Pages 126 to 128 develop the idea that different fraction numerals may be used to symbolize a part of a whole. Thus the child learns that $\frac{1}{8}$, $\frac{2}{4}$, $\frac{1}{2}$, $\frac{8}{16}$, and $\frac{16}{32}$ may all be used to symbolize the same part (or "fraction") of a piece of paper.

The work on pages 129 and 130 applies this idea to a group of things. In this connection, keep in mind that when considering a part of a group, the group must be thought of as a whole. Note on pages 129 and 130 how the background over the objects helps the children to think of the green portion in each picture as a part of a whole.

Also remember that the work in this lesson does *not* include the idea of computational reduction. All that is intended is to give the children some background for understanding equivalent numerals, that is, different names for the same fraction.

Teaching the whole class

You may wish to remind the children that they have used numerals in pairs, written one over the other, before. They have learned to make ratios to express rate and also to express comparison. Now they are going to learn that a pair of numerals written in this way may be a fraction numeral that expresses a part of something. Most of the children will have had previous experience with reading and writing fraction numerals, and all of them will have had much oral experience with fraction expressions. Use such experience as background when discussing what a fraction is (a number that tells how much there is in a part of something) and what a fraction numeral is (the symbol for the fraction). Be sure to use the words *numerator* and *denominator* in your discussion so that the children will become familiar with these names for the terms of a fraction.

Page 126: Ask into how many parts the piece of paper in Picture A has been marked off and how many of these parts are red. Have someone read the

text. Then discuss with the class what the 4 and the 8 in the fraction numeral mean. Point out that the numeral representing the total number of parts (the denominator) is written below the line, and the numeral representing the number of red parts (the numerator) is written above the line. Explain that $\frac{4}{8}$ is called a "fraction numeral," and that it means we are to think of 4 of the 8 equal parts into which the whole piece of paper has been divided.

Point out that fraction numerals are read in a special way. Explain that when 4 over 8 means 4 of the 8 equal parts, the fraction numeral is read "four eighths." Also tell the children that each term of a fraction has a special name. Help them learn the words *numerator* and *denominator* by asking questions that require them to use the words when answering.

When the children study Picture B, point out that this piece of paper is the same size as that in Picture A. The red portion is also the same size as that in Picture A. Let the children use tracings to verify these facts if they wish to. Explain that because Picture B has been marked off into a different number of equal parts from Picture A, the fraction numeral that tells how much is colored red is also different. Have the children note that the denominator expresses the total number of equal parts and the numerator the number of these parts that are colored red. They should see that since both $\frac{4}{8}$ and $\frac{2}{4}$ tell how much of the paper is colored red, the two fraction numerals are different ways of expressing the same thing.

Be sure to give the children some pointers on how to read fraction numerals. Point out that the numerators are read in the usual way, but that the denominators are read using the words thirds, eighths, etc.

Page 127: For Pictures C and D and the accompanying text, adapt the teaching procedures suggested for page 126.

Page 128: For Picture E and its accompanying text, adapt the teaching procedures suggested for the preceding pages. Then let the children study Picture F. Before they read the text and answer the questions, they may be helped to summarize their learning by carrying out the following activity.

Tell the children that now they are going to fold a sheet of paper in such a way that it will show the fractional parts they have been studying. They may use notebook paper or any kind of paper available that is uniform in size and easy to fold. Demonstrate each step before the children do the folding.

Have them fold the sheets in half, so that there are two equal parts, as shown in Picture C. Have them draw a heavy line on the fold and then color (or shade or crosshatch) the bottom half. Tell the children to write, on a separate sheet of paper, the fraction numeral ($\frac{1}{2}$) that tells how much is colored.

Now ask the children to fold the sheet again so that there are four equal parts, as shown in Picture B. Let them observe that the sheet is now folded into four equal parts and that the colored portion, which is the same size as before, has two of these parts. Have the children write the fraction numeral ($\frac{2}{4}$) that can now be used to tell how much is colored.

Have the children fold the sheet three more times. Each time they make a fold, have them unfold the sheet, observe that the colored portion has more parts, and write the fraction numeral that can now be used to tell how much is colored. Point out that each time the sheet is folded into smaller parts, a new fraction numeral can be used to express the number of parts in the colored portion. Emphasize that the various fraction numerals are merely different ways of telling how much of the sheet is colored.

Now have the children take turns reading the text and answering the questions for Picture F. Ask them to relate each of the five fraction numerals at the bottom of the page to the picture with which it belongs. Make sure the children understand that a fraction is a number that stands for a part of a whole and that a fraction numeral is the written symbol for it.

Page 129: When teaching the work on this page (and also on page 130), keep the pupils' attention centered as much as possible on the whole group of objects. The shaded background in the pictures on these pages is provided to help the pupils think first of one big group of pinwheels. The rings separate the pinwheels into equal groups, and it is the number of these groups (*not* the number of objects in a group) that is important at the moment. The idea that $\frac{4}{6}$, $\frac{2}{3}$, and $\frac{8}{12}$ can all be used to express the same portion of the total group of pinwheels can be developed without reference either to the total number of objects or to the number in each of the equal groups.

Have the children read the first paragraph of the text for Picture G. Point out how the term *fractional part* is used to describe a part of the total quantity. Ask someone to tell how many groups there are in the total quantity. Ask how many are green.

Have one of the children write on the chalkboard the fraction numeral that shows what fractional part of the groups is green. Then call on someone to read the rest of the text and relate each term of the fraction to the picture. Emphasize use of the terms *numerator* and *denominator*.

Tell the children to examine Picture H and determine how many groups there are altogether and how

many groups are green. Ask them to explain how they can tell that the groups in green are the same fractional part of Picture H as of Picture G. Call on a child to read the text accompanying the picture and to relate the terms of this fraction to the picture. After the children have read the fraction numeral, let one or more of them supply the numerals that belong where the screens are at the bottom of the page.

Page 130: Adapt the procedures suggested for page 129 to Picture I and its accompanying text.

The rest of the material on the page shows Pictures G, H, and I again and summarizes what has just been learned about fractional parts of a group. Have the children study the pictures and verify that the same fractional part of the pinwheels is green in each. Then have them read the text and answer the questions. Make sure the children can relate the three fractions at the bottom of the page to the correct picture.

Page 131: Discuss each of the first six exercises (A through F) before the children write the fraction numerals required. Be prepared for variations in judgment, but require the pupils to justify their viewpoints.

Providing for the able pupil

The able pupils may be encouraged to make pictures (geometric shapes, simple objects, etc.) that will illustrate various fractions. They may use the pictures on pages 126 to 131 of the pupils' book as suggestions, but encourage them to use their ingenuity in selecting fractions for illustration. Each drawing should be labeled to show the fraction represented. Display the finished drawings.

These children might also draw diagrams on 3″ x 5″ cards and shade a fractional part of each diagram. The slow learners could use these cards for identifying the fraction illustrated and then writing the fraction numeral.

Some of these children might like to pick fractions at random between $\frac{1}{2}$ and $\frac{1}{100}$ and then write the fraction name in words opposite each fraction numeral. Again, this work would make a good bulletin board display.

The flannel board described in Activity 34, page 375, may be used to provide additional work in recognizing fraction numeral equivalents.

Helping the slow learner

Give the slow learners many opportunities to write fraction numerals associated with pictures. For this purpose you may provide worksheets containing pictures divided into equal parts, with a portion shaded. Keep these pictures simple, much like those on pages 126-131.

If the able pupils have made the 3″ x 5″ cards to illustrate various fractions, let the slow learners use these cards for practice.

Activities with the flannel board involving fractions (see Activity 34, page 375) may be adapted for use with slow learners.

132-133 Thinking straight

Lesson Briefs for this lesson are on pages 125-127.

Expanded Notes are not considered necessary for this lesson, except for suggestions for the able pupil and the slow learner.

Providing for the able pupil

At the conclusion of the work on page 133, the able pupils might enjoy the following activity. Put pairs of numerals on the chalkboard ($\frac{3}{4}$ or $\frac{5}{8}$ or $\frac{6}{10}$, etc.) and ask the pupils to describe or show with objects three different situations (fraction, comparison, rate) that might be expressed by each pair of numerals. Write the numerals in each pair one above the other with a horizontal line between them. The slower children may listen to the descriptions and watch the demonstrations. At the same time, they will be developing the ability to discriminate between fraction situations and rate or comparison situations. Permit discussion about the pupils' reasons for thinking of a situation as involving a fraction, a rate, or a comparison.

Helping the slow learner

Try to discover what special difficulties these pupils have. Then repeat any or all of the activities suggested for slow learners in connection with the work on pages 112-117, 119-123, and 126-131 of the pupils' book.

134-135 Using arithmetic; Keeping skillful

Lesson Briefs for these lessons are on pages 127-129.

Expanded Notes are not considered necessary for these lessons.

136-137 Thinking straight

Lesson Briefs for this lesson are on pages 129-131.

Expanded Notes are not considered necessary for this lesson, except for suggestions for the slow and able pupils.

Providing for the able pupil

These children will be challenged by an activity in which they must give examples of numbers that are

divisible by 2, 3, 4, or 5. To get the activity started, have the children number off. Those with odd numbers are on one team, and those with even numbers are on the other. The leader of one team begins by asking the members of the other team to write, for example, a number expressed by a three-figure numeral that can be divided by 3. When each child has written a numeral, the leader calls on a child to read the numeral he has written. If his numeral is correct, it is his turn to ask the members of the first team to write, for example, a four-figure number that can be divided by 4. He then calls on a child to read his numeral. When a player misses, he gives a member on the opposing team a chance to answer. No dividend requested should exceed five places. If you wish, you may impose a time limit for giving answers. Adjust the time limit to suit the ability of the pupils.

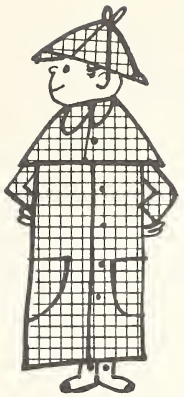
The able pupils can also benefit from an introduction to the concept of prime numbers. A prime number is a number that is divisible only by itself and 1. You might ask the children to test numbers by this definition, and help them to see that the numbers 1, 2, and 3 are prime numbers. Then ask someone to explain why no even number, except 2, is a prime number. The 26 prime numbers between 1 and 100 are listed below. Have the pupils find as many of these numbers as they can. After the pupils have made their lists, write the 26 numbers on the board and let each pupil see if he included some numbers that are not prime numbers and note any omissions. If a non-prime number has been included, ask the pupils to try to find what numbers the number can be divided by.

(Prime numbers between 1 and 100: 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97)

If any of the children want to extend their list of prime numbers beyond 100, let them do so. When a pupil suggests that a number is a prime number, let the class test the number to see if it has any divisors. Of course, they cannot prove that it is a prime number, but they may be able to prove that it is not a prime number by finding a divisor.

Helping the slow learner

The first activity suggested above can be adapted for slow learners in the following manner: The leader of one team gives a member of the other team a number expressed by a numeral with two or three figures. This second pupil is to tell whether it can be divided by 2, 3, 4, or 5. He must also explain how he knows. The activity can then continue in the way described above. Dividends used should not have more than three figures.



CHARTING THE COURSE

Multiplicative situations in problems

In arithmetic, children have in the past learned to solve certain types of problem situations without really understanding them. Repeated experience with selected examples of a given type has enabled some children to solve other examples of a given type. Suppose, for example, the problem reads as follows: Mary opened some packages of doughnuts. There were 8 in each package. When she counted the doughnuts, she found she had 24. How many packages had she opened?

Under traditional teaching, the pupil is told to solve the problem by finding how many 8's are in 24. It is true that the division of 24 by 8 produces the correct answer, but many pupils will fail to understand why this is so. Moreover, the structure of this situation may be correctly viewed as being of the *multiplicative* type with the number of groups unknown. (See page 138 of the pupils' textbook.) If visualized in this way, it is even more difficult to see why the division process is used in solving problems of this type.

In *Seeing Through Arithmetic 5,* pages 138 and 139 are devoted to showing pupils how to solve problems when they are viewed as belonging to the multiplicative-division type. The aim is to help the pupil write the equation if he happens to approach the problem from the multiplicative point of view, and then to understand why division is used to solve it.

At this point the teacher should note that problems of this type may also be approached from a more general point of view. Fundamentally, multiplicative situations involve the idea of rate. In a later sequence of lessons that begins on page 215 of the pupils' textbook, pupils will be taught how to solve such situations by using the rate idea. Eventually, when the idea of rate is

seen to be a fundamental aspect of all multiplicative situations, the approach used on pages 138 and 139 may be viewed as merely a short cut. This point will be developed more completely in later sections of these notes.

138-139 Exploring problems

Lesson Briefs for this lesson are on pages 131-133.

Expanded Notes are not considered necessary for this lesson. See Charting the Course above.

CHARTING THE COURSE

Reduction of ratios and fraction numerals

Pupils were introduced on page 113 to the idea that different ratios can be used to express the same rate. By means of pictures showing the grouping of objects, they learned that the rate 12 per 15 can also be expressed as 4 per 5 and as 8 per 10. At that time no attempt was made to show that the rate could be "reduced" by dividing or multiplying the number in each of its terms by the same number. Similarly, pupils learned on later pages (see page 120) that different ratios can be used to express the same comparison. Still later (see page 126) they were taught that the same fraction can be expressed by different fraction numerals. In none of these examples were computational methods used to change from one pair of numerals to another equivalent pair.

Another way of stating the situation could be used. We could say that "$\frac{4}{5}$" and "$\frac{8}{10}$" are two different names for the same fraction. This is similar to saying that "4," "four," and "IV" are different names (or symbols) for the same number.

This form of expression, although probably not appropriate for pupils at this stage, is mathematically sound and emphasizes the distinction between the concept and different symbols, or "names," for it.

Computational methods of replacing ratios by equivalent ratios are taught on pages 140-142. Similarly, computational methods of replacing fraction numerals by equivalent fraction numerals are taught on pages 143-146. On these pages (140-146) the pupil is shown how he can find an equivalent number pair by dividing both terms of a given pair by the same number. For example, the pair 6/10 is "reduced" to 3/5 by dividing both 6 and 10 by 2. Also, the pair 6/10 can be replaced by the equivalent pair 12/20, which is found by multiplying both 6 and 10 by 2. This change is also usually referred to as a "reduction." Since equivalent number pairs are different symbols for the same relationship or fraction, they may be used to form equations such as $6/10=3/5$.

Often a particular ratio is given, and at the same time one of the terms of an equivalent ratio is specified. The problem of finding the other term of the equivalent ratio then arises. The same situation occurs, of course, with fraction numerals. *In Seeing Through Arithmetic 5*, pupils are shown how to find the unknown term of a number pair in such situations. The importance of the ability to do this can hardly be overestimated, since it enters fundamentally into the solution of many types of problems. To cite but one example, it is used in replacing fraction numerals by others which have a common denominator before the fractions are added or subtracted.

140-142 Moving forward

Lesson Briefs for this lesson are on pages 133-135.

Teaching the whole class

Before beginning work on this lesson, you may wish to remind the children that they know that more than one ratio can be used to express the same rate or comparison. Groups of objects may be arranged as shown on pages 112-117 and 119-123 in the pupils' book, and you or the pupils can demonstrate, for example, that 4/6, 8/12, and 2/3, while different ratios, all express the same rate (or comparison).

You may also tell the pupils that now they will learn how to compute to replace a ratio by another equal ratio. Continue to emphasize that the rate or comparison remains the same.

This computational procedure is known as "reducing" a ratio, and the word *reduce* is used when a given ratio is replaced by an equal ratio with either higher or lower terms. That is, the ratio 4/6 is said to be "reduced" when it is replaced by either 2/3 or

12/18. However, do not expect the children either to understand or to use "reduce" in this sense.

As the lesson proceeds, some of the children will realize that the computational procedures they are learning for reduction of ratios will apply also to the reduction of fraction numerals. Do not proceed with reduction of fraction numerals at this point, however.

Keep the pupils aware of the fact that when a ratio is reduced, it still expresses the same rate or comparison. Stress, too, that when a ratio is reduced, *both* terms must be multiplied or divided by the same number. The use of objects will help the children see that related groups (whether in rate or comparison situations) must be operated upon in the same way. Thus 4 pencils at a cost of 10¢ also can be expressed as 2 pencils at a cost of 5¢ or 20 pencils at a cost of 50¢.

Page 140: Have the children read the statement beside the heading "Moving forward." Tell them that they are going to study ratios that express rates or comparisons. Remind them that the same rate or comparison can be expressed by many ratios.

Have the children read the text for Picture A and examine the picture. Then have them interpret the ratio in terms of the picture.

Now go on to the first part of the text for Picture B. Make sure the children note the two equal groups and understand why one group is dimmed off. Emphasize that 3/5 expresses the rate of "cards per pennies" in terms of only one of the equal groups. With the help of the second part of the text, get the children to see that because the postcards and pennies have been divided into two equal groups, the corresponding computational procedure is to divide the 6 and 10 by 2. Then we have a new pair of numbers that expresses the same rate.

Going on to Picture C, the children should see right away that they are now concerned with two groups of 6 postcards. Since the number of postcards has been doubled, the number of pennies must also be doubled. Have the children read the last two lines of text, supplying the numerals that belong where the screens are.

Page 141: Now the rate at which Sally bought the cards shown in Picture C (page 140) is expressed by a new ratio. Remember that the rate has not changed, but different numbers are being used to express the rate. Read together the text explaining the computation. Many children will see intuitively why 2 is used as the multiplier. Others will be helped by referring to Picture C.

Discuss Exercises A to E with the class and let the pupils take turns in giving answers. Keep emphasizing the fact that the new ratios they find, whether by division or by multiplication, are all equal to the original ratio 12/8. By the time Exercise E has been completed, some of the pupils will begin to see that there is no limit to the new equal ratios that can be found by multiplication, but there is a limit to those found by division (as long as fractions are not introduced). You may need to help some of the pupils by using objects and setting up a rate or comparison situation to show that $12/8=3/2=24/16$, etc.

The children should work independently on Exercises A to T. Give them plenty of time for this work. You may point out that since the ratios they are to write for Exercises A to H involve smaller numbers, they must divide. In each case they must choose a divisor that is common to each term of the given ratio because each term must be divided by the same number. Do not insist that they use the largest common divisor, however.

For Exercises I to T, point out that the new ratios they are to write involve larger numbers. Consequently, they must multiply each term of the given ratio by the same number. This is a good time to ask, "Why can we find as many equal ratios as we want to replace these ratios?"

The slower children may need to use objects. Allow them to do so if it seems necessary.

It is important to discuss each exercise with the class. Let the children explain and justify the ratios they have written. Some of them may wish to use objects when explaining their thinking.

Have the children read the first six lines of Exercise A. Explain that they are to find the numeral that belongs where the screen is in the ratio equal to 9/6. (Note that this missing numeral must bear the same relation to 6 that 18 does to 9.) Ask whether 9 is multiplied or divided to get 18. Ask by what number it is multiplied or divided. When the children see that 9 can be multiplied by 2 to get 18, they will realize from their previous experience that they must multiply 6 by 2.

Page 142: Work through Exercises B, C, and D with the class. In Exercise B, point out that first it is necessary to find out what number can be multiplied by 4 to get 16. When this number has been established as 4, ask someone to explain what must be done to the 3 in 3/4 (multiply it by 4).

In Exercises C and D, note that first it is necessary to find out by what number the 10 in 10/15 and the 56 in 35/56 have been divided. Not until this has been done can the same operation be performed on the 15 and the 35. Be sure the children realize that when one of the numbers represented in a ratio has

been divided, the other number in the ratio must also be divided by the same divisor.

Exercises A to R should be used for independent written work. Supply answers and let the pupils verify their work. If necessary, use objects to clarify any exercises that seem especially difficult.

The exercises in the "Keeping skillful" section should be assigned as written work as a separate lesson.

Providing for the able pupil

Prepare mimeographed sheets on which you have written the ratios that are written lightly in the sample shown below. Note that the terms of the first

$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12} = \frac{7}{14} = \frac{8}{16} = \frac{9}{18} = \frac{10}{20}$

$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15} = \frac{12}{18} = \frac{14}{21} = \frac{16}{24} = \frac{18}{27} = \frac{20}{30}$

$\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = \frac{15}{20} = \frac{18}{24} = \frac{21}{28} = \frac{24}{32} = \frac{27}{36} = \frac{30}{40}$

$\frac{4}{5} = \frac{8}{10} = \frac{12}{15} = \frac{16}{20} = \frac{20}{25} = \frac{24}{30} = \frac{28}{35} = \frac{32}{40} = \frac{36}{45} = \frac{40}{50}$

$\frac{5}{6} = \frac{10}{12} = \frac{15}{18} = \frac{20}{24} = \frac{25}{30} = \frac{30}{36} = \frac{35}{42} = \frac{40}{48} = \frac{45}{54} = \frac{50}{60}$

$\frac{6}{7} = \frac{12}{14} = \frac{18}{21} = \frac{24}{28} = \frac{30}{35} = \frac{36}{42} = \frac{42}{49} = \frac{48}{56} = \frac{54}{63} = \frac{60}{70}$

$\frac{7}{8} = \frac{14}{16} = \frac{21}{24} = \frac{28}{32} = \frac{35}{40} = \frac{42}{48} = \frac{49}{56} = \frac{56}{64} = \frac{63}{72} = \frac{70}{80}$

$\frac{8}{9} = \frac{16}{18} = \frac{24}{27} = \frac{32}{36} = \frac{40}{45} = \frac{48}{54} = \frac{56}{63} = \frac{64}{72} = \frac{72}{81} = \frac{80}{90}$

$\frac{9}{10} = \frac{18}{20} = \frac{27}{30} = \frac{36}{40} = \frac{45}{50} = \frac{54}{60} = \frac{63}{70} = \frac{72}{80} = \frac{81}{90} = \frac{90}{100}$

ratio in each row differ by 1. Note also that the ratios in each row are equal and that the multipliers used are 2, 3, 4, successively, up through 10. Each series can be carried as far as desired, of course.

Give one sheet to each pupil and tell the children to discover the system used for finding each row of equal ratios and then to write the ratios that will complete each row. The sample shows a completed sheet.

The able pupils will probably make various observations about their completed arrangements. For example, while the terms of the first ratio in each row differ by 1, the terms of each second ratio differ by 2, those of each third ratio differ by 3, and so on.

"Spin-a-rate," Activity 32, on page 373, is a game devised to give children practice in recognizing among various ratios those which express a specific rate or comparison. You may want to make up more than one game by using rates or comparisons other than the ones suggested in the activity.

Helping the slow learner

The slow learners can also use the mimeographed sheet described above for able pupils, but it should be modified somewhat. Multipliers used probably should not exceed 5, and it might be wise to limit the work involved by stopping with the ratio 4/5.

When the slow learners begin to handle ratios more easily, let them join the whole class in a game of "Spin-a-rate," Activity 32, page 373.

143-146 Moving forward

Lesson Briefs for this lesson are on pages 136-138.

Teaching the whole class

Before you have the children open their books, draw on the chalkboard three circles and shade within each a portion equal to what has been shaded in the drawings on page 143. Cover or erase all dividing lines.

Begin the lesson by asking the children to decide whether the shaded portion within one circle is more than or less than the shaded portion within the other two. Ask them if they can tell by looking at the drawings just how much of each drawing is shaded. Let them conclude that it is difficult to decide on a number to describe the shaded portion. They know only that "this much" is shaded and that in every case the "this much" that is shaded looks the same size. They may note that, if all three drawings could be laid one on top of the other, the shaded portions would coincide.

You might proceed somewhat in this manner:

Now let us see how we can tell how much of each drawing is shaded. First of all we would say that "a fraction of the drawing" is shaded, and we think that for all three drawings the fraction that tells how much is the same. We want to know what this fraction is.

If you remember, when we worked with ratios, we learned that more than one ratio can be used to express a rate or a comparison. We will soon see that we can call the fraction that tells how much of each drawing is shaded by more than one name. For example, if we divide the area within a circle into 3 equal parts and we find that 2 of the 3 equal parts are shaded, we call the fraction "⅔." [Show this by a drawing on the board.] Or, if we divide the area within the circle into 6 equal parts and see that 4 of the 6 equal parts are shaded, we can call the fraction "⅙." [Show this by a drawing.] Or, we may also divide the area within the circle into 12 equal parts and this time see that 8 of the 12 equal parts are shaded, and we say that 8/12 is shaded. Thus, there are at least three different pairs

of numbers that we can use to tell the part that is shaded, or to name the fraction.

Page 143: Let the children read the text and count the equal parts in Picture A. Call on someone to answer the question and then ask the children if they think Picture A is like any of the drawings you have made on the board. If you divided the first drawing into thirds, the second into sixths, and the third into twelfths, Picture A and the second drawing on the board should be alike.

Then have the children study Picture B to determine if it shows more than one way to express the shaded part. Point out the heavy lines that divide the paper into thirds and the lighter lines that divide it into sixths. Point out that $\frac{2}{3}$ and $\frac{4}{6}$ both express the fraction that tells how much is shaded. Go on to Picture C and help the children see that $\frac{2}{3}$, $\frac{4}{6}$, and $\frac{8}{12}$ all tell how much of the paper is green. Emphasize tnat the colored part of the paper is the same in each case, and so one fraction tells how much is green. Since all three fraction numerals refer to the same fraction of the paper, they are equal. Emphasize the idea that they are different *numerals* that name the same *fraction*. The children are dealing with the same fraction in all three pictures, but they are not dealing with the same fraction numerals.

Once you have established the idea that the fraction numerals are different and the fraction is the same, you are ready to show the children the computational method of producing equal fraction numerals, which are different names for the same fraction.

As an aid in explaining the computational procedure, you can duplicate ahead of time for each child in your class three disks drawn on one sheet of transparent paper—one divided into sixths, one into thirds, and one into twelfths. Make the disks the same size so they can be laid on top of one another. Have the children first work with the disk divided into thirds and color $\frac{2}{3}$ of it yellow. Then tell them to color green $\frac{4}{6}$ of the disk divided into sixths, and color red $\frac{8}{12}$ of the other disk. Then ask them to cut out the disks.

Ask the children to lay the disk divided into thirds in front of them on their desks. Discuss the number of equal parts in the disk and also the fraction that tells how much is shaded. Write the fraction numeral $\frac{2}{3}$ on the board. Now have the children lay the disk divided into sixths on top of the disk divided into thirds. Let them see that the fraction remains the same, but that now the equal parts are smaller and there are more of them. After they tell you that in this case the numeral that expresses the fraction is $\frac{4}{6}$, write the numeral on the board opposite the $\frac{2}{3}$. Discuss what can be done to the 2 and the 3 in $\frac{2}{3}$ to change $\frac{2}{3}$ to $\frac{4}{6}$. After you point out that where there was one part before there now are two parts, someone will see that 2 is the multiplier used in changing the terms of the fraction. Thus, by multiplying both the numerator 2 and the denominator 3 by 2, the fraction numeral $\frac{4}{6}$ is found as another name.

Next, have the children place the disk divided into sixths on their desks and place on top of it the disk divided into twelfths. Again they will see that for each sixth there are now two smaller equal parts. Hence, if both the numerator and denominator of $\frac{4}{6}$ are multiplied by 2, the new fraction numeral is $\frac{8}{12}$.

You may want to go a step farther and show the children that $\frac{2}{3}$ and $\frac{8}{12}$ are equal. Have them place the disk divided into twelfths on top of the disk divided into thirds. This time there are 4 smaller equal parts for each third; and, therefore, the multiplier is 4.

So far you have shown the children how $\frac{2}{3}$ equals $\frac{4}{6}$ and $\frac{8}{12}$, and how $\frac{2}{3}$ equals $\frac{8}{12}$. You have shown them how reduction works when you begin with a few large equal parts and change them to a greater number of smaller equal parts. Since the result is a larger number of equal parts than there were to begin with, the process of multiplication is used to reduce the fraction numerals.

Now you will want to show them how to start with $\frac{8}{12}$ and obtain $\frac{4}{6}$ and $\frac{2}{3}$. In this case you begin with small equal parts and group them into larger equal parts. Since the result is fewer equal parts than there were to begin with, the process of division is used to reduce the fraction numerals.

The children will see this better if you have them place the disk divided into sixths on the desk in front of them and then on top of it put the disk divided into thirds. Explain that to change the greater number of smaller equal parts to a smaller number of larger equal parts, they combine groups of one sixths. Every time they imagine two of the one sixths put together, they form one third. Thus to reduce the fraction numeral $\frac{4}{6}$ to $\frac{2}{3}$, the numbers represented by both the numerator and the denominator of $\frac{4}{6}$ must be divided by 2.

In like manner, you can show how $\frac{8}{12}$ can be reduced to $\frac{4}{6}$ or $\frac{2}{3}$. To get $\frac{4}{6}$, the twelfths must be combined in groups of two to get a sixth. So both the numerator 8 and the denominator 12 are divided by 2. To change $\frac{8}{12}$ to $\frac{2}{3}$, the twelfths are combined in groups of four to get thirds. So the numerator 8 and the denominator 12 are divided by 4.

The children might summarize the above explanation by writing each fraction numeral on the board and

then showing by which process (multiplication or division) it is changed to each of the other fraction numerals. The summary might look like this:

$\frac{2}{3} = \frac{4}{6}$ by multiplying $\frac{2 \times 2}{2 \times 3} = \frac{4}{6}$

$\frac{2}{3} = \frac{8}{12}$ by multiplying $\frac{4 \times 2}{4 \times 3} = \frac{8}{12}$

$\frac{4}{6} = \frac{8}{12}$ by multiplying $\frac{2 \times 4}{2 \times 6} = \frac{8}{12}$

$\frac{4}{6} = \frac{2}{3}$ by dividing $\frac{4 \div 2}{6 \div 2} = \frac{2}{3}$

$\frac{8}{12} = \frac{4}{6}$ by dividing $\frac{8 \div 2}{12 \div 2} = \frac{4}{6}$

$\frac{8}{12} = \frac{2}{3}$ by dividing $\frac{8 \div 4}{12 \div 4} = \frac{2}{3}$

Page 144: Reduction is here shown with two different fractions and a new kind of diagram. The procedures suggested for page 143 may be adapted for this page.

Page 145: To prepare the children for the work on this page, discuss with them the idea that each fraction numeral has many other fraction numerals equal to it. Explain that when both the numerator and the denominator are divided by the same number, the terms of a new fraction numeral are produced. Thus, if both 12 and 16 in the fraction numeral $\frac{12}{16}$ are divided by 2, a fraction numeral equal to $\frac{12}{16}$ will be produced. The number 4 may also be used as a divisor to get another fraction numeral. Show the children that many other equivalents may be found by multiplying the terms of a fraction numeral by any number from 2 up. The numerator and denominator of $\frac{12}{16}$ may be multiplied by 2, or 3, or 4, or 5, or 6, etc.

Point out that in Exercises A to P, at the bottom of Column 1 on page 145, the computational method of reducing fraction numerals by division is to be used, and in Exercises A to L (at the top of Column 2) fraction numerals are to be reduced by multiplication. Let the children work independently on these exercises, but go over their work with them and discuss it. If a child has difficulty in understanding how to correct an error, illustrate your explanation with diagrams cut from transparent paper, as suggested in the notes for page 143.

In Exercises A to D in the second column, part of the equal fraction numeral is shown. Point out that the pupils must determine what process (multiplication or division) can be used to get the missing numeral.

Page 146: Have the children work independently on Exercises A to R at the top of the page.

In the section "Thinking straight," make sure the children understand what is meant by expressing a fraction in its simplest form. When they begin to compute with fractions, the pupils will often be told to give answers involving fractions in simplest form. This means that they are to reduce a fraction numeral so that it expresses the fraction by the smallest possible numbers.

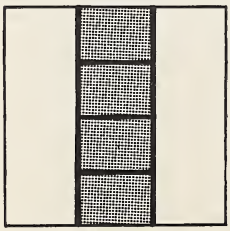
Providing for the able pupil

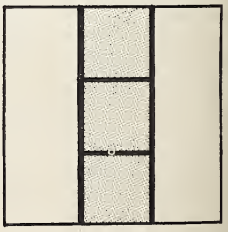
Experience in computing to find new fraction numerals may be given by choosing ten or twelve fraction numerals that are not in simplest form. Then direct the children to find the ones that are equal to others. Finally, direct the pupils to write for each of the original fraction numerals two equal fraction numerals. One equal fraction numeral must involve larger numbers than the original one, and one must involve smaller numbers.

These pupils will also profit by playing "Fraction row," described in Activity 35, page 376. This game will help them to recognize equivalent fractions quickly and correctly.

Helping the slow learner

Supply a set of "fraction papers" as follows. Use five pieces of transparent paper (about 8″ x 8″) marked to show fourths. One of the four sections in all but one paper is then subdivided so that a different fraction of the whole paper is illustrated. For example, one paper will show eighths of the whole paper in one of the quarter sections, one will show twelfths, one will show sixteenths, and one will show twentieths. The remaining paper should show only the original four sections. Use 5 transparent papers to show halves in the same manner. Divide all 5 into halves, and then subdivide one of the halves on each of the 4 separate papers as follows: one to show fourths, one to show sixths, one to show eighths, and one to show tenths. Use five papers for thirds. Divide each paper into thirds, and then subdivide one of the thirds on each





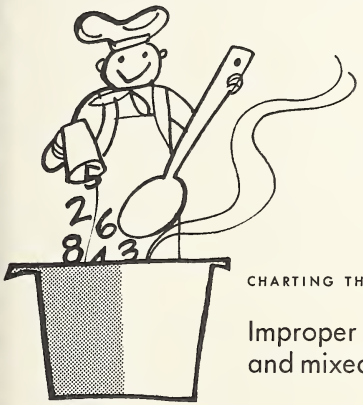
of 4 separate papers as follows: one to show sixths, one to show ninths, one to show twelfths, and one to show fifteenths. Color or shade the subdivided section of each paper.

Pupils can use these papers to experiment with equal fraction numerals. Show them how to impose one paper over the other and get them to write or say such statements as "$\frac{1}{4}$ equals $\frac{3}{8}$," "$\frac{1}{4}$ equals $\frac{4}{16}$," and "$\frac{3}{8}$ equals $\frac{6}{16}$." These statements would be appropriate when papers showing one fourth with eighths in a subdivided section are put over papers showing one fourth with sixteenths in a subdivided section.

147 Thinking straight

Lesson Briefs for this lesson are on pages 139-140.

Expanded Notes are not considered necessary for this lesson.



CHARTING THE COURSE

Improper fractions and mixed numbers

A fraction, in its primitive or basic meaning, represents one or more of the equal parts of one thing. From this it appears that the interpretation of the number pair $\frac{7}{5}$ as "7 of 5 equal parts of 1 thing" is meaningless, and $\frac{7}{5}$ cannot represent a "proper fraction." Moreover, fractions differ from natural or "whole" numbers in an important way that must always be kept in mind. The same fraction may represent quantities that vary in size. For example, two fifths of one pie may be much less pie than two fifths of a different and larger pie. In contrast, the number 5 represents the same "numerousness," regardless of the size, shape, or other physical properties of the objects. The same fraction—two fifths, for example—represents the same quantity in two or more examples only when whatever 1 represents is the same for all. Informally, we may say that two fifths of one pie is equal to two fifths of another only when both pies are the same size.

Suppose, now, we do have two or more quantities that are the same or "equal" in some respect. If all of them are partitioned into the same number of equal parts, all of these parts will also be equal. For example, suppose two circular disks that have equal diameters are both partitioned into 5 equal parts. In this case the number pair $\frac{7}{5}$ can be used to represent 7 of these equal parts, and by an extension of the primitive meaning of fractions, $\frac{7}{5}$ may be regarded as a fraction numeral that represents a so-called "improper fraction."

In every case, the quantity represented by an improper fraction can also be represented by the sum of a natural number and a fraction. For example, $\frac{7}{5}=1+\frac{2}{5}$. Usually $1+\frac{2}{5}$ is abbreviated to $1\frac{2}{5}$, and numerals such as $1\frac{2}{5}$ represent "mixed numbers."

In *Seeing Through Arithmetic 5* these ideas are introduced on pages 148-150 by methods based on visual perception of concrete situations. Similar methods are used on pages 151-153 to teach computational procedures for replacing an improper fraction numeral by an integer plus a proper fraction—that is, by a "mixed number." The concrete representations are designed to help the pupil understand that although the quantity which is being considered does not change, the symbols used to represent it may take several different forms. In other words, pupils should understand that equal fraction numerals are really different names for the same fraction.

148-150 Moving forward

Lesson Briefs for this lesson are on pages 140-142.

Overview

This lesson reviews what the child has learned about improper fractions and mixed numbers. He learns that both an improper fraction and the equivalent mixed number can represent the same quantity. The terms *proper fraction*, *improper fraction*, and *mixed number* are introduced.

Teaching the whole class

To give the children many opportunities to *see* what mixed numbers and improper fractions represent, a flannel board can be used to demonstrate the meaning of mixed numbers and improper fractions. Activity 34, page 375, tells how to make a flannel board.

If you prefer having all the children (or a group, such as the slow learners) work at the same time at

their seats, disks of oaktag or cardboard cut into fractional pieces can be used. The children can manipulate these as you direct, or they may use them to make their own discoveries. The able pupils can certainly enjoy discoveries made on their own.

Page 148: The first fraction considered on this page is a proper fraction. The term *proper fraction* is introduced and defined. To review what the children know about a proper fraction, call attention to the disk in Picture A. Ask questions that lead the children to see that the disk is partitioned into fifths and that two of these fifths are colored red. Then discuss the questions in the text at the right. Bring out that in the fraction numeral, the 5 that is the denominator shows that the disk is divided into 5 equal parts, the 2 that is the numerator shows that two of the equal parts are being considered. The fraction numeral itself is read *two fifths*. Also call attention to the way $\frac{2}{5}$ is written in words.

Have the children explain how they know that $\frac{2}{5}$ is less than 1 by referring to the disk in Picture A. Point out that any fraction like $\frac{2}{5}$, that stands for less than a whole thing or less than 1, is a *proper fraction*. Let the children give other proper fractions. If you have a flannel board (see Activity 34, page 375), you might let them put fractional parts of disks on it to represent the proper fractions they suggest. Confine the fractional parts to those represented by such commonly used fractions as halves, thirds, fourths, fifths, sixths, eighths, etc.

Next have the children read the first three lines of text opposite Picture B and refer to the picture. Then go on to a consideration of the fraction $\frac{7}{5}$. Be sure the children understand that the denominator, 5, shows that each disk was cut into fifths, and that the numerator, 7, shows how many of the fifths we are considering. Have them note how $\frac{7}{5}$ is written in words.

The two disks in Picture B should help pupils see that $\frac{7}{5}$ is more than 1. Then discuss the term *improper fraction*. Ask the children to explain why $\frac{7}{5}$ is an improper fraction and have them give examples of a few other improper fractions.

You may wish to use the flannel board to illustrate other improper fractions. To do so, you will need two disks of the same size cut into the same number of equal parts for each improper fraction you wish to show. On the flannel board put enough equal parts to make one complete disk, but keep the fractional parts separated a little (see Picture B). Then beside it put as many parts of the other disk as are needed to indicate the improper fraction you are illustrating. Let the children take turns expressing these quantities as improper fractions.

You can also let the children demonstrate improper fractions that you suggest. They can put the fraction pieces on the flannel board or use oaktag disks at their desks. To save work in preparing material, confine the latter activity to improper fractions based on fifths so that only two disks are required for each child. (If the children prepare the disks, they may use cardboard models as a guide.) You can ask the children, then, to show $\frac{6}{5}$, $\frac{7}{5}$, $\frac{8}{5}$, and $\frac{9}{5}$.

Page 149: To begin this work on mixed numbers, ask the pupils to compare the disks in Picture C with those in Picture B on page 148. The children should observe that the disks are alike, but in Picture C the fifths are not separated from one another. Tell the pupils that they are going to learn another way to think about the two disks. Ask why the top disk can be thought of as one whole. Then ask how many fifths are colored in the bottom disk.

Next turn attention to the symbolization and text at the right. Have the pupils read what is said about each of the numerals used to write the mixed number. Let them find in the picture the quantity represented by each numeral. Also have them note how the mixed number $1\frac{2}{5}$ is written in words. Then discuss the term *mixed number*. Be sure the pupils realize that a mixed number always stands for more than one whole thing and includes a part of one whole thing.

If you wish to reinforce the ideas conveyed by Pictures B and C, put one whole disk on the flannel board and beside it place two fifths of another disk of the same size to show $1\frac{2}{5}$. Then on the flannel board put five fifths of a disk of the same size and beside it place two fifths. Guide the pupils to interpret the top disks as $1\frac{2}{5}$ disks, and the bottom disks as $\frac{7}{5}$ disks. Write the numerals on the board and help the children see that $1\frac{2}{5}$ and $\frac{7}{5}$ are equal because they stand for the same quantity.

Give the pupils, especially the slower ones, many opportunities to work at the flannel board to show other mixed numbers. They will use the fractional pieces you have prepared and also whole disks of the same size. If the pupils at their desks can be supplied with one complete disk, in addition to the disk cut into fifths that they already have, they can show the mixed numbers $1\frac{1}{5}$, $1\frac{2}{5}$, $1\frac{3}{5}$, and $1\frac{4}{5}$.

Picture D on page 149 shows the pupils that a fraction like $\frac{9}{9}$ is also an improper fraction. First let the children determine the number of equal parts into which the cake has been cut. Then let them decide that each part is $\frac{1}{9}$ of the cake. Ask one pupil to count the parts by ninths. The class should see that $\frac{9}{9}$ of the cake is on the plate and understand that $\frac{9}{9}$ represents the whole or 1. Discuss such improper fractions

as ⅔, ⅝, ⅞, etc. If necessary, use the flannel board to demonstrate improper fractions equal to 1. Guide the pupils to see that in improper fractions, the numerator may be larger than the denominator or equal to it, but never smaller.

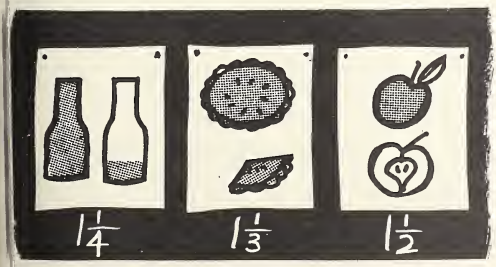
Page 150: Discuss Exercises A to X. Be sure the pupils can explain why a given numeral expresses a proper fraction, an improper fraction, or a mixed number. Get them to generalize that the numerator of a proper fraction is smaller than the denominator, and that the numerator of an improper fraction is equal to or larger than the denominator.

Exercises A to C should be written. When the children are through, let some of them put their work on the board so that the class can discuss it. The work in Exercises A to P should also be written.

The work done for Exercises A to F will help the children to summarize what they have just studied. Be sure that the children describe the fractional parts shown in green in each picture by using both an improper fraction and a mixed number. Before assigning Exercises A to F as written work, you may wish to discuss Exercise A with the class to make sure the children understand why the green portions can be expressed as ⅝ and also as 2½. Then tell the pupils to write the letters A to F on their papers and after each letter, to write a numeral for the mixed number and a numeral for the improper fraction that express the green portion of the corresponding picture. Discuss the answers after the pupils have completed their work.

Providing for the able pupil

Able children can make a wall panel of pictures that illustrate a sequence of fractions, whole numbers, and mixed numbers. These pictures should show the numbers according to size, beginning with the smallest. Each picture should be labeled to show the number it represents. A section of such a panel is shown below. At this point, perhaps the fractions should be restricted to ones that are easily identifiable as to relative sizes.



Helping the slow learner

Slow learners will benefit most from the use of visual aids. If the flannel board and fractional part cutouts are available, let these children take turns making concrete representations for the fractions and mixed numbers found in the exercises in Block 1 (A to X) on page 150. Have the fractional parts of the disks mixed up so that the child, if he cannot recognize a fractional part (⅕, for example), must first build a complete disk of 5 equal parts. Then he can proceed to remove fractional parts from the disk, include other fractional parts, or put a whole disk on the flannel board. What he does depends upon the number (proper fraction, improper fraction, or mixed number) that he is illustrating. An able pupil can supervise this work.

As another activity, divide the participating pupils into two teams. An able pupil (or you) can name a proper fraction, a mixed number, or an improper fraction and ask a member of one team to illustrate it on the flannel board. If that pupil cannot do it, after a time limit of perhaps thirty seconds, he must sit down, and a member of the other team has a chance to illustrate the fraction or mixed number. The winning team is the side with the most players left at the end of a designated time.

151-153 Moving forward

Lesson Briefs for this lesson are on pages 143-144.

Teaching the whole class

The computational procedure for changing an improper fraction to an equivalent mixed number or to a whole number is presented in this lesson.

It might be helpful for the children to use fractional parts of disks to match the action shown in the pictures on pages 151, 152, and 153. If you have prepared a flannel board and fractional pieces as suggested in the previous lesson (and in Activity 34, page 375), they can be used for this lesson as well. Or if the children have prepared fractional pieces of paper or oaktag for use at their desks (also suggested in the previous lesson), all the children can participate. They will need disks divided into fourths, fifths, and sixths.

Page 151: Before working with this page, review with the children what they have learned about improper fractions and mixed numbers. Be sure the children realize that the same quantity may be represented in both ways by equal numerals. Then tell them to read the statement after the heading "Moving forward" to find what they are going to learn in this lesson.

Tell the pupils that all the pieces of pie shown in Picture A are equal in size and originally came from a number of equal-sized pies. Ask them to read the first sentence at the right to find what fractional part of a whole pie each piece is. Then call on someone to count the fourths to verify that there are 14 pieces, or fourths. Next discuss the improper fraction numeral and the accompanying text at the right of Picture A. Emphasize the meanings of 14 and 4.

Tell the pupils that they are to find how many whole pies and extra fourths, if any, there are in 14 fourths. Point out that Picture B shows how many fourths are needed to make one pie. Put fourteen of the fourths of disks on the flannel board. Move four of these pieces together to show the children that one complete disk does result. If the children have fractional pieces for fourths at their desks, have them do the same thing. They should see that four fourths make one whole. Also direct them to the text opposite Picture B in their books. Then remind them that they have 14 fourths to be grouped into as many whole pies as possible. Tell them that by using what they know—that 4 fourths make 1 whole—they can find not only the number of whole pies, and any parts that may be left over, but also the mixed number or whole number that represents this amount.

Ask a pupil to come to the flannel board and separate the 14 fourths into groups of 4 fourths. He will find that 2 fourths are left over. Remind the class that these pieces are all fourths and that they are finding how many 4 fourths there are in 14 fourths. Then have a child at the flannel board put each of the groups of 4 fourths together to make 1 whole. The class can then see that there are 3 wholes and 2 fourths left over. Have them look at Picture C in their texts to verify this.

Next discuss with them how they compute to find the answer. Bring out that essentially they are dividing a group into equal smaller groups. Ask someone how many there are in the total group [14]. Then have him tell the number in the equal groups into which it is to be divided [4]. Finally, ask a pupil to divide 14 by 4 on the chalkboard. When he gets the answer "3 and 2 remainder," remind the pupils that the 3 means 3 wholes or ones, as Picture C and the work at the flannel board show, and that the 2 remainder represents the 2 fourths. Have pupils finish by reading this same information in the book.

Page 152: Pupils should understand that Picture D shows the same 3 pies and 2 fourths of another that were illustrated in Picture C. Have them look carefully at the third line that shows that $1\frac{1}{4}=3$ *ones* and 2 *fourths*. Let someone point out the 3 ones and 2 fourths in the picture. Then have the class look at the large numerals printed in green, as a pupil reads the accompanying text.

Ask a pupil how $\frac{2}{4}$ can be changed to simplest form. Then have someone read the last line of text. Ask the class if Picture D also shows that $\frac{1}{2}$ of a pie is left over.

Now draw attention to Picture E. Tell the class each piece of cake is one sixth of a cake. Let them count the total number of sixths. Then have someone read aloud the text at the right and supply the missing numerals.

Picture F shows the 17 sixths arranged into as many whole cakes as possible. Have someone count the number of sixths in one whole cake. On the flannel board put 17 sixths of a disk. Ask the pupils to imagine that these are pieces of cake. Call on a pupil to divide these 17 sixths into groups of 6 sixths. Then ask him to put each group of 6 sixths together to make a whole disk or "cake." Then ask him how many sixths are left over. Draw attention again to the picture of the whole cakes in the book so that the children can see that this agrees with what they found in their work on the flannel board. Let pupils take turns reading the text at the right and supplying the missing numerals. Ask a child to show the division computation on the board and explain what the resulting 2 and 5 remainder represent.

Page 153: Point out to the children that Picture G shows the same cakes as Picture F. Get a pupil to explain why the 5 remaining pieces of cake can be expressed as 5 sixths. Then call on pupils to relate the various numerals in the accompanying text to Picture G. Be sure the pupils understand that $\frac{5}{6}$ is the simplest form of the fraction numeral. Remind the pupils that they, in effect, divided the numerator, 17, by the denominator, 6.

With Picture H, ask the pupils to imagine first that the pieces of cake are unassembled, just as fractional pieces were first shown for the pies and cakes in the other situations. Have someone count the number of fifths in one cake. Then let someone count the total number to verify that there are 15 fifths in all. Then ask what improper fraction numeral expresses the total number of fifths. Ask someone to write 15 over 5 on the board and ask the class to explain why 15 is divided by 5 to find the number of wholes, or ones. Let the class divide to find the answer. Ask if the result is a mixed number or a whole number and how they know. Let them verify their answer by looking at Picture H.

Exercises A to D may be used orally. Exercises A to X should be assigned as written work. Discuss each

exercise that caused trouble. Use fractional parts of circles, if necessary, to clear up any difficulties.

Providing for the able pupil

Able pupils might like to construct a chart showing equivalent forms for improper fractions and mixed numbers, using denominators through ninths. Each child can take one or more series of improper fraction numerals with the same denominators and with numerators through ten and compute to change them to the equivalent mixed number form. Then all the numerals can be arranged on a chart for display. Such a chart will give the children an idea of the order (and relative size from smallest to largest) of a few improper fractions. (Order and relative size of mixed numbers should already be understood if the wall panel suggested in the previous lesson has been made.) At the same time the children will begin to acquire the ability to recognize certain equivalents without computing. A completed chart is shown below:

halves	$\frac{3}{2}$ $1\frac{1}{2}$	$\frac{4}{2}$ 2	$\frac{5}{2}$ $2\frac{1}{2}$	$\frac{6}{2}$ 3	$\frac{7}{2}$ $3\frac{1}{2}$	$\frac{8}{2}$ 4	$\frac{9}{2}$ $4\frac{1}{2}$	$\frac{10}{2}$ 5
thirds	$\frac{4}{3}$ $1\frac{1}{3}$	$\frac{5}{3}$ $1\frac{2}{3}$	$\frac{6}{3}$ 2	$\frac{7}{3}$ $2\frac{1}{3}$	$\frac{8}{3}$ $2\frac{2}{3}$	$\frac{9}{3}$ 3	$\frac{10}{3}$ $3\frac{1}{3}$	
fourths	$\frac{5}{4}$ $1\frac{1}{4}$	$\frac{6}{4}$ $1\frac{1}{2}$	$\frac{7}{4}$ $1\frac{3}{4}$	$\frac{8}{4}$ 2	$\frac{9}{4}$ $2\frac{1}{4}$	$\frac{10}{4}$ $2\frac{1}{2}$		
fifths	$\frac{6}{5}$ $1\frac{1}{5}$	$\frac{7}{5}$ $1\frac{2}{5}$	$\frac{8}{5}$ $1\frac{3}{5}$	$\frac{9}{5}$ $1\frac{4}{5}$	$\frac{10}{5}$ 2			
sixths	$\frac{7}{6}$ $1\frac{1}{6}$	$\frac{8}{6}$ $1\frac{1}{3}$	$\frac{9}{6}$ $1\frac{1}{2}$	$\frac{10}{6}$ $1\frac{2}{3}$				
sevenths	$\frac{8}{7}$ $1\frac{1}{7}$	$\frac{9}{7}$ $1\frac{2}{7}$	$\frac{10}{7}$ $1\frac{3}{7}$					
eighths	$\frac{9}{8}$ $1\frac{1}{8}$	$\frac{10}{8}$ $1\frac{1}{4}$						
ninths	$\frac{10}{9}$ $1\frac{1}{9}$							

If you decide to use the game "Fraction match" suggested below for the slower pupils, you should transfer the numerals from the chart above to sets of cards used in playing the game. The able pupils can help you in preparing the cards.

Helping the slow learner

Practice in changing numerals for improper fractions to the equivalent form for mixed numbers can be gained from playing the game of "Fraction match" described in Activity 36, page 376.

These children also will enjoy "Make one." See Activity 37, page 377.

154-155 Using arithmetic

Lesson Briefs for this lesson are on pages 145-146.

Expanded Notes are not considered necessary for this lesson.

155 Checking up

Lesson Briefs for this lesson are on pages 146-147.

Expanded Notes are not considered necessary for this lesson.

156 Thinking straight

Lesson Briefs for this lesson are on pages 147-148.

Expanded Notes are not considered necessary for this lesson, except for suggestions for the able pupils and slow learners.

Providing for the able pupil

The game of "Call the average" (see Activity 30, page 372) can be used here. Encourage the able pupils to use mental calculation when finding the totals before estimating averages.

Able students should also enjoy playing "Cork drop" (see Activity 11, page 361), using three-figure or four-figure numerals on the corks and keeping score by adding mentally. One player should act as a "checker" to verify the scores.

Helping the slow learner

The game of "Cork drop" (see Activity 11, page 361) may be used for the slow learners by confining the numerals on the corks to two figures and allowing plenty of time when scoring is done by mental calculation. An able pupil should act as a "checker" to verify the scores. Let the pupils take turns doing the checking.

A "mental arithmetic spelldown" can provide practice in adding and subtracting mentally. On small cards write two-figure and three-figure numerals, one to a card. Prepare three or four times as many cards as there are players. Place the cards in a box. Divide the participating members into two teams. Let the first child on Team 1 think of a number expressed

by a two-figure or three-figure numeral and tell it to the first child on Team 2. The Team 2 member draws a card from the box. He reads the numeral aloud and then gives the sum of the number told him by the child on Team 1 and the number he drew. Members of Team 1 should challenge him if they think his calculation is incorrect. If he has added incorrectly, he is out of the game. The card is now replaced in the box. The next child in line on Team 1 gives a new number to the next child in line on Team 2. If the Team 2 member computes correctly, he continues by suggesting a number to the next in line on Team 1, and so on. The team with the most players left (at the end of a specified time or when interest lags) wins. You will get the greatest amount of pupil participation if the teams consist of no more than four pupils each.

157 Thinking straight

Lesson Briefs for this lesson are on page 149.

Expanded Notes are not considered necessary for this lesson.

CHARTING THE COURSE

Common denominators

Fractions would not be useful as numbers if it were not possible to tell when one fraction is larger than another, or to add two or more fractions, or to subtract one fraction from another. One basic idea underlies the comparison, the addition, and the subtraction of fractions—namely, the use of a common denominator.

If one fraction is to be compared with another, both fractions are first expressed by fraction numerals that have the same denominator. Then the numerators may be compared, and the larger numerator belongs to the larger fraction. Similarly, if fractions are to be added, they are first expressed by fraction numerals with the same denominator. Then the numerators are added, and their sum is used as the numerator of a new fraction numeral which also has the common denominator. This new fraction numeral represents the sum of the fractions. Subtraction is, of course, also accomplished by using the common denominator idea.

Children must learn these principles of arithmetic. In *Seeing Through Arithmetic 5,* the common denominator idea is taught *before* the children are asked to add fractions, rather than as a step of the process itself. If children first gain a clear understanding of the common denominator idea, and some skill in changing fraction numerals so that they have a common denominator, then learning the process of adding or subtracting fractions is greatly simplified.

In *Seeing Through Arithmetic 5,* the pupils learn about common denominators as they learn to compare fractions—which is in itself an important ability, but is simpler than the addition of fractions. The development is organized into two stages. At first, attention is focused on comparison (pages 158-159). Fraction numerals are displayed in order on a "number line," so that it is possible to tell by observation whether one fraction is larger or smaller than another. Later (on pages 162 and 170), attention is given to comparing fractions on the basis of the numbers only.

The "number line" should be regarded as more than a convenient device for this lesson. It plays an important rôle from this point on in the study of mathematics. It is also used, for example, as a scale in various graphical representations. When extended "backward" it provides an introduction to negative numbers. This extension of the number line to negative numbers will be introduced in Book 6.

As the second stage of development, *Seeing Through Arithmetic 5* focuses attention on the common denominator idea as a simple way to determine whether one fraction is larger or smaller than another (pages 160-162). Two adjacent number lines are used to provide an introduction to the idea, but purely numerical methods are soon suggested.

The development of the common denominator idea as presented in this book should not be regarded as merely a set of lessons preliminary to the addition of fractions. It has important values in its own right in that it teaches pupils the fundamental procedure for comparing fractions and it introduces them to the number line idea.

158-159 Moving forward

Lesson Briefs for this lesson are on pages 150-151.

Overview

In this lesson children learn to compare one fraction with another by using a number line. As they study the number line, they can see that if the point assigned to one fraction is farther from the point of origin (0 on the number line) than the point for another fraction, then the first fraction is larger than the second one. They can also see that sometimes more than one fraction numeral may be used to designate the same point on the line and that these fraction numerals are equal, or that they are different names assigned to the same point.

At this stage the child's concept of a fraction is likely to be limited to something that is a part of an object. He must now learn to think of fractions as a part of the number system.

For purposes of simplification, the number lines in this lesson have been limited to the range from 0 to 1. Children who can extend the range beyond 1 should be encouraged to do so. A realization of the indefinite extensibility of a number line should be a part of the able pupil's understanding of the number system. For example, he might make a number line for the range from 0 to 2 or 3 or 4, etc., putting in the fractions between the points for the numbers. Be sure the pupils realize that these points represent mixed numbers, and that they should be read accordingly. No pupil should get the idea that a number line must take a particular position. For this reason, a horizontal line is shown on page 158 and a vertical line on page 159.

Although a number line is limitless, for practicality it is limited in length. Examples are the ruler, the yardstick, and the circular number line on the face of a clock. Since almost any kind of linear measuring instrument can be considered a number line, some children may be able to suggest additional examples, such as a thermometer, a tape measure, or the bar on a weighing machine.

A ruler is an excellent example of a number line put to practical use. As children study their rulers, you might point out that 0 belongs on the first point on the ruler. Let the children discover how many divisions there are in an inch and assign fraction numerals to these points. Then let them continue naming the points as they occur after 1: $1\frac{1}{16}$, $1\frac{2}{16}$ ($1\frac{1}{8}$), $1\frac{3}{16}$, $1\frac{4}{16}$ ($1\frac{2}{8}$, $1\frac{1}{4}$), etc.

Teaching the whole class

Page 158: Direct attention to the number line and let someone tell how many equal parts make up this line. Have someone tell what numeral marks the beginning. Ask if there is more than one name for the halfway point, and let a child tell what these names are.

Have the pupils take turns working through Exercises A to I orally. In discussing Exercises J and K, be sure that the pupils consider position on the number line in determining their answers.

Page 159: Let the class compare the number line on this page with the one on page 158, explaining in what ways they differ and in what ways they are alike. Have the pupils do Exercises L to T orally. For Exercises U, V, and W, the pupils will need a ruler and a piece of paper about 9 in. by 12 in. Be sure everyone understands that the scale on the ruler begins at 0. Now tell the children to make a line 10 in. long on their paper and divide it into 10 equal parts by marking intervals of 1 in. Then tell them to label each of these points with as many fraction numerals as they can find to represent them. Some children may need help in thinking of all of the equivalents. It would probably be wise to reserve Exercise W for the able pupils.

Providing for the able pupil

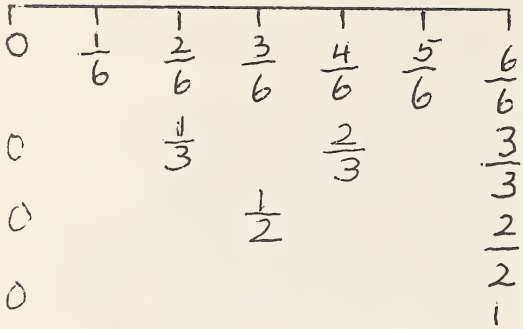
The able child should be encouraged to extend the number line beyond 1 and to make number lines using various subdivisions, such as twelfths, fourteenths, etc.

Able pupils should realize that there is no limit to the number of possible fractions, any more than there is a limit to whole numbers. To demonstrate this, they can draw a long line on the chalkboard with an arbitrary number of equidistant points. They can then double the number of points by putting points halfway between each of the points already there. For example, a line originally divided into twelfths can be subdivided into twenty-fourths, forty-eighths, etc. This dividing of the line can be continued until the pupils realize that the number of points on a number line is without limit.

Helping the slow learner

Let these children make a very simple number line, divided into no more than six equal parts to begin with. Then let them make one divided into eight equal parts. The six-part line will look like the one in the picture on page 318.

If they do not understand why one point shows both $\frac{2}{6}$ and $\frac{1}{3}$, let them make another line 6 in. long on transparent paper, divide it into 3 equal parts, and superimpose it on the line they have already made. They will then see that the $\frac{1}{3}$ mark is at exactly the same point as the $\frac{2}{6}$ mark and that both numerals represent the same length.



160-162 Moving forward

Lesson Briefs for this lesson ore on poges 152-154.

Overview

In this lesson the pupils learn to compare fractions that are expressed with different denominators. They learn that to compare fractions they must express the fractions with a common denominator. On page 160 they see how to find out how much longer $\frac{2}{3}$ of a piece of string is than $\frac{3}{5}$ of a piece of string of the same length. On page 161 they compare part of the interior of one rectangle with part of the interior of an identical rectangle. They also find common denominators by computation. Page 162 provides practice in finding the common denominator as a means of comparing fractions.

Teaching the whole class

Page 160: When using the illustrations on this page, the children should see immediately that the red and brown strings are equal in length and that all three pictures show the same pair of strings. They should first note that the brown piece of string is divided by the dots into fifths and the red string into thirds. Explain that the dotted lines crossing both strings in each picture divide both strings into the same number of equal parts. Point out that the lines divide the strings into fifths in Picture A, into tenths in Picture B, and into fifteenths in Picture C, and that the dots divide the strings into fifths and thirds in all three pictures.

Make certain the pupils understand that they want to find a fraction that will express exactly how much longer $\frac{2}{3}$ of the red string is than $\frac{3}{5}$ of the brown string.

You may begin the discussion by having someone read Exercise A aloud. In discussing Picture A; have the children note that the brown string is divided into fifths, and each fifth of the string is labeled consecutively from $\frac{1}{5}$ to $\frac{5}{5}$, starting from the bottom. Then discuss the string that is divided into thirds in a similar way.

Next have the pupils find the dot marked $\frac{3}{5}$ on the brown string and the dot marked $\frac{2}{3}$ on the red string. It should be immediately obvious to them that $\frac{3}{5}$ of the brown string is shorter than $\frac{2}{3}$ of the red string. They should also see that they cannot determine exactly how much shorter $\frac{3}{5}$ of the brown string is than $\frac{2}{3}$ of the red string.

In connection with Exercise B, help the pupils see that the dotted lines crossing both strings divide them into fifths. Point out that on the brown string the dotted lines pass through the dots on the string, since both the dots and the lines divide the string into fifths. The fact that the dotted lines do not pass through the dots marking off thirds on the red string should indicate to the pupils that no basis is shown for comparing thirds and fifths.

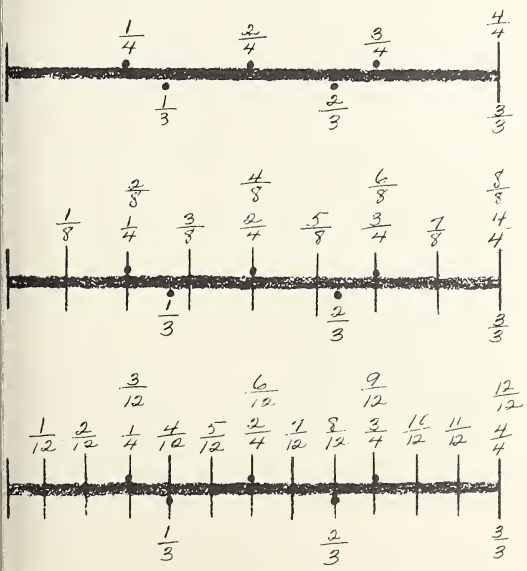
In connection with Exercise C and Picture B, have the pupils verify the statement that the dotted lines divide both strings into tenths. You might ask what fraction numeral with 10 for a denominator could also be written for $\frac{5}{5}$ and $\frac{3}{3}$ ($\frac{10}{10}$). Then have the pupils try to determine the number of tenths in $\frac{3}{5}$ and in $\frac{2}{3}$. They will find that they cannot make an accurate comparison between $\frac{3}{5}$ and $\frac{2}{3}$ in terms of tenths. They can say only that $\frac{3}{5}$ is the same as $\frac{6}{10}$ and that $\frac{2}{3}$ is almost as much as $\frac{7}{10}$.

Exercise D, relating to Picture C, will help the children see that when the same strings are divided into fifteenths, it becomes possible to determine how much shorter $\frac{3}{5}$ of one string is than $\frac{2}{3}$ of the other string. Direct attention to the fact that the dotted lines, indicating fifteenths, pass through the dots that indicate both thirds and fifths. Discuss the division of the strings into fifteenths in much the same way as you did for the division into tenths. Then have the pupils find the number of fifteenths in $\frac{3}{5}$ and in $\frac{2}{3}$. They will find that they can express $\frac{3}{5}$ as $\frac{9}{15}$ and $\frac{2}{3}$ as $\frac{10}{15}$. They can see that $\frac{9}{15}$ is $\frac{1}{15}$ less than $\frac{10}{15}$. Point out that it was not until both fractions ($\frac{3}{5}$ and $\frac{2}{3}$) were expressed as fifteenths that an accurate comparison could be made.

Page 161: The first three exercises on this page help the pupils review what they learned on the preceding page. In discussing Exercise E, ask a pupil to give the equivalent in tenths for each fifth ($\frac{1}{5}=\frac{2}{10}$, $\frac{2}{5}=\frac{4}{10}$, $\frac{3}{5}=\frac{6}{10}$, $\frac{4}{5}=\frac{8}{10}$). Similarly, for Exercise F have someone give the equivalents in fifteenths for the fifths and thirds ($\frac{1}{5}=\frac{3}{15}$, etc.; $\frac{1}{3}=\frac{5}{15}$, etc.). Discuss Exercise G carefully, since it introduces and explains the term *common denominator*. Be sure the pupils understand that "common" means "alike for all."

Then have the pupils look again at Pictures A, B, and C on page 160. Point out that the dotted lines divide the red and brown strings into a different number of equal parts in each picture. This was done to find other fraction numerals equal to $\frac{3}{5}$ and $\frac{2}{3}$ so that these fractions could be compared. Ask if there was a good reason for dividing both strings into fifths at first. The pupils should see that this represents an attempt to express both $\frac{2}{3}$ and $\frac{3}{5}$ as fifths. If $\frac{2}{3}$ could have been expressed as a number of fifths, the problem would have been solved immediately. But another denominator had to be tried, and in Picture B, tenths were tried. Get the pupils to see that tenths were tried because it is obvious that fifths can be changed to tenths; that is, $\frac{3}{5}$ is equivalent to $\frac{6}{10}$. But, again, it was found that $\frac{2}{3}$ could not be changed to tenths; none of the lines dividing the strings into tenths passes through the $\frac{2}{3}$ mark. In Picture C the strings were divided into 15 equal parts because fifths can also be changed to fifteenths; $\frac{3}{5}$ is equivalent to $\frac{9}{15}$. This time, it is seen that $\frac{2}{3}$ can also be changed to fifteenths; the line indicating $\frac{10}{15}$ coincides with the line for $\frac{2}{3}$. Point out that in finding the common denominator, a number divisible by 5 was always used until a denominator was found that was also divisible by 3. Five was used because it is the larger of the two denominators, 5 and 3.

As a class project you might wish to set up another situation like the one on page 160 in which pupils must compare fractional parts of 2 pieces of string of equal length. Instead of thirds and fifths, the comparison might be between thirds and fourths, for example. Then the problem could be to find how much longer $\frac{1}{3}$ of a piece of string is than $\frac{1}{4}$ of a piece of string of the same length. This problem could be worked at the board or individually on paper. The lines representing the strings should be of equal length, and the first pair should show one divided into fourths, the other into thirds. The illustration shows how the completed project should look and indicates that $\frac{1}{4}$ of one string is $\frac{1}{12}$ shorter than $\frac{1}{3}$ of the other.



The children should now be ready to continue with Exercise H and Pictures D through G in their books. The four rectangles on this page are to be used with Exercises H through L to help them compare $\frac{3}{4}$ of the area of one rectangle with $\frac{5}{6}$ of the area of an identical rectangle. First have someone verify that Rectangle D is divided into fourths, Rectangle E into sixths, and Rectangles F and G into twelfths. The answers for Exercise H are obvious.

In working with Exercises I and J, point out that the problem is similar to the one about the strings. Then go on somewhat like this:

We want to find a common denominator to replace the 4 of $\frac{3}{4}$ and the 6 of $\frac{5}{6}$. Do you remember that on page 160 we started with the larger of the two denominators? In comparing $\frac{3}{5}$ with $\frac{2}{3}$, we tried to use 5 as a common denominator. In comparing $\frac{3}{4}$ with $\frac{5}{6}$, what should we try first? [6] *Do you think we can express $\frac{3}{4}$ as a number of sixths? Let's see if we can. What do you do in changing a fraction numeral to an equivalent fraction numeral?* [*Multiply or divide both numbers by the same number.*] *Do you know how to multiply the denominator, 4, by any number to get 6?* [*No*] *So 6 is not a common denominator for $\frac{3}{4}$ and $\frac{5}{6}$.*

Pictures F and G show each rectangle divided into how many equal parts? [*12*] *Do you know why? Remember, on page 160, when it was found that 5 was not a common denominator, 10 was tried next because it is the next larger number divisible by 5. What is the next larger number after 6 divisible by 6? Do Pictures F and G show that $\frac{3}{4}$ and $\frac{5}{6}$ can be changed to twelfths? Count the twelfths in $\frac{3}{4}$ and in $\frac{5}{6}$. How would you change $\frac{3}{4}$ and $\frac{5}{6}$ to twelfths?*

In connection with Exercise L, the children should see that, to change $\frac{5}{6}$ to twelfths, they must multiply both the denominator, 6, and the numerator, 5, by 2. To change $\frac{3}{4}$ to twelfths, they must multiply both the denominator, 4, and the numerator, 3, by 3. After they have discovered that $\frac{5}{6}=\frac{10}{12}$ and $\frac{3}{4}=\frac{9}{12}$, they should immediately see that $\frac{5}{6}$ is $\frac{1}{12}$ more than $\frac{3}{4}$.

When discussing Exercises M, N, and O, have the class compute to see whether or not the suggested denominators of 18, 24, 36, and 48 can be used as common denominators to replace the 4 of $\frac{3}{4}$ and the 6 of $\frac{5}{6}$.

Page 162: When the children answer Exercise P, have them refer to their work with the last three exercises on the preceding page. They should see that the smallest common denominator is 12. Help them see that it is possible to compare $\frac{5}{6}$ and $\frac{3}{4}$ by means of any of the common denominators suggested in Exercise O, but that it frequently is easier to work with the smallest common denominator.

Exercises Q to S involve finding out which is larger, $\frac{3}{4}$ or $\frac{7}{10}$. Have the children take turns answering the questions in these exercises.

Exercise T should help the pupils see that sometimes they can use the denominator of one of the two fractions being compared as a common denominator. In such a case, only one fraction numeral needs to be changed before a comparison can be made. Exercises U and V give the class directions for comparing the two fractions $\frac{5}{6}$ and $\frac{7}{8}$.

The exercises labeled A to P should be assigned as written work. Tell the pupils to label each set of fraction numerals with their corresponding letter and then to express each pair of fraction numerals as equivalent fraction numerals with a common denominator. Instruct the children to write the numeral for the smaller fraction after each set of equivalent fraction numerals. Remind the children to use the smallest common denominator, but do not penalize them for failing to select the smallest denominator. Afterward, discuss the work.

Assign the "Keeping skillful" exercises as written work at another time.

Providing for the able pupil

You might let the abler children experiment with common denominators by working from the smaller denominator instead of the larger in finding the common denominator. Let them again compare $\frac{3}{4}$ and $\frac{7}{10}$. They may work at the chalkboard and show the steps needed to find 20 as a common denominator if they start with $\frac{3}{4}$. The pupils will probably multiply 4 by 2 first. Then when they find that 8 will not do as a common denominator, they will multiply 4 by 3, then by 4, and finally by 5 before they find that 20 is a common denominator for $\frac{7}{10}$ and $\frac{3}{4}$. This will show them that they can find a common denominator by starting with the smaller denominator, but that it requires more work.

Pairs of able pupils also can practice finding common denominators and determining which of two fractions is larger by using the activity "Finding the common denominator," described in Activity 38, page 378.

Helping the slow learner

Slow learners will probably need help in understanding why, for exact comparison, fractions must be expressed with a common denominator. The illustrations in the text show how to make comparisons involving fractions of lines and partial areas of rectangles. These pupils should find it helpful to work with fractions of disks. The flannel board (described in Activity 34, page 375) can be used for this.

Since it is difficult to superimpose felt or flannel fractional parts one upon another for purposes of comparison, the parts can be compared by being placed beside one another. If the flannel board was used as suggested in connection with the lesson on pages 148-150, you should already have fractional parts of disks cut in halves, thirds, fourths, etc. You may need to prepare disks divided into other fractional parts, too.

The pupils might begin by comparing $\frac{1}{2}$ of one disk with $\frac{1}{3}$ of another disk of the same size. Have someone put a whole disk made of halves on the flannel board and beneath it put another disk of the same size made of thirds. Ask a pupil to point out $\frac{1}{2}$ of the first disk and $\frac{1}{3}$ of the other one. Then discuss what has to be done to compare the two fractional parts. The pupils should decide that the fractions must be expressed with a common denominator. They should then decide that 6 is the smallest common denominator. Put a disk made of sixths on the flannel board. Then let a pupil take enough sixths from it to equal $\frac{1}{2}$ of a disk and enough sixths to equal $\frac{1}{3}$ of the other disk. Have him take these sixths from the full disk and put the $\frac{3}{6}$ beside the $\frac{1}{2}$ so that the class can see that they really are the same size. Similarly, have him put the $\frac{2}{6}$ beside the $\frac{1}{3}$ so that the class can see that these are also the same size. Then have the class observe that $\frac{1}{2}$ of a disk is $\frac{1}{6}$ of a disk larger than $\frac{1}{3}$ of a disk by comparing the $\frac{3}{6}$ with the $\frac{2}{6}$.

Complete the activity by having the children work through the comparison by computation—expressing both fractions in terms of sixths.

If you have cardboard disks, the children can use them at their desks in the same way. Or you can have one pupil demonstrate for the others. The advantage over the flannel board is that the parts to be compared can be superimposed, and a direct comparison can be made.

If some of the children have difficulty in changing a fraction numeral to an equivalent fraction numeral, have them review the work on pages 143-146.

163-164 Using arithmetic

Lesson Briefs for this lesson are on pages 155-156.

Expanded Notes are not considered necessary for this lesson.

CHARTING THE COURSE

Addition and subtraction of fractions and mixed numbers

The processes of addition and subtraction with fractions have traditionally been regarded as very difficult to learn and to remember. In part, this is due to the fact that teaching procedures have until recently relied chiefly upon connectionist learning theories. Here again each process was analyzed into an extensive set of "specific habits" which were then taught one by one through drill methods.

In contrast to these procedures, *Seeing Through Arithmetic 5* puts the emphasis on developing an understanding of the general principles of adding and subtracting fractions before the pupil is expected to develop skill through practice. Children are first shown by visual, rather than computational methods, what adding or subtracting fractions means (pages 165-168). Addition is associated with the act of combining of parts of concrete objects. Subtraction is associated with the act of taking part of an object away from a larger part or from the whole object. Since the idea of a common denominator has already been introduced, it can be used, but the presentation is not complicated by giving attention to the details of how fraction numerals that have the same denominators may be found computationally.

When readiness for adding fractions has been developed by the methods described above, the process itself can be learned without experiencing undue difficulty. In the past, some of the difficulty has usually been postponed by using at first only examples in which the fraction numerals have the same denominator. *Seeing Through Arithmetic 5* introduces the process with examples in which the fraction numerals do not have the same denominator. This is in conformity to the principle that, to promote the development of insight, processes should be introduced by examples that are as representative of the complete process as possible.

Similar principles govern the teaching of the addition of mixed numbers. In the first example the sum of the fractional parts of the two mixed numbers exceeds 1, so that a form of the process of "carrying" is required. Once this major feature of the process is understood, the teacher does not need to be careful to select only examples that do not require "carrying," and the pupil may proceed without encountering later a fundamental modification of the process.

The teaching of the process of subtraction of fractions and mixed numbers is organized according to the plans outlined above for addition. Although pupils at this stage should not be expected to achieve immediately either perfect accuracy or a high rate of performance in adding and subtracting fractions and mixed numbers, there is no reason why they should not be expected to have a good understanding of what they are doing. When this is reasonably well established, attention may be turned, in this and subsequent grades, to the development of acceptable skill with the processes.

165-168 Moving forward

Lesson Briefs for this lesson are on pages 157-159.

Overview

In this lesson the children learn that fractional parts of things can be combined or separated in the same way that whole things can be combined or separated. Pictures help them see that before fractions can be added or subtracted, they must be expressed with a common denominator. The *process* of adding or subtracting fractions is not taught in this lesson. Rather, the pictures and accompanying text show the *meaning* of adding and subtracting fractions. Addition and subtraction follow in the next lessons.

Teaching the whole class

Page 165: After the opening statement has been read, you may wish to review the combining and separating of objects regarded as wholes. On a table where everyone can see, first put a group of 3 small objects; then put 2 more objects with the original group. Ask a pupil to describe what happened and what the result is. Be sure the class understands that this is an

additive situation and that 3 objects plus 2 objects equal 5 objects. Now ask pupils to look at Movie A and read its accompanying text to find out about combining fractions. (Be sure pupils understand that each of the two one-half pies is the same size.) The pupils should know from experience that combining the two one-half pies will make one whole pie, but if they do not, the second scene will show them this. They should also see that combining 2 halves of pies to get one whole pie involves the same process as combining 2 objects with 3 objects. That is, they should see that the idea of adding fractions is similar to adding whole numbers.

In Movie B the pupils see that ⅛ of one cake can be combined with ½ of another cake, but the total amount of cake is not immediately recognized. Something must be done before we can tell exactly how much cake there is. In asking for suggestions as to what to do, you might remind the class that in the previous lesson they learned that to *compare* fractions, the fractions had to be expressed by fraction numerals with a common denominator. Movie C shows the one-half cake changed to eighths of the whole cake. Ask why 8 is used as the common denominator instead of 2. Bring out the meaning of changing the ½ of a cake to eighths. Suggest that the pupils must first think of the whole cake and then think of it as divided into 8 parts, each equal to ⅛ of the whole cake, as in Scene 1. The class can then see from Scene 2 of Movie C that ½ equals ⅝. Then, in Movie D, they can see that ⅜+⅛=⅝. Discuss the equation in the text beside Movie D. As a step toward understanding addition of fractions, to be taught later, you might ask what fraction numeral equivalent to ½ could be substituted for ½ in the equation. Discuss whatever answers the pupils give.

Use of the flannel board will emphasize what is being taught on this page. First put ½ of a disk on the board. Below it put ⅛ of a same-sized disk. Let a pupil place the ⅛ piece with the ½ piece and tell what must be done to find what fraction is now illustrated. Have available enough other eighths to assemble 8 eighths (a whole disk) on the flannel board. Have a pupil do so, and then have him find the number of eighths that are equal to the ½ disk already on the board. (He can place these beside the ½ disk for comparison.) Then ask him to remove the ½ disk and substitute the ⅜, put the original ⅛ of a disk with the ⅜, and count the total number of eighths to find how many there are in all. Ask him to write the completed equation on the chalkboard.

If the pupils have oaktag pieces of disks in halves and eighths, they may perform this same experiment at their desks, superimposing the eighths on the half disk.

Page 166: Pupils must realize that the three cups in Picture E are all the same size. They should see that ¼ of one cup is also equal to ¼ of another cup, that ⅓ of one cup is equal to ⅓ of another, etc. The unlabeled cup in Picture E shows the amount of liquid obtained when the liquids in the two cups are combined. Pupils can see that the markings for fourths and thirds do not tell them exactly what part of a cup of liquid there is in all.

In considering Picture F, discuss with the pupils why they should imagine the ⅓ cup and the ¼ cup divided into twelfths. Have someone explain why 12 was chosen as the common denominator. Let the pupils count the number of twelfths in ¼ and ⅓.

Explain that each cup in Picture G shows the combined contents of the two cups in Picture F. Have someone count the total number of twelfths. Explain that the cup on the right is the same as the one on the right in Picture E. You might also ask a pupil to write equivalents in twelfths for ¼ and ⅓ and substitute them in the equation in the text opposite Picture G. In discussing the questions about eighths, be sure the children realize that they could not know how much liquid there is in all if the cups were marked in eighths, because thirds cannot be expressed as eighths.

Page 167: Before beginning this section of the lesson on subtraction, demonstrate a subtractive situation with objects so that the children can review the separating action. Also ask a child to write on the board the equation describing your demonstration.

Then direct attention to Picture H and its text. Most of the children will not be able to tell what fractional part of a pie is left either from the picture or the text at the right of Picture H. (A few of the able pupils, however, may know intuitively that separating ⅙ from ⅚ leaves ⅘.) The class should see in Picture I that they must first know how many sixths were in the whole pie. Then they can see the ⅚ they had to begin with. In Picture J, when the ⅙ is removed, they can count the number of sixths left to find that there are ⅘.

This activity can be demonstrated at the flannel board or with oaktag pieces at the pupils' desks.

The next problem deals with the removal of a fractional part of a bar from the whole bar. Picture L helps the pupils see that they must think of the whole candy bar as divided into 3 equal parts, or thirds. Then when ⅓ is removed, they see that ⅔ is left.

Page 168: The subtractive situation on this page involves subtracting fractions expressed with unlike

denominators. Tell the pupils that the white portion in Cup 1 is milk. From this cup, ½ cup of milk is to be poured into Cup 2. Ask someone to point to the mark on Cup 2 that will be reached when the milk from Cup 1 has been poured into it. Pupils must understand that the problem is to find how much milk will be left in Cup 1. Discuss why ½ and ⅔ must be expressed by fraction numerals with the same denominator.

Ask for suggestions as to what denominator should be used and why. Pupils can see in Picture N that both cups have been divided into sixths. First, have them use Cup 1 to count the number of sixths in ⅔. Then, to find the number of sixths in ½, they should use Cup 2. When they find that there are ⅜ in ½, have them count off 3 sixths downward from the ⅙ mark on Cup 1. This will bring them to the ⅙ mark, which is the same mark that the milk reaches. In this way they can see that the answer is ⅙.

Before assigning Exercises A to X as written work, discuss Exercises A and B. Tell the pupils to give only the common denominator they would use in adding or subtracting. Later you may ask the pupils to explain why they chose the common denominators they did. If, in the written work, some of the able pupils finish quickly, let them actually express the fractions with a common denominator. Do not require them to find the smallest common denominator, however.

Providing for the able pupil

The activity "Finding the common denominator" (see Activity 38, page 378) can be adapted to give the able pupils practice in finding common denominators as preparation for adding or subtracting fractions. To prepare for this modification of the activity, remove all cards with fraction numerals on them that total more than 1. Then let pairs of children begin in the way described. After a card with two fraction numerals on it has been turned up and the fraction numerals have been expressed by equal fraction numerals with the same denominator, the children may use the flannel board (or oaktag fractional pieces at their desks) to find the total when the two fractions are combined. For example, if ⅙, ⅔ appear on the first card turned up, the children will replace the numeral ⅔ by an equal fraction numeral so that there is a common denominator (⅙ and ⅙). Then at the flannel board whoever found the common denominator first should assemble the parts and find the total.

For practice in preparation for subtraction of fractions, the children should be instructed to compare the smaller fraction with the larger one to see how much smaller it is. To compare ⅙ and ⅔, for example, they will first replace the fraction numeral ⅔ by an equal fraction numeral so there is a common denominator, and then they will put ⅔ of a disk on the flannel board, exchange the thirds for sixths, so that there will be 4 sixths, and remove ⅙.

Helping the slow learner

These children should have many opportunities to work with fractional pieces, combining and separating fractional parts. The work may be done at the flannel board or with fractional pieces made of oaktag that the children use at their desks. Either you or some of the able children may set up the situations you want the children to solve by manipulation. (Do not provide sets of fractional parts that make more than 1 whole when combined.) For example, on the flannel board put up ½ of a disk and ¼ of an equal-sized disk and ask that these be combined. Let the children experiment to find out that they must change the ½ to fourths. When they accomplish this, they should remove the ½ of a disk, replace it with two fourths, and then combine the ¾ and the ¼ to find that the answer is ¾. Let the pupils work in pairs with various combinations of fractions that require a change of denominator in one of the fractions.

169 Keeping skillful

Lesson Briefs for this lesson are on pages 159-160.

Expanded Notes are not considered necessary for this lesson.

170 Thinking straight

Lesson Briefs for this lesson are on pages 160-161.

Expanded Notes are not considered necessary for this lesson.

171-176 Learning how

Lesson Briefs for this lesson are on pages 162-165.

Teaching the whole class

Page 171: Begin the lesson by reading the text for Picture A and having the children relate the fractions to the two pieces of cake. Discuss the meaning of the equation. Then go on to the text for Picture B and explain that, to find how much of the cake Mrs. Page gave away, it is necessary to put the two pieces of cake together. Explain that if Mrs. Page had given away 2 whole cakes and 3 whole cakes, it would be possible by looking at pictures of the cakes to tell how

much cake she gave away. Illustrate this fact on the chalkboard by drawing 2 cakes and 3 cakes.

Then explain that in this case Mrs. Page gave ½ of a cake to one friend and ⅓ of it to another. On the board draw one third of a cake and one half of the same cake. Then ask the children if they can tell just by looking at the picture on the board what fraction numeral they would use to express how much of the cake Mrs. Page gave away.

To illustrate this, you may use a large whole disk cut from white paper, a red disk of the same size cut into halves, a blue disk of the same size cut into thirds, and a yellow disk of the same size divided (not cut) into sixths. Use transparent paper for the yellow disk. Place a red half and a blue third of a disk on top of the whole white disk, and show the children in this way that they must do something else before it is possible to know what fraction they can use to express the sum of the two fractional parts.

Read the first paragraph of text for Picture C and point out that fractions expressed with 6 as the denominator are chosen as another way to express ½ and ⅓. Discuss the reasons for using sixths. To make the reasons clear, place the yellow disk divided into sixths on top of the half of the red disk and the third of the blue disk. Of course, the red half and the blue third are lying side by side on top of the whole white disk. Be sure the children see that the sixths divide the whole disk into equal parts, and that they also divide into equal parts the ½ red disk and the ⅓ blue disk. Therefore they can say that fraction numerals with 6 as the denominator can be used to express the fractions of the cake given away.

At this point, you may need to review the procedure of multiplying both the numerator and denominator of a fraction numeral by the same number to arrive at an equivalent fraction numeral. Relate the reduction process as used with comparison of fractions to the addition of fractions.

Finish the text on page 171 and discuss the equation. You will want to point out that there are other fraction numerals that can also be used for ½ and ⅓, such as $\frac{6}{12}$ and $\frac{4}{12}$, but that computation is usually simpler when a smaller denominator is used. Some of the children may not clearly understand this, and you may wish to illustrate how the fractional parts shown in Picture C can be divided into twelfths.

Page 172: The children know that ½ is equal to $\frac{3}{6}$ and that ⅓ is equal to $\frac{2}{6}$. Have them note that Picture D shows $\frac{3}{6}$ of the cake and $\frac{2}{6}$ of the cake put together. Now, since all of the parts are the same size, it is possible to look at the picture and tell how much cake Mrs. Page gave away.

Read the text and emphasize that the denominator 6 in each fraction numeral indicates that the original cake was cut into six equal parts. Help the pupils understand what the numerator of each fraction indicates and why the numerators are added to find the sum of the two fractions.

Your explanation of the second SEE step will be facilitated if you bring glass measuring cups to class and demonstrate the steps of the problem as you go along. Use sugar, sand, or water colored with vegetable dye. Note that this part of the lesson shows the addition of three fractions.

Read the first part of the text and ask the children to match the fractions in the equation with the corresponding cups in Picture A. If you have brought measuring cups to class, let the children examine them for the division marks and then fill each one with the correct amount of "sugar." Discuss how a portion of sugar in a measuring cup differs from a portion of the cake they have just studied. You will want the pupils to conclude that the pieces of cake have definite form and shape, whereas sugar or water does not; yet the latter materials can also be measured. The fractions tell how much of whatever kind of unit of measurement is used—in this case, the cup.

Ask the children to look at the equation and tell in what way the fractions differ from one another. Then have them study the markings on each cup. They should see that, since ⅓, ½, and ¾ have different denominators, they cannot find the sum of the three fractions by using the fraction numerals in their present form. They will have to find a new way to express each fraction. Verify this discussion by reading the rest of the text and studying the picture accompanying it. If you have a two-cup measure, you will want to repeat the action in the picture for the children. If the materials have been carefully transferred from cup to cup, the total amount in the large cup will be somewhat below the 1⅔ mark, but there will be no marking on the cup that will tell the children just by looking what numeral to use to express the amount of sugar Mrs. Page used. Explain that they must find a common denominator for all three fraction numerals.

Page 173: Review with the children the idea that the common denominator is a number that is divisible by the denominators of all three fractions. Help the children decide that they will first try 4, the largest of the denominators, and they will next try 2×4, or 8. Then they will try 3×4, or 12. Have them look at Picture C. Ask them to count the number of small equal parts into which each cup has been marked. They will see that there are twelve parts in each cup.

Have the children note also that these division lines coincide with the red division lines for the halves, thirds, and fourths.

Point out that the children must now find out how many twelfths there are in $\frac{3}{4}$, in $\frac{1}{3}$, and in $\frac{1}{2}$. First let them count the number of twelfths for each of these fractions, as shown in Picture C. Then review the computational method for finding equivalent fractions. Have the children supply the missing numerators for the three fractions. Now read all of the text for Picture C and give particular attention to the equation showing the parts of cups expressed by equivalent fraction numerals with a common denominator. Make sure the children understand that the new fraction numerals enable us to add the fractions.

Then go on to Picture D and look first at the equation. Relate the fractions in it to those represented in Picture A on page 172. Let the children discover that if they count the total number of twelfths or add the numerators of the three fractions, they will arrive at the numerator of the answer, 19. Call on one of the pupils to read the text and explain why only the numerators have been added. Discuss with the class how many twelfths there are in one cup. Then refer to Picture D again and bring to the attention of the class that when the sugar is poured together, there is more than one full cup. Explain that the answer $\frac{19}{12}$ is more than one cup and will be easier to understand if it is expressed as a mixed number. When the children write the answer, it should show that there is 1 cup and also $\frac{7}{12}$ of a cup more.

If you have been demonstrating the ideas with measuring cups, pour the contents of the large cup back into the 1 cup measures. Let the children note that there is one full cup and part of another cup. Have the children study Picture E and count the number of twelfths taken up by the sugar in the cup that is partly full. Let them refer back to Picture D and count the number of twelfths there are besides the $\frac{12}{12}$ for a full cup. The count they have made for both pictures gives them $\frac{7}{12}$ more than one full cup.

Read the text and pay particular attention to the reduction of $\frac{19}{12}$, printed in red. Discuss the computational method of changing $\frac{19}{12}$ to $1\frac{7}{12}$. End the discussion of this problem by having several children read the equation at the bottom of page 173 and summarize in their own words the steps they have taken to arrive at the answer.

Page 174: You might begin by saying to the class: *For this problem the fraction numerals do not have to be replaced before you can add. Look at the three pictures and decide why you think this is true.*

The children will see that all the pieces are alike in size in Pictures A, B, and C and that they are working with fractions expressed with the same denominator.

Read the text for Picture A and have the children relate the fractions in the equation to the pieces of chocolate in the picture. Have them count the number of pieces there are in all and explain that since the problem tells that there is $\frac{3}{8}$ of a bar in one package and the picture shows 3 pieces, each piece must be $\frac{1}{8}$ of a bar. Say:

Look now at Picture B and read the text opposite the picture. There are how many eighths in a whole bar? If 8 eighths are equal to one bar, was Mary right when she said she had found enough to make a whole bar of chocolate? To find out if she was right, $\frac{3}{8}$ and $\frac{5}{8}$ must be added. Why is the numeral 8 in both the numerator and denominator in the answer? Picture C will tell us what $\frac{8}{8}$ means. Do you think that $\frac{8}{8}$ is the same as 1?

Have the children relate the numeral $\frac{8}{8}$ in the first line of text (opposite Picture C) with the pieces of candy in the box. Explain that the picture shows $\frac{8}{8}$ as equal to one whole bar. When $\frac{8}{8}$ is reduced, it is written as 1, meaning 1 whole.

Some children in your class may not grasp immediately that a fraction numeral whose denominator and numerator are the same always represents 1. You can use other sets of numbers like $\frac{5}{5}$, $\frac{7}{7}$, etc., to explain this. Demonstrating with sheets of paper will help to clarify it. Hold up a whole sheet of paper. Then mark it into equal parts (sixths). Explain that you have marked the paper into 6 sixths. Write the fraction numeral $\frac{6}{6}$ on the board. Cut the paper into the sixths and show the children how various combinations of the sixths can be put together ($\frac{2}{6}+\frac{4}{6}$, $\frac{3}{6}+\frac{3}{6}$, $\frac{1}{6}+\frac{5}{6}$), all resulting in $\frac{6}{6}$ or 1.

Page 175: Examples A to F (E and F are on page 176) include various combinations of denominators met in addition of proper fractions, as well as various types of situations where reduction is needed. Note that both the traditional computational (vertical) arrangement of the fraction numerals and the equation (horizontal) form are shown. The vertical or computational form has no real advantage when all of the fractions are proper fractions, but it does have an advantage when mixed numbers are to be added. Discuss both arrangements and get the children accustomed to both.

Study first the equation form for Example A. Have the children read the text and explain why 10 is the common denominator chosen for the two addends. Discuss how the numerator and the denominator are

found for $1\frac{3}{10}$ and how this answer is reduced to $1\frac{3}{10}$. Then study the computational (vertical) form.

Adapt the procedures given for Example A to Examples B, C, and D. In Example C, you will note that the denominators are the same and that there are two steps in the reduction of the answer.

If any of the children have difficulty understanding the computation for Examples A to F, use concrete materials, or pictures on the chalkboard, to illustrate the work.

Page 176: Adapt the procedures given for the examples on the preceding page to Examples E and F. Note that Example E contains three addends and that the answer is a whole number. Note that in reducing the answer for Example F, $5\frac{1}{24}$ can be thought of as two groups of $2\frac{1}{24}$, with $\frac{3}{24}$ left over, and that a second step is necessary to reduce the $\frac{3}{24}$ to its simplest form.

When the children understand Examples A to F thoroughly, let them work the exercises in the DO step independently. You will want to make sure that all the children know how to use both the equation form and the vertical form of showing their work. You may use the two blocks of exercises in the ways suggested in the Comments (page 162) or in any other way you choose. Verify each child's work at his desk and give any further explanation that may be required.

Providing for the able pupil

The children may make up their own exercises to include various types of denominator patterns. For example, some exercises may have denominators that are the same, others may include fractions in which one denominator is a multiple of the other, and still others may have different denominators with neither one a multiple of the other. If the children illustrate the steps of the computation by pictures, the papers would make a good bulletin board display.

Activity 37, page 377, may be used if the children need some extra work in reducing fractions.

Helping the slow learner

Examples A to F on page 175-176 provide a good outline of the work for addition of proper fractions. Give the slow learners a fuller explanation of these pages if they need it. You may also have them illustrate the computational steps by pictures.

Activity 37, page 377, is especially useful in helping slow learners understand the relation of fractions to wholes.

After these children understand how fractional parts may be combined to make one whole, then you may wish to have them use Activity 39, page 378, in which they find the number of fractional parts in one or more wholes.

177-182 Learning how

Lesson Briefs for this lesson are on pages 166-169.

Overview

In learning to add mixed numbers, the children begin with a representative situation. The addends they work with have fractions with different denominators, and when common denominators are found and the fractions are added, the sums result in improper fractions. The children learn to add to the ones the whole numbers that result when these improper fractions are reduced.

Teaching the whole class

Since the work in this lesson requires the reduction of improper fractions, you may wish to review briefly the idea that 8 eighths are needed to make 1 whole, that 6 sixths are needed to make 1 whole, and so on. Some of the children will be helped if this idea is demonstrated by means of the flannel board, oaktag fractional pieces, etc.

Page 177: The pupils should first identify the groups of apples in Picture A in terms of the fractions mentioned in the problem. Be sure they understand that because the situation is additive, the fractional parts of apples as well as the whole apples must be combined and some way found to express the total. Discuss the equation in relation to Picture A.

When Picture B is discussed, be sure the children understand why the groups of whole apples are dimmed (they are not being considered at the moment). Get the pupils to observe that in Picture B the $\frac{1}{2}$ apple is changed to $\frac{2}{4}$ apple. Have them explain why this change has been made. Then have them look at the way the computation is written. Let them note that the mixed numbers are written first as they appear in the problem. Then they should observe that the mixed numbers are rewritten, with the fraction numerals changed to equal fraction numerals with a common denominator. Discuss why 4 was chosen as the common denominator. Then tell the pupils to write the mixed numbers in computational form on their papers.

Next have the pupils observe that in Picture C the fractions of apples (fourths) have been combined. Let them find the total. They may now add the fractions they have written on their papers. Since the answer, $\frac{5}{4}$, is an improper fraction, ask them not to write it, but to remember it or write it on scratch paper. Ask someone to explain how he knows that it is more than 1.

Page 178: Have the pupils read the text and relate it to Picture D. Discuss where the whole new appl

came from and ask why ¼ of an apple is left over. Have the pupils use the picture to explain how the improper fraction 5/4 can be changed to the mixed number 1 ¼. Have them note where the ¼ is written. Tell them to remember the whole number and write the ¼ on their papers. Turn attention next to Picture E, the text, and the computation. Remind the pupils that only the ¼ has been recorded so far. Have them note where the 1 (representing the whole new apple in the picture) is written in the ones' column. Point out that 4/4 has been expressed as 1, and this 1 has been carried to the whole number column. Direct attention, too, to the way this procedure parallels that used in the addition of whole numbers.

Then have the children look at Picture F and see that the groups of whole apples are being combined. Ask someone to tell how many whole apples there are in all. Have the children next add the whole numbers, including the carried 1, and note where the 11 is written. Let them write the carried 1 above the column of whole numbers on their papers, and then add. Call on someone to read the answer. Then have Picture G examined and the final equation read.

Page 179: Proceed with the work on this page in much the same way as was outlined for the SEE step. Now, however, the children should answer the questions provided in the text as a guide to their thinking. The only new aspects here are the number of addends (3) and the fact that one of the addends is a proper fraction. The procedures for adding are no different from those developed for the addition of mixed numbers only. Remind the children, when they come to the last line on the page, that the 17/8 must be remembered or recorded on scratch paper.

Page 180: Continue adapting the procedures suggested for page 178. Remember that the children are to answer the questions in the text. They should note that 2 is the whole number being carried here. Be sure they can explain, in connection with Picture E and the computation beside it, where the 2 carried to the ones' column came from.

Page 181: When the children have supplied the numeral that replaces *n* (opposite Picture G), have them reread the problem on page 179 to see if the question has been answered. The children should use Exercises A to G on this page to see how well they understand the addition of mixed numbers. Tell them to try each exercise and if they have difficulty in adding, to look at the worked-out example and compare their work with it. Tell them that they may use scratch paper and write any improper fractions they feel they may not remember or do any computing that they find necessary.

As the children work, observe their progress. Help those who seem to be having trouble. Afterward, discuss the examples briefly. Ask questions that will enable the children to make the following observations: The sum of the fractions may be a proper fraction that need not be expressed as a mixed number (Example B); the number to be carried does not always need to be written above the ones' column (Example C); proper fractions sometimes should be reduced to simplest form (Example D); sometimes the reduction of the improper fraction results in a whole number instead of a mixed number (Example F); and sometimes it is not necessary to find a common denominator (Example G).

Page 182: Assign Exercises A to Z in the DO step as written work. Remind the children to use the vertical form when they compute. Also remind them that sometimes they will not need to find a common denominator. Afterward, have the solutions put on the board. If any of the work is incorrect, let the class discuss the solution and find what is wrong. The pupils who made many errors may, after the class discussion, try Exercises A to H. Those who still continue to make many errors will need the help suggested in the section "Helping the slow learner."

The three blocks of work in the "Keeping skillful" exercises may be used as one lesson or as separate lessons. Supply answers, and discuss the equations and computation with the pupils if you think it is necessary.

Providing for the able pupil

The game "Fraction race," described in Activity 40, page 379, will give able pupils practice in adding mixed numbers.

Helping the slow learner

To help the slow learner, you will have to observe his work very closely to see what his specific difficulty is. If it is inability to find common denominators, review with him the lesson on common denominators, pages 160-162; if necessary, go back to page 143 and review changing fraction numerals to equivalents. Also let him try the games and activities suggested for these lessons. If his difficulty is inability to change an improper fraction to a mixed number, have him review the work in the lesson beginning on page 151, and let him try again the activities suggested. For practice in reducing fractions to simplest form, have him work again the exercises in "Thinking straight" on page 146. If he needs help in adding proper fractions, ask him to review the lesson beginning on page 171. Let him use objects at his desk or the flannel board wherever possible.

183-184 Using arithmetic

Lesson Briefs for this lesson ore on pages 170-171.

Expanded Notes are not considered necessary for this lesson.

185 Moving forward

Lesson Briefs for this lesson ore on pages 171-172.

Expanded Notes are not considered necessary for this lesson.

186-189 Learning how

Lesson Briefs for this lesson ore on pages 172-175.

Overview

In this lesson the children learn how to subtract proper fractions. Pictures help them see that before they can tell exactly how much remains they must think of the original quantity and the quantity removed as divided into equal-sized parts. They then see that the fractions must be expressed with a common denominator before one number can be subtracted from the other.

Teaching the whole class

Page 186: After you have started this lesson by having a pupil read the introductory statement aloud, you can review with the class what they already know about subtracting fractions. Remind them that in an earlier lesson they used pictures to compare two fractions to see if one was larger or smaller than the other. Recall that they were able to tell how much larger or smaller one fraction was by changing the fraction numerals so that they had a common denominator. (Refer them, if necessary, to the lesson beginning on page 160.) Remind them, too, that in the lesson beginning on page 165 they learned that fractions of things can be involved in subtractive situations. Here, too, they saw that before one fraction can be subtracted from another, the fractions must be expressed with a common denominator.

When calling attention to the first problem on page 186 and its accompanying picture, be sure the pupils understand that $\frac{2}{3}$ of a pie is shown in the two top scenes. One half of the original *whole* pie is to be used for lunch. Most of the children should by this time have a good idea of what $\frac{1}{2}$ of a disk (or pie) looks like and should have no trouble finding the part that is the $\frac{1}{2}$ pie in the second scene. The children can see the piece that is left in both the second and third scenes, but they have no way of knowing exactly what fraction of the whole pie it is. Then point out how the accompanying equation relates to the illustrated situation. Have someone explain each symbol in the equation.

Continue by asking for suggestions as to how to find what fraction of a whole pie is left. By this time the children should realize that it is necessary to think of the original part of a pie and the part to be taken away as though they were divided into the same equal-sized pieces. Have them look at Picture B to see that the whole pie has been divided into six equal parts, that $\frac{2}{3}$ is equal to $\frac{4}{6}$, and that $\frac{1}{2}$ is equal to $\frac{3}{6}$.

Now discuss what should be done with the numerals in the equation to make them correspond to the changes in the picture. In working with the fractions, the pupils should see that they must find a common denominator. You might ask:

What are the two fractions we want to express with a common denominator? [Write them on the chalkboard.] We know that to change a fraction numeral to an equivalent fraction numeral, we can multiply or divide both the numerator and the denominator by the same number. Remember that we want a common denominator for both fractions. Should we start working with the larger or the smaller denominator? Why?

3 is the larger denominator. Let's see if we can multiply it by some number and get a number that can be divided by the other denominator, 2. Try multiplying 3 by 2. What do you get? Can you multiply the denominator 2 by any number to get 6? What is the number? So 6 can be used as the common denominator. But we also must multiply the numerator by the same number by which we multiplied the denominator. By what number do we have to multiply the 2 in $\frac{2}{3}$? What do we get? When we multiplied the 3, what did we get? By what new fraction numeral can $\frac{2}{3}$ be replaced? Now let's consider the $\frac{1}{2}$. We decided we could multiply the denominator, 2, by what number to get 6? So we should multiply the numerator by what number? What is the new fraction numeral expressed as sixths, that is equal to $\frac{1}{2}$?

Have the pupils look at the fraction numerals in the text to verify the changes. Be sure they see that the equation can be rewritten with these fraction numerals in place of $\frac{2}{3}$ and $\frac{1}{2}$.

Page 187: Next call attention to Picture C. The pupils should see that $\frac{1}{2}$, or $\frac{3}{6}$, of the pie is being removed from the $\frac{2}{3}$, or $\frac{4}{6}$, of the pie. They can see that $\frac{1}{6}$ of the whole pie is left. Have them read the

text at the right carefully to find out that the same answer can be obtained by subtracting the numerators of the fractions now expressed as sixths, and writing 6 as the denominator. Have them note how and where the answer is written.

You may want to have this example demonstrated on the flannel board, using thirds, halves, and sixths of disks. Let the children write the fraction numerals on their papers, make the necessary changes, and do the computation.

The problem in the THINK step can be handled in much the same way. Use the questions in the text to help the pupils think through the steps. Be sure to let the pupils explain how to select 12 as the common denominator for the fractions and how to change them to equivalent fraction numerals with the same denominator.

Page 188: Finish the problem by having the pupils answer the questions and find the missing numerals for the text.

For the examples in the TRY step, discuss thoroughly the computational arrangement presented in Example A. You can point out to the pupils that since only the numbers in the numerators are subtracted, the equation form does have an advantage because no numerals separate the two numerators. Then let the children write the example in the vertical or computational form and express the fractions with fraction numerals that have a common denominator, as shown in the text. Have them try Examples B through D (D is on the next page) and compare their work with that in the book after they have finished each example. In the discussion afterward, point out that in Example B they do not have to find a common denominator because the fraction numerals already have a common denominator.

Page 189: In discussing Example D, point out that the fraction numeral in the answer should be reduced to its simplest form.

For the DO step, have the pupils write their work and answers for the exercises labeled A to T. Discuss the work afterward. Then, for those pupils who seem to be having difficulty, assign the second block of exercises, labeled A to L. Work with these pupils if necessary.

The "Keeping skillful" exercises provide practice in adding fractions and mixed numbers and in the addition and subtraction of whole numbers. The three blocks of work may be assigned at the same time or at different times, as you prefer. Refer the pupils back to the lessons on problem solving if they encounter difficulties.

Providing for the able pupil

All children will benefit by seeing concrete illustrations of subtraction of fractions. The able pupils can help out here especially by doing demonstrations on the flannel board. After discussion of the examples in the DO step on page 189, they can take turns illustrating those examples that seemed to cause trouble for some of the slower children. Such demonstrations and explanations by the abler pupils may help clear up difficulties for the slow children, and also will benefit those who explain and demonstrate.

For an example like $\frac{3}{4}-\frac{1}{3}$, an able pupil should proceed somewhat as follows: Put $\frac{3}{4}$ of a disk on the flannel board and then explain that he cannot yet remove $\frac{1}{3}$ of the disk. He should discuss finding a common denominator for $\frac{3}{4}$ and $\frac{1}{3}$ and then tell why he chooses 12. On the chalkboard he should show the class how he computes to replace $\frac{3}{4}$ and $\frac{1}{3}$ by fraction numerals with the same denominator. Then on the flannel board he should put $\frac{9}{12}$ of a disk to replace the $\frac{3}{4}$ of a disk. Next he should take away $\frac{4}{12}$, explaining that this is equivalent to taking away $\frac{1}{3}$. He can let a classmate who had trouble with the example count the remaining parts to find that there are $\frac{5}{12}$.

Helping the slow learner

Activity 38, page 378, "Finding the common denominator," can be adapted to give the slow learners practice in subtracting fractions. The same pack of cards with two fraction numerals on each card may be used in this version. As with the original game, the children play in pairs. One child turns up a card, and all the children write in equation or computational form the fraction numerals they see preparatory to subtracting. Whoever first succeeds in writing these fractions with fraction numerals that have a common denominator, subtracting, and finding the correct answer, wins that round. The children may keep track of the number of rounds they win. An able child might act as a checker.

190-195 Learning how

Lesson Briefs for this lesson ore on pages 175-178.

Overview

In this lesson the children are taught how to subtract mixed numbers. They will find that on occasion they must use a technique similar to the borrowing used in the subtraction of whole numbers. If, after they have expressed the fraction numerals as equal fraction numerals with the same denominator, they find that the fraction to be subtracted is larger than

the one from which it is to be subtracted, they "borrow" from the whole number. The children are taught to take 1 from the whole number and think of it as an improper fraction having the same denominator as the fraction they already have, and to which it is then added. Then they can subtract the fractions as well as the whole numbers. The material in this lesson is presented by the four-step teaching method.

Teaching the whole class

Page 190: Pictures A and B and the accompanying text should be used in much the same way as were the opening steps for the lesson on addition of mixed numbers (page 326). Point out that in Picture B the $\frac{2}{3}$ of a pound of butter is marked to show $\frac{8}{12}$. The additional marks on the wrapper show that the whole pound is thought of as divided into twelfths. The $\frac{3}{4}$ of a pound to be removed is thought of as $\frac{9}{12}$. Ask the children if they can subtract $\frac{9}{12}$ from $\frac{8}{12}$. When they see they cannot, refer them to Picture C to find out what can be done. Here they can see that 1 of the whole pounds of butter has been marked into 12 equal parts. Have the pupils read the text to find that these $\frac{12}{12}$ are going to be put with the $\frac{8}{12}$ of a pound. Ask why it is reasonable to think of the whole pound as $\frac{12}{12}$ rather than as $\frac{6}{6}$ or $\frac{9}{9}$.

Page 191: The changes that were made in Picture C are shown in the computation beside Picture D. Discuss the computation carefully. The pupils should understand why it is sometimes necessary to rewrite a mixed number in a different form. In this case they cannot subtract until $4\frac{8}{12}$ has been rewritten as $3\frac{20}{12}$.

Now the pupils should see that they are ready to subtract and all they have to do is to subtract the fractions and the whole numbers in turn.

This problem situation can be demonstrated. You may use oaktag or colored construction paper disks or felt pieces at the flannel board. To begin this activity, 4 equal-sized disks and $\frac{2}{3}$ of another one should be used. Then to take away $2\frac{3}{4}$ disks, the pupils will need at least 20 pieces that represent twelfths of disks of the same size. Let the children select the necessary pieces, superimposing them on the other fraction pieces or placing them side by side on the flannel board. Let the children demonstrate the entire subtraction operation.

Page 192: Discuss the new problem and the equation with the class. Ask the children how they would go about taking a fraction of a pound of lard, like $\frac{7}{8}$ of a pound, from the whole pounds. Most of the children will realize that a whole pound will have to be opened and cut. Then proceed to Picture B and the accompanying text. Point out that what has happened in the picture is indicated by the small black numerals in the computation; that is, the 4 pounds are now being thought of as $3\frac{8}{8}$ pounds. Ask why the 1 pound is thought of as $\frac{8}{8}$, rather than as $\frac{4}{4}$ or $\frac{10}{10}$, for example. Pupils should see that it is now possible to remove the $\frac{7}{8}$ of a pound. The computation opposite Picture C shows that the fractions can now be subtracted.

Page 193: The remaining work now involves only the subtraction of the whole numbers and should present no difficulty.

In the problem in the THINK step, the pupils should see immediately that the fraction numerals already have a common denominator, but that it is again necessary to "borrow" from the whole number, since $\frac{3}{4}$ cannot be subtracted from $\frac{1}{4}$. Have the children answer the questions in the text so that they will know why their thinking must parallel the action shown in Picture B.

Page 194: For the work on this page, follow the questions in the text and have the children supply the numerals that belong where the screens are.

Be sure the children observe that the answer is a proper fraction, not a mixed number. They should have no difficulty in explaining this.

Page 195: With the TRY step, tell the pupils to try to solve each of the examples without any help from the book. Tell them to verify their answers and their work by referring to the book. Discuss Example D of the TRY step, since here there is a fraction in the minuend from which nothing is to be subtracted, so it is merely brought down as a part of the answer. Since nothing was borrowed from the whole number, the numeral expressing it stays the same, and subtraction of the whole numbers can take place.

When all the children have finished the first block of work in the DO step, discuss the exercises thoroughly. Those children who still have difficulty may need additional help, as suggested in the section "Helping the slow learner." After such work, you might assign Exercises A to L in the second block to them.

Providing for the able pupil

If the playing board, fraction cards, and markers suggested for Activity 40, "Fraction race," have been made and have been used to practice the addition of fractions and mixed numbers, the same materials may be used and the game adapted to the subtraction of fractions and mixed numbers. (For a description of the materials, see page 379.)

In the subtraction version of the game, the children try to move their markers on the number line from

4 to 0 by subtracting the fraction on one card at a time. As usual, 7 cards are distributed to each of 2, 3, or 4 players. The first player draws a card from the top of the pile and tries to subtract the fraction shown on one of his cards from 4 so that the remainder equals one of the places marked on the line on the board. He will, of course, try to use a fraction card that moves his marker the farthest. He moves his marker to that place, puts down the card with the fraction he subtracted in front of him, and explains his answer to the other players. Each player should have scratch paper and pencil available so that he can figure his own moves and verify those of his competitors. Anyone so checked and found to be in error loses his play and cannot move his marker that round. After moving his marker, the first player then discards one card from his hand. If he cannot make a play, he simply discards.

Each player in turn does the same thing. On the second time around, the player must subtract one of his fractions from the number at the point where his marker is. For example, if he reached the $3\frac{5}{8}$ mark the first round, then he must subtract from $3\frac{5}{8}$ on the second round, and so on. The round is over when a player's marker rests at 0. His final answer must be exactly 0. The round is also over when one player has put down all his cards. This will happen more often. However, this does not mean that the player who lays down all his cards first wins. The winner of each round is the one who has advanced to the lowest number. Only the score of the winner is recorded for each round. A game should consist of five rounds.

Helping the slow learner

Slow learners will have to be watched closely to determine just where their individual weaknesses lie. Give them help by discussing proper methods with them, then have them restudy the original lessons in the textbook, and finally, let them engage in the special activities planned to give them practice.

The trouble may still be in finding common denominators. Refer them again to the lesson on pages 160-162, and if necessary, to the one on replacing fraction numerals by equivalent fraction numerals, pages 143-146. Also let them play the game, "Finding the common denominator" (Activity 38, page 378).

If the difficulty lies in borrowing from the ones' column and replacing the mixed number by an equivalent form so that the fractions can be subtracted, refer the pupils to the lesson on page 185.

If reducing fraction numerals to simplest form is bothering some pupils, have them look again at "Thinking straight" on page 146. Let the children use objects at their desks or work with the flannel board wherever possible. Such work can be done under the supervision of an able pupil.

196-197 Using arithmetic

Lesson Briefs for this lesson ore on poges 179-180.

Expanded Notes are not considered necessary for this lesson, except for suggestions for the able pupils and slow learners.

Providing for the able pupil

An activity able pupils might enjoy involves supplying mixed numbers to complete a magic square. The children must both add and subtract mixed numbers in working with this activity. See Activity 41, page 380, for complete details.

Details of an activity for pupils who can read music will be found in Activity 42, page 381.

Have able pupils write, for a series of denominators, fractions that differ from 1 by the smallest possible fraction. For example, they could write $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, etc., or $\frac{24}{25}$, $\frac{25}{26}$, $\frac{26}{27}$, etc.

Helping the slow learner

Working through the four types of problem situations with objects may help the slow learners.

Use whatever fractional parts you have—felt pieces on the flannel board, or fraction pieces made of construction paper for the pupils' individual use. For Problem A, for example, discuss the correctness of the equation given and help the pupils see that they do not know what has been subtracted from $6\frac{1}{2}$. Put down $6\frac{1}{2}$ disks to help them see what can be done. They are to pretend that the disks are bushels. Discuss the fact that $3\frac{1}{4}$ bushels were left after some bushels were planted, and that if the group of $3\frac{1}{4}$ is removed, the number of bushels left will represent the group that was planted. Ask what they have to do to remove $3\frac{1}{4}$ bushels from $6\frac{1}{2}$ bushels. They should see that first they must change the $\frac{1}{2}$ disk into fourths. Have fractional parts available so that this change can be made. From the $6\frac{2}{4}$ disks, the children should then be able to remove $3\frac{1}{4}$ disks. They should see that $3\frac{1}{4}$ disks are left, representing the $3\frac{1}{4}$ bushels of potatoes that were planted. The other types of problem situations can be worked through in much the same way as this one.

197 Checking up

Lesson Briefs for this lesson ore on pages 180-181.

Expanded Notes are not considered necessary for this lesson.

198-199 Side trip

Lesson Briefs for this lesson are on pages 181-183.

Teaching the whole class

Page 198: Before you begin this lesson, make a special effort to help the children understand that the rotation of the earth is from west to east, and that although the sun appears to move across the sky as though it were going around the earth from east to west, it is really the earth that is moving. It may be helpful to use some device to show the children that the rotation of the earth causes part of its surface to be turned away from the light of the sun while part of the earth is toward the sun; that when it is night in one place, people on another part of the earth's surface have daylight. Some children may already have performed such experiments as holding a flashlight beam steady (to represent the sun) while slowly turning a globe (to imitate the rotation of the earth) to simulate daylight and darkness on the earth. (Make it clear that it is the turning of the earth on its own axis that causes day and night, not the revolution of the earth around the sun.) To help the children understand that the apparent motion of the sun is an illusion, you can discuss other illusions, such as the sensation of passengers in a stationary train that is being passed by a moving train. (The passengers on the stationary train know they are not really moving, but if they believe the evidence of their eyes, they feel that they are moving past a stopped train.)

When you are sure the children understand why noon cannot occur at the same moment everywhere on the earth, direct attention to the map and discuss Exercises A and B. Let a pupil read the time shown on each clock, starting at the Pacific Zone and going east. Let another pupil tell what the time difference is between each clock and the next one directly to the east. Then let another child start with the clock in the Atlantic Zone and read the times shown on the clocks going westward. Let someone tell what happens to clock time as you go from east to west. Ask another pupil to tell what the difference in time is between San Francisco and New York. Then have the pupils read Exercise C. Let someone volunteer the answer. Have Exercises D, E, and F discussed.

Two children at a time might then participate in a "long distance telephone call" activity. The rest of the class can "listen in" and be ready to correct any mistakes. The child pretending to place the call tells what city he is in and what city he is calling. He then tells what time it is in his city and asks for the correct time at that moment in the city he is calling. The other child gives this information.

In discussing Exercise G, you can tell the children that in the years at the beginning of the eighteen hundreds, each community *did* have its own time, based on the position of the sun overhead. Through appropriate questions, let the children discover that as long as people stayed near home and had little communication with other communities, wide variations in local time did not matter, but that when railroads were built and people began traveling on them, it was necessary to know, for example, not only what time a train would leave, but also what time it would arrive at its destination. It might interest children to know that before time was standardized, some railroad timetables had to list 68 different kinds of local time!

Page 199: Many adults are confused about the precise limits of the A.M. and P.M. hours. (This may be caused in part by the fact that there really was a period when noon was considered to be the beginning of the new day.) Explain that now there is an international agreement about when the day is considered to begin and end, so that we know what we are talking about when we describe any particular hour.

Because many persons erroneously think of the hours of darkness as being entirely P.M. hours, and the hours of daylight as being entirely A.M. hours, you should pay special attention to the proper use of these terms. If you like, you can explain that these letters are abbreviations for Latin words that stand for "before noon" (*ante meridiem*) and "after noon" (*post meridiem*). Explain that the A.M. hours are those between midnight and noon, and let a child name them. Then explain that the P.M. hours are those between noon and midnight, and let another child name these hours. Since midnight may be thought of as either the end of the old day or the beginning of the new one, and noon may be thought of as either the end of the morning or the beginning of the afternoon, have children avoid using A.M. and P.M. in distinguishing between noon and midnight. Tell them to say "Twelve noon" and "Twelve midnight."

Let someone tell why we need such expressions as "A.M." and "P.M." to distinguish time before noon from time after noon. Then you might ask, "Would it be easier to express time if the figures we use on the clock did not have to do double duty?" A question such as this could lead into a discussion of the 24-hour method of expressing time and the fact that four figures are used to express time in this system. Show the pupils how this method avoids misunderstanding, particularly at midnight, since 2400 is used to express the end of the old day, while 0000 is used to express the beginning of the new day—even though

both events are shown simultaneously on the face of a clock. After you have explained the 24-hour system, perhaps you might have the pupils make charts with information on them similar to this:

1 minute before midnight	2359
Midnight, end of old day	2400
Midnight, start of new day	0000
1 minute past midnight	0001
30 minutes past midnight	0030
59 minutes past midnight	0059
First hour after midnight	0100
10 hours after midnight (10 A.M.)	1000
1 minute before noon	1159
Noon	1200
1 minute past noon	1201

In explaining how "daylight-saving" time works, let the children think of a working day from 9 A.M. to 5 P.M. (Standard Time). Ask if it is light at 9 A.M. in summer. Ask when it begins to get dark. Explain that if the clocks are set ahead in summer, the span of hours from 9 A.M. to 5 P.M. occurs in *full* sunlight, since these "daylight-saving" hours are really the "standard" hours between 8 A.M. and 4 P.M.

Providing for the able pupil

Able pupils can look up the time zones for Hawaii and Alaska, and work out the time difference between cities in these places and various cities in the United States and Canada. They may also find the time differences between great cities of the world and key points in the United States and Canada. (Pan American Airways supplies a "time selector" device on request.)

Able children may enjoy an activity such as the following. Make a set of cards with the names of well-known cities in the United States and Canada on them. (If you use cities not shown on the map on page 198, include the name of the time zone.) To vary the activity, some cards can ask for "daylight-saving time" and others "standard time." Let each child draw five cards. The child then writes on paper the names of the cities on his cards and after each the time it is at that moment in that city, based on the time shown by the clock in the classroom.

To enrich understanding of the 24-hour method of expressing time, able children could practice expressing such times as 2:05 P.M., 7:50 A.M., etc., in four figures.

The able pupils can make a "Time Board" to show the correct time in important cities of the world, such as Washington, Honolulu, London, etc., when it is 1 P.M. in their own community. This time board can be made with real alarm clocks or with paper clocks with movable hands. Each clock should be labeled with the name of the city whose time it represents.

200 Thinking straight

Lesson Briefs for this lesson are on pages 183-184.

Expanded Notes other than provisions for able pupils are not considered necessary for this lesson.

Providing for the able pupil

There are many short cuts in computing that the able pupils should be able to understand and use. A few of these short cuts are described below. Discuss these with the children to give them a starting point for thinking up other methods.

To multiply a number by 25, simply multiply the number by 100 (that is, annex 2 zeros) and divide the result by 4. For 25×36: $100\times36=3600$; $3600\div4=900$. Ask the children why this short cut works [$25=100\div4$].

To multiply a number by 9 (or 99), multiply the number by 10 (or 100) and then subtract the original number from this result. For 9×85: $10\times85=850$; $850-85=765$; for 99×85: $100\times85=8500$; $8500-85=8415$. Why this short cut works should be obvious.

To multiply a number by 15, first multiply the number by 10 and then divide this result by 2. Next add this new number to the first answer. For 15×24: $10\times24=240$; $240\div2=120$; $240+120=360$.

Show children a procedure for multiplying any two-figure number by 11. You know your answer is most likely to have three figures. To multiply 45 by 11, think of the 4 and the 5 in 45 as the first and last figures in the 3-figure answer. Then find the sum of 4 and 5, the figures in 45. Use the sum, 9, as the middle figure of the answer. Thus $11\times45=495$. To multiply 87 by 11, think of the 8 and the 7 as the first and last figures in the 3-figure answer. Then find the sum of 8 and 7 to get the middle figure in the answer. Use the 5 from the sum 15 as the middle figure. The 1 of 15 must be added to 8, the first tentative figure in the answer. The answer is 957. Encourage the children to multiply 87 by 11 in the usual way and to use their work to explain why the "trick" works.

201-202 Checking up

Lesson Briefs for this lesson are on pages 185-186.

Expanded Notes are not considered necessary for this lesson.

203 Keeping skillful

Lesson Briefs for this lesson are on page 187.

Expanded Notes are not considered necessary for this lesson.

CHARTING THE COURSE

Estimating the quotient

One of the reasons that the process of long division is considered difficult is that it is continually necessary to estimate partial quotients. The method of teaching long division that is presented in the *Seeing Through Arithmetic* program not only emphasizes understanding the process as a whole, but also makes it unnecessary for the pupil to find the maximum quotient at each step before he can proceed to the next step. Then the pupil is shown gradually how to estimate the partial quotient figure so that the amount of work required appears to be shorter. Thus pages 77 and 91 of *Seeing Through Arithmetic 5* give some helpful suggestions about estimating the quotient, and pages 204-206 help the pupil to refine a technique for this purpose. Actually, the amount of work saved is less than it appears to be because much thinking and mental computation are expended on estimating the quotient. This fact provides an additional justification for the organization outlined above in which the technique of estimating the quotient is subordinated to more fundamental features of the complete process.

All techniques of estimating the quotient are based upon the "rounding off" of the dividend and the divisor, but this fact is not always made clear to the pupil. Now the "rounding off" of numbers is an important idea in its own right and has many uses other than estimating the quotient. For this reason, and also because an understanding of "rounding off" is essential for understanding the technique of estimating the quotient, *Seeing Through Arithmetic 5* explains a technique in terms of "rounding off." Formal rules such as those associated in the past with the techniques commonly called "the apparent quotient method" and the "increase-by-one method" are not taught in this book because they are not needed.

In *Seeing Through Arithmetic 5*, the pupil is first taught how to round numbers up and down. For example, 285, when rounded to the next higher hundred, is replaced by 300, and in this example 300 is also the nearer hundred. When rounded to the lower hundred, 285 is replaced by 200. With these ideas in mind, the pupil is shown how an example such as $180 \div 30$ may be replaced by $18 \div 3$. He is then in a position to understand the technique of estimating the quotient in a division example such as $64{,}895 \div 871$.

In *Seeing Through Arithmetic 5*, the pupil is taught to round the divisor upward. In the example above, 871 is mentally replaced by 900. As in all techniques in common use, the dividend is rounded down. Thus, the dividend 64,895 is replaced by 63,000 because this number is divisible by 900. These replacements reduce the problem to an easier one whose quotient can be found mentally, and this quotient is then used to carry on the division process with the original numbers by the methods taught earlier.

This technique has the following features in its favor. First, it yields the true quotient more frequently than the techniques of rounding the divisor down. Second, if the estimated quotient is not the true quotient, it is always smaller than the true quotient. Consequently, no erasures are necessary and division computation by the method taught in this book can proceed without interruption. Third, this technique avoids the necessity of teaching certain auxiliary "rules" that must be applied in special cases that arise when other techniques are used. It is, therefore, what is often referred to as a "one-rule" technique. It is, in fact, more genuinely deserving of this designation than other techniques because the auxiliary rules mentioned above are not necessary.

Research and experience show that in actual practice children and adults use a wide variety of techniques for estimating the quotient. The authors of *Seeing Through Arithmetic 5* do not expect children in the earlier stages of learning long division to master an efficient or "mature" technique. Instead, the primary objective is a good understanding to which skill may gradually be added. Techniques of estimating quotients will be further developed in Book 6. The objective is to enable the child to be resourceful.

204 Thinking straight

Lesson Briefs for this lesson ore on pages 188-189.

Expanded Notes are not considered necessary for this lesson. See "Charting the Course" on page 334.

205-206 Thinking straight

Lesson Briefs for this lesson are on pages 189-191.

Expanded Notes are not considered necessary for this lesson.

207-208 Using arithmetic

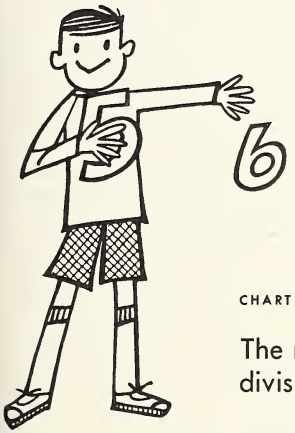
Lesson Briefs for this lesson ore on pages 191-192.

Expanded Notes are not considered necessary for this lesson.

209 Keeping skillful

Lesson Briefs for this lesson ore on poge 193.

Expanded Notes are not considered necessary for this lesson.



CHARTING THE COURSE

The remainder in division

The remainder in a division example can always be used to form a fraction, but the number so obtained is not always meaningful. In *Seeing Through Arithmetic 5,* the interpretation of the remainder is carefully taught to help pupils avoid answers which are not sensible. Pages 105 and 106 are devoted to examples in which the remainder can in some cases be used to round off the answer to a whole number. Pages 210-214 show pupils how the remainder may in other situations be used to form a fraction, and they help him to develop judgment as to when this is or is not a sensible thing to do.

The way a remainder is used depends solely upon the problem situation from which the need for a division comes. If the situation involves objects that can be partitioned into fractional parts, the remainder is often used to form a fraction. Two boys may share 11 candy bars equally if each takes $5\frac{1}{2}$ bars. However, if three girls have 16 balloons, it is not sensible to share them equally; to offer $5\frac{1}{3}$ balloons as an answer to such a problem reveals a lack of attention to and understanding of the situation. The common practice of instructing children, as soon as they have studied fractions, to use the remainder to form a fraction is not good teaching. Instead, children should be taught to treat the remainder in ways appropriate to the situation and in common-sense fashion.

Teachers should be aware of the fact that competent writers sometimes use remainders in ways that are not easily interpreted. Averages are an example, and one may see in print statements such as "the average number of persons was 91.3." A common-sense interpretation would demand that this answer be expressed as 91. However, these exceptions to "common-sense" expressions that arise in statistical and similar applications need not be discussed with children at this stage of their learning.

210-212 Moving forward

Lesson Briefs for this lesson are on pages 194-196.

Expanded Notes are not considered necessary for this lesson.

Overview

Be sure to read the "Charting the Course," which precedes, so that all of the implications of this lesson are understood. Note that the thinking which the child must do in making the fraction in the quotient differs for the two divisive situations. In one, page 210, the object is divided. In the other, page 211, remaining objects form a fraction of a unit.

213-214 Exploring problems

Lesson Briefs for this lesson are on pages 196-197.

Expanded Notes are not considered necessary for this lesson.

CHARTING THE COURSE

Rate and comparison situations

Among the problems in which arithmetic is used perhaps no types are more important than those that involve rate. For example, suppose candy bars are priced at 10¢ for 3 bars. Kathy wants to buy 6 bars. How much will she have to pay for 6 bars? In this example the rate, which is 10¢ per 3 bars, is given. The problem calls for finding the cost of 6 bars at this known rate. Teachers should recall that the meaning of rate and problems in which the rate was to be expressed in symbols were introduced earlier on pages 112-118 of *Seeing Through Arithmetic 5*. Pages 215-220 show pupils how to deal with situations in which the rate is known, but is to be expressed in different terms.

Traditionally, problems such as the example above have usually been taught as "multiplication problems." One common instructional procedure (the "method of unitary analysis") is to tell the pupil something like the following: "First, find the cost of 1 candy bar. Divide 10 by 3. One candy bar would cost $3\frac{1}{3}$¢. To find the cost of 6 candy bars, multiply $3\frac{1}{3}$ by 6. Then $6\times3\frac{1}{3}¢=20¢$, which is the cost of 6 candy bars."

Another procedure is to tell the pupil something like the following: "The candy is priced at 10¢ for 3 bars. Kathy wants 6 bars; $6\div3=2$, so Kathy wants to buy two times as many. Then $2\times10¢=20¢$, which is the cost of 6 bars."

It should be noted that these instructional procedures are highly verbal and consist principally of giving a set of rules. After solving many examples of this type in one of these ways, most pupils become able to deal with such problems when they arise. However, usually pupils do not understand the basic principle involved or recognize that its generality extends far beyond problems of buying.

In the example above, and others of the same general type, the idea of rate is basic. The rate is known and is expressed by writing a pair of numerals as a ratio. The problem requires this same rate to be expressed by another pair of numerals, and one of these is also known. The other member of the pair is not known, and must be found. In general, two "steps" are required. In one of them the process of division is used. In the other, the process of multiplication is used.

Reference again to the specific example above, and to pages 215-220 of *Seeing Through Arithmetic 5*, will make this clear. The rate in the example is expressed by the ratio 10/3. In the new form, the cost term is at first unknown and may be temporarily represented by n. The number of bars is 6, so the new ratio is $n/6$. Both ratios must express the same rate, so $10/3=n/6$. Here, alternative procedures may be used to find n.

One procedure that is valid mathematically but is too advanced for use at this grade level, is to replace the ratio 10/3 by an equivalent ratio in which the second term is 1. To accomplish this, both 10 and 3 may be divided by 3, and the result is the ratio $3\frac{1}{3}/1$. Then $3\frac{1}{3}/1=n/6$, and to find n, the ratio $3\frac{1}{3}/1$ is replaced by an equivalent ratio in which the denominator is 6. To accomplish this, both $3\frac{1}{3}$ and 1 may be multiplied by 6, and the result is the ratio 20/6. Therefore, $n=20$. This is the basis of the "method of unitary analysis."

It can hardly be overemphasized that the traditional presentation of this method consists of a sequence of rules for short cuts. The apparently greater simplicity of such "explanations" is a consequence of their failure really to explain the procedure in a fundamental way. The truth of this statement is likely to be more apparent to pupils than to teachers who, through long experience with the short-cut explanation, have come to believe they understand it.

The second procedure is shorter. It makes no attempt to replace the ratio 10/3 by an equivalent ratio in which the second term is 1. This step is unnecessary. Instead, since $10/3=n/6$, the idea is merely to replace 10/3 by an equivalent ratio whose second term is 6. Both the first term 10 and the second term 3 must be multiplied by the same number. This number is found by dividing 6 by 3. When the result, 2, is used as a multiplier, the equivalent ratio is 20/6, which shows that $n=20$. This is the basis of the "2 times as many" short cut.

Seeing Through Arithmetic 5 uses this second procedure and provides a full explanation of it so that children can understand. The authors believe that "short cuts" and verbal rules are not conducive to the development of understanding, and these have been avoided throughout the initial presentations.

Similar procedures are followed when the problem situation involves a comparison. The solution of problems of this type is explained in detail on pages 221-226 of *Seeing Through Arithmetic 5*. For example, suppose "John earns 25¢ a day, and Bob earns 15¢ a day. When John has earned $1.25, how much will Bob have earned?" (See page 221.) In this example the ratio 25/15 expresses the comparison of the amount John earns in a day with the amount Bob earns in a day. This ratio must be replaced by one in which the first term is 125 and the second term is unknown, or $125/n$. As in the rate problems, a multiplier must be found by division, and in this example, it is 5. Then $5\times25=125$ and $5\times15=75$, so the new ratio is 125/75. This shows that $n=75$, which is the amount Bob will have earned. The solution consists of expressing the same comparison with different numerals.

Finally, these procedures also apply to problems of a type which has long been included in arithmetic programs. The following is an example. "Sally found 16 shells, and Kathy found 8 shells. Sally found how many times as many shells as Kathy?" (See page 227 of *Seeing Through Arithmetic 5*.) Basically, this is not a different type of problem, but "times as many" is a form of verbal expression associated with the use of a particular ratio to express a comparison—namely, a ratio whose second term is 1. In the example, the ratio 16/8 expresses the comparison of the number of Sally's shells with the number of Kathy's shells. This ratio may be replaced by another whose second term is 1. Temporarily, this new ratio may be expressed as $n/1$. Since $16/8=n/1$, the problem can be solved by dividing both 16 and 8 by 8. This yields the equivalent ratio 2/1, which shows that $n=2$. The new ratio means that Sally found 2 shells for each shell that Kathy found. It has become customary to state this in a modified form by saying "Sally found 2 times as many shells as Kathy." Pages 227-230 of *Seeing Through Arithmetic 5* provide experiences to help pupils understand and learn to solve such problems.

It is important to note that the methods outlined above serve to unify or bring under one fundamental procedure several types of problems that have usually been taught separately. Although the details differ from example to example, the basic ideas are the same throughout. Moreover, these same basic ideas will be essential for work in later grades when problems requiring the multiplication or division of fractions are taught. The methods used in *Seeing Through Arithmetic 5* are not chosen because they may seem novel or different to those familiar with traditional arithmetic instruction. On the contrary they are chosen and presented solely because they are fundamental and in spite of their novelty at this level.

The introduction of these fundamental ideas at the points where they are first needed will keep arithmetic from becoming merely a collection of short cuts learned by drill methods.

215-220 Exploring problems

Lesson Briefs for this lesson are on pages 198-201.

Overview

The concept of rate may be applied to many kinds of problems in the arithmetic of everyday life. Hence the writing of ratios to express rates and then using these ratios to solve problems is a very useful mathematical technique. But all too frequently children have been given little or no help in learning to solve rate problems, except that they have been taught to treat these problems in general as multiple-step problems. They have rarely been given a mathematical procedure that applies to all rate situations.

It should be noted that in all rate problems, two ratios, both expressing the same rate, are involved. Of the four terms in these two ratios, one term is unknown, and its value must be found by computation. Examination of pages 215-220 in the pupils' book will show how equal ratios are written to describe a problem situation and how the computation for finding the value of n is performed. Since the children have already had experience in writing ratios to express rates and in finding the missing term in a pair of equal ratios, the only really new thing in this lesson is the writing of the proper equation. (Note that equations of the type used in these situations [$10/3=n/6$] are frequently referred to as "proportions.")

Keep in mind (and emphasize throughout the teaching) that it does not matter which of the two numbers involved in a rate has its numeral used as the first term in the ratio. However, once the first ratio

has been established, then the relationship determined by the position of the terms must be maintained when the second ratio is written.

Teaching the whole class

Page 215: Before proceeding with the work on this page, you may wish to review briefly the changing of ratios to equivalent forms. Write a ratio such as 3/4 on the board. Ask the children to think of this ratio as expressing a rate and to give concrete examples of such a rate (3¢ for 4 lollipops, for example). Then write the equation $3/4=6/n$ on the board. Get someone to explain what $6/n$ expresses [a ratio equal to the ratio 3/4, but with one term unknown]. Let someone explain how to find the missing numeral. Then interpret the completed equation in terms of the original problem. Thus, $3/4=6/8$ can be interpreted as meaning that, at the rate of 3¢ for 4 lollipops, 6¢ would buy 8 lollipops or that the rate of 3¢ per 4 lollipops is the same as 6¢ per 8 lollipops.

Now direct attention to the problem and Picture A. First get the children to state the rate. Then have them explain what the 10 and the 3 stand for in the ratio 10/3. Get them to see that they must find another ratio to express this rate and that 6 must be a term of this new ratio.

Direct attention to the text opposite Picture B. Get the children to see that in the new ratio the cost (as yet unknown) is written *above* the 6, which stands for the number of bars. They should understand that the original rate could have been expressed as the ratio 3/10, but that then the new equal ratio would have to be $6/n$.

Emphasize that when the numeral that replaces n is known, it will be a numeral that will make $n/6$ equal to 10/3.

Page 216: Point out in Picture C the 2 groups of 3 bars that make the 6 bars Kathy wants to buy. In the equation at the right it can be seen that 3 can be multiplied by 2 to get 6. Now get the children to see how Picture D, the accompanying equation, and the text show that since 3 can be multiplied by 2 to get 6, the upper term in 10/3 must also be multiplied by 2 to find the numeral that replaces n. Be sure the children realize that the 20 in the new ratio represents 20¢, just as the 10 in 10/3 represents 10¢.

Play money and objects representing candy bars can be used to demonstrate this situation and its solution. A pupil should match a group of 10 pennies to each group of 3 candy bars. As an additional activity, you might ask someone to find out how much 9 candy bars would cost, at the rate of 3 bars for 10¢, again using play money and objects. When the answer has been found, ask another child to write an equation that describes the situation on the board and then tell how he would find the numeral that replaces n.

The problem in the second "See" step, at the bottom of the page, also involves a rate. Now, however, the children are to find the cost when the number of bars of soap is fewer than the number given in the original ratio. Have a child relate Picture A to the problem and to the ratio 60/8.

Page 217: Discuss the new ratio with the children. Since in the original ratio the cost is the upper term, in the equivalent ratio the cost must again be the upper term, and the 2, which stands for the number of bars is written below the line.

Get the children to explain that the equation $60/8=n/2$ is correct for this problem because the two ratios express the same rate.

Next discuss the solution of the equation in the "Compute" step. Point out that since the problem asked for the cost of 2 bars, it will be helpful to think of the 8 bars as divided into 4 groups of 2 bars. And, since the rate for 2 bars is the same as the rate for 8 bars, it is possible to think of the cost as also divided by 4. Picture D shows that 60¢ can be divided into 4 equal groups of 15¢ and that each 15¢ can be matched to one of the four groups of bars. In the equation, discuss the fact that since the 2 is found by dividing 8 by 4, the 60 also must be divided by 4, and thus $n=15$. The children should interpret this 15 as 15¢ and realize that the rate of 8 bars for 60¢ is the same rate as 2 bars for 15¢. In other words, the cost of 2 bars is 15¢.

A pupil may demonstrate the solution of this problem with play money and objects. You can also let an able pupil find the cost of 4 bars of soap at the rate of 8 bars for 60¢. He should divide the objects into 2 groups of 4 bars. Then he should separate the money into 2 equal groups to find that 4 bars would cost 30¢.

Page 218: Have the problem at the top of this page read and Picture A examined. The children should be able to explain why 19 and 57 are the upper terms of the equal ratios and 2 and n are the lower terms.

Picture B shows that it takes three groups of 19¢ to use up the total 57¢. Then, by matching a group of 2 grapefruit to each group of 19¢, the total number of grapefruit bought (6) is found. Since 19 can be multiplied by 3 to get 57 in the equation, the 2 has to be multiplied by 3, also.

In the problem at the bottom of the page, let the children answer the questions in the text about the equation. Ask the class how they know that fewer cans can be bought for 38¢ than for 76¢, if the rate remains the same.

Page 219: Get the pupils to explain why, in Picture B, the 76¢ is separated into groups of 38¢. They should be able to infer now that the cans have to be separated into 2 equal groups and that there will be 3 cans in each group. Discuss the corresponding computation that must be performed to find the value of n in the equation.

Each of the remaining problems on this page should be discussed. Be sure that the pupils can explain what each numeral or letter stands for in each equation. They should be able to justify their choice of a multiplier or divisor in each case. Have them express each answer as a rate (325 mi. per 10 hr., for example). In Problem D be sure the pupils understand that the 140 in the original ratio means 140 cents and that the answer, 35, represents cents.

Page 220: Two equations are shown for each of the first four problems on this page. Discuss with the children why either of the two equations can be used, but stress that in each case the arrangement of the data in the second ratio must correspond to the arrangement used for the first ratio. Let children take turns explaining each equation and finding the missing term.

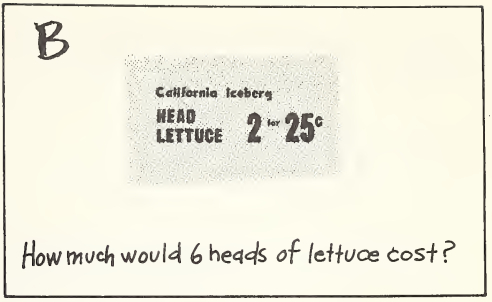
Assign Problems I to O as written work for all the children. (Problem P should be assigned only to the more able children.) Provide answers and let the pupils verify their work. Give individual help as needed and discuss with the class those problems that caused special trouble.

Providing for the able pupil

Able pupils may be asked to bring from home newspaper advertisements in which merchandise is offered for sale at a rate of more than 1 item for a given price ("2 heads of lettuce for 25¢," for example). Such ads can be cut out and pasted on 3″ x 5″ cards and problems made up about them. Each card should be lettered for identification. Then the pupils may exchange cards and solve one another's problems. However, be sure to examine the cards before they are used to make sure that no question has been asked that involves working with a fraction or that involves finding how much one item will cost.

Encourage the pupils to use the full set of four types of questions: "How much will 6 cost (at 2 for 25¢)?" "How much will 2 cost (at 6 for 75¢)?" "How many can I buy for $1.00 (at 2 for 25¢)?" "How many can I buy for 25¢ (at 6 for 75¢)?" A sample card is shown at the right above.

Instruct the children to write on their papers the letter that identifies the card, to read the ad and the accompanying question, and then to write the equation and solve it.



Helping the slow learner

Give the slow learners many opportunities to use objects in situations involving rates and then finding equivalent ratios to express the same rates. For such activities you will need play money and various objects, with 10 to 15 of each kind.

Write on the chalkboard a phrase to describe a rate and ask two children to show with objects the rate expressed. For example, if you wrote "8 pieces of candy for 10¢," they should show the following with objects:

Then ask pupils to show this rate in another way with objects. For example, ask them to show the number of candies that could be bought for 5¢, or for 20¢. They should express each rate as a ratio on the chalkboard. An able pupil might supervise the work.

The slow learners may also need more work in replacing ratios by equivalent ratios, to help them think more clearly about such changes when they are involved in a problem situation. For this type of exercise write several sequences of ratios on the board. Each ratio should be easy to derive from the ratio at the left of it. For example, you might write:

$$\frac{4}{6} = \frac{n}{3} = \frac{8}{n} = \frac{24}{n}$$

$$\frac{2}{10} = \frac{6}{n} = \frac{24}{n} = \frac{n}{60}$$

Let the children explain what they must do to find the value of n and why. Then have them erase the n in each case and write the numeral to replace n.

221-226 Exploring problems

Lesson Briefs for this lesson are on pages 202-205.

Overview

In this lesson the children learn how to make equations for problems involving comparisons, and they learn how to solve these equations. Exactly the same method is used as was used for problems involving rates. If the children know how to solve problems involving rate, they should have no trouble with the solution of problems involving comparison. The use of play money and other objects for illustrating comparison situations will be most helpful, and such items should be collected or prepared beforehand.

Teaching the whole class

Page 221: First review the idea of comparison and its expression with ratio. If necessary, refer back to pages 119 to 123. Then write a ratio such as 4/8 on the board. Ask the pupils to make some comparisons using 4 and 8. Then tell them that in this lesson they will learn how to solve problems that involve comparison. Next have them turn to the problem on page 221.

First of all they must see that this is a comparison situation, and that the earnings of two boys are being compared. Let them then relate the coins in Picture A to the boys' earnings in one day. Also have them relate the figures in the ratio to the amounts mentioned in the problem and to the coins in Picture A. Be sure to get the children to read the ratio as "25 to 15" and to note that the numeral representing John's earnings is above the line and that the numeral for Bob's is below the line.

The next step is to make the pupils realize that they need to make another ratio equal to this ratio. Get them to see that the problem gives an indication of this new ratio. (The final total amount that John earned, \$1.25, is given, and, as they learned previously, this amount must be written in the same position in the new ratio as the numeral for John's earnings was written in the original ratio. This amount becomes the upper term of the new ratio.) Point out that the pupils should think of \$1.25 as 125 cents or pennies and write it as 125 in the ratio. Lead the pupils to see that since they do not know the total amount that Bob will have earned, they can write n as the lower term of the new ratio. Then discuss how to write the two ratios in an equation, as shown at the bottom of the page. Let a pupil explain each symbol in the equation, particularly explaining the position of 125 and n in the second ratio.

Page 222: Point out to the pupils that John earns 25¢ a day and that he earned a total of \$1.25. Ask the children to look at Picture C to see how many quarters it takes to equal \$1.25. Then have them look at the equation and figure out how 125 can be obtained from the 25 [by multiplying 25 by 5].

Next have the children recall that for every 25¢ John earned, Bob earned 15¢. Get them to see that Picture D shows 15¢ matched with each 25¢. 5 groups of 15¢ are needed. Looking at the equation, they know they multiplied 25 by 5 to get 125. So the lower term, 15, must also be multiplied by 5. Then the numeral 75 should replace n in the second ratio. Remind the pupils that 75 means \$.75. Have them count the value of the 5 groups of 15¢ in Picture D to see if the total equals \$.75.

For slow learners, this problem may be demonstrated with play money. Ask a pupil to set up the basic comparison of 25¢ to 15¢ as in Picture A on page 221. Then tell him to lay down enough additional quarters to make \$1.25. Since Bob earns 15¢ every time John earns 25¢, tell the pupil to match 15¢ with each quarter. It will now be clear that \$.75 is the total amount that Bob will have earned. Have the children note that 5 quarters and 5 amounts of 15¢ were laid down. Ask them to multiply 25¢ by 5 and 15¢ by 5 to see that the total amounts can be obtained this way, too. Then ask someone to write the equation on the board and to explain what must be done to find the numeral to replace n.

The new problem at the bottom of the page may be handled in much the same way. Get the pupils to see that the problem states a comparison between the number of quarts that Tony picked and the number Bill picked. Then help them understand that they are to express this same relationship with smaller numbers, one of which is not given in the problem. Discuss the ratio 12/8 and relate it to Picture A.

Page 223: Discuss the making of the equal ratio and the equation as you did for the previous problem. Since the children must now compare the number of quarts that Tony picked with each 2 quarts that Bill picked, help them see that first of all Bill's 8 quarts should be separated or divided into groups of 2. When they find the number of groups of 2 quarts, they can divide Tony's 12 quarts into the same number of groups for matching purposes and find how many are in each group. Let someone work this out with objects representing the quarts of berries. Also discuss how the same answer is arrived at by division

Page 224: In discussing the problem at the top of the page, be sure the pupils understand what groups are being compared, and that the second ratio expresses the same comparison as the first ratio. Point out that since the second ratio indicates larger groups than the first ratio, multiplication is indicated. You may wish to demonstrate the solution of this problem.

In discussing the problem and equation at the bottom of the page, help the children see that the terms of the second ratio must be the result of division, since the numerals represent smaller numbers.

Page 225: In completing the solution of the problem that was begun on page 224, have a child work with objects to duplicate the actions indicated by Picture B and the text.

The remaining problems on this page should be discussed with the class. Be sure the children can explain the position and the meaning of each symbol in each equation. Let children take turns finding each missing term. Wherever possible, let the children use objects to dramatize the problems and the solutions.

Page 226: Treat the first four problems on this page as you did the last four on the preceding page. However, point out that for each problem two equations have been written to describe the situation. Remind the pupils that they have already discovered that it does not matter which quantity is expressed in the upper term of the first ratio, provided it is expressed in the same position in the second ratio. Have each pair of equations solved at the chalkboard and explained by the pupils. Assign Problems I to N as written work.

Providing for the able pupil

Pupils of this grade are usually interested in collections, hobbies, making money, etc. Any of these activities can be the basis of problems on comparisons created by the able pupils. Such original problems may be used in several ways:

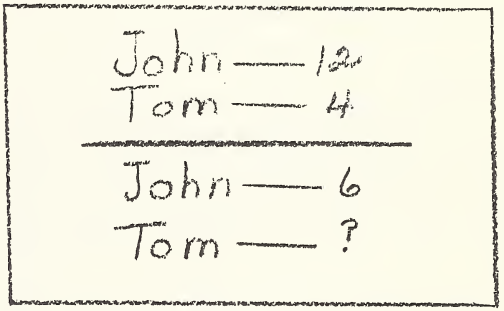
(1) You may type them on 5″ x 3″ cards, to be used by the children in the way suggested in Activity 14, page 362. Prescreening allows you to remove any problems of this type whose solution involves fractions.

(2) They may be pictured (as in the text, pages 224-225) by pupils to show comparisons between two groups of objects. Such drawings may be displayed on the bulletin board, or collected by the children in individual notebooks. On each drawing or notebook page the children should write the problem and write the equation, first with the letter n in the proper place, and then with the proper numeral inserted.

Helping the slow learner

Slow learners usually need many opportunities to work with objects in comparative situations. The activity described below will give them experiences in solving comparative problem situations.

On 3″ x 5″ cards write information as shown. Explain to the pupils that the set of numerals above the line shows how many objects each of a pair of children has now and that the set below the line is to show how many they will have at a later date. Explain that the relationship must be kept the same and that they are to find the missing numeral.



Provide a large group of objects (buttons will do). Place these objects on a desk or table in a central location. Let the children pair off and allow one of the children of each pair to draw a card. He is Pupil 1 (John on the card). Tell him to take as many objects as the card indicates above the line (12). His partner is designated as Pupil 2 (Tom on the card). Pupil 2 should take as many objects as the card indicates (4). Pupil 1 should put his objects in a row on the desk, and Pupil 2 should put his objects in a row *beneath* them. Ask Pupil 1 to compare the two groups he now sees. He should say, "I have 12 objects to Tom's 4." Direct Pupil 1 to look at the numerals below the line and state the comparison in terms of these numerals, as, "I shall have 6 objects to how many of Tom's?" Have him take 6 more objects and place them in a group beside his original group of 12. Then Pupil 2 must take the correct number of objects (2) and put them in a group beside his original 4. He can use any means he wants to find how many he should take. Suggest that first he write on the chalkboard the equation that describes the situation [$12/4=6/n$]. Then if he wants, he can work with the objects. He can first divide his partner's 12 objects so there are 6 in a group. This way he finds there are 2 groups of 6. Then he can divide his 4 objects into 2 groups to match the 2 groups of 6. He finds there are 2 in each of his 2 groups. Thus he sees

that his partner has 6 objects to his 2. He should take 2 more objects and place them beneath the 6 of his partner's. Or he can work out the solution computationally and then take the number of objects he finds and put them beside his first group of 4. He should state the new ratio as 6 to 2.

On the cards, you should vary the position of the missing numeral in the set of numerals below the line. For example, you might write "Sally—5" and "Ruth —10" above the line; "Sally—?" and "Ruth—20" below the line.

227-230 Exploring problems

Lesson Briefs for this lesson are on pages 205-208.

Overview

Problems involving "times as many" are really a special kind of comparison situation in which the ratio showing the comparison between two differently sized groups is changed to show what the comparison would be when one of the groups consists of only one object. Traditionally, this type of problem has been solved directly by multiplication or division—a procedure that is really a short cut made possible because the second term of one of the ratios is the numeral 1. (See the discussion under "Comments" in the Lesson Briefs, pages 205-206, and the "Charting the Course," page 337.)

This lesson shows the children how to solve such problems by writing an equation based on two equal ratios in which one term of one ratio is the numeral 1. Then the children learn that the equation is solved by exactly the same procedures as were used for solving equations in the preceding lesson.

Teaching the whole class

By the time the children have reached fifth grade, they have had opportunities to hear and to use expressions such as, "two times as many," "four times as many," "twice as much," and so on. Some of them will have a very clear idea of what is meant. Others will not. Before the work on page 227 is begun, it will be well to discuss the meanings of these expressions. Have available two kinds of objects, such as black buttons and white buttons. On a table place 1 white button and 2 black buttons. Ask a pupil to tell what ratio expresses the comparison of black buttons to white buttons [2 to 1]. Write this ratio on the chalkboard in the usual way [2/1]. Then explain to the pupils that the statement "There are two times as many black buttons as white buttons" also expresses the same comparison. Next have a pupil show the ratio 3 to 1 with the buttons. Ask him to give this same comparison using the expression "times as many." Guide him to say, "Three times as many black buttons as white buttons." Then ask another pupil to show with the buttons "4 times as many black buttons as white buttons." Ask him both to write and to read this as a ratio [4/1 and 4 to 1]. Point out that the comparison is always with 1. Conclude by asking the class to give examples of other things that can be expressed using the phrase "times as many." For example, "There are 3 times as many blue balloons as green balloons." "There are four times as many yellow pencils as red pencils." Then the pupils should be ready for the work on page 227.

Page 227: To begin this lesson, have the pupils read Problem A. Have them note the now familiar term "times as many." Then go on somewhat as follows:

> *Does the problem tell us how many times as many shells Sally has as Kathy? It doesn't. But we have seen that the expression "times as many" is used when we make a comparison based on* one, *haven't we? Jane, read the first statement below the problem. When we know how many shells Sally found for each shell Kathy found, we will know how many times as many Sally found. Do you see any information in the problem that we can use to compare Sally's shells with Kathy's shells? Can you state this comparison by using a ratio?* [*Discuss the suggested ratios.*]

Then have the pupils read in the book to see how this ratio is written and also to look at Picture A to see the ratio illustrated. Suggest that they replace this ratio by an equivalent ratio in which one of the terms is 1. Remind the pupils that this second ratio will show how many shells Sally found for each one that Kathy found. Discuss the fact that the numeral representing Sally's shells (16) is used as the upper term of the first ratio in the book (16/8). Ask the pupils whose shells should be represented as the upper term of the new ratio. Since they do not know the number, they must use the letter *n*. Point out that the numeral representing Kathy's shells is used as the lower term of the first ratio. Since they want the new ratio to show a comparison with 1 of Kathy's shells, the numeral 1 should be used as the lower term of the new ratio. Have the pupils look at this ratio at the bottom of Column 1.

Next have the pupils look at the two ratios written in an equation, as shown at the top of Column 2. Point out that this equation is just like those in the preceding lesson. Have the pupils discuss how the 1 in the new ratio can be found [By dividing 8 by 8]. Ask them if 16 should also be divided by this number (8). Have someone divide 16 by 8.

Point out that the numeral 2 can replace the letter *n* in the new ratio. Have the pupils see that this new ratio is illustrated in Picture B. Then remind them that there is another way to describe the situation when the ratio is 2 to 1. Have them find this statement in the book (2 times as many).

Page 228: Problem B on this page is somewhat different from the preceding problem. Have the pupils read it and note that they are told immediately how many times as many buns Ruth bought as Ann. The children should know that "3 times as many" means there is a comparison of 3 to 1. Ask them to look at Picture C where they can see Ruth's buns laid out to show 3 of hers to 1 of Ann's. Have the pupils note in the equation that describes the problem situation that the ratio they know, 3/1, is written first. Since the numeral representing Ruth's buns is used as the upper term of the first ratio, a numeral representing Ruth's buns must also be used as the upper term of the second ratio. Since the number of buns Ruth bought is unknown, the letter *n* is written in place of it. The numeral 12, representing Ann's buns, is written as the lower term. Let the pupils look at the equation and determine the relationship between 1 and 12 in the two ratios (1 can be multiplied by 12 to get 12). Ask the pupils what 3 should be multiplied by [12]. 36 can then replace the letter *n*. So the ratio 36 to 12 means that Ruth bought 36 buns to Ann's 12.

Problem C can be discussed in the same way. Be sure the children understand why the known ratio is 4/1 and why the ratio $20/n$ is equal to 4/1.

Page 229: Discuss the fact that before the children can find the value of *n*, they must first find how 20 can be obtained by using 4. They should understand that 4 can be multiplied by 5 to get 20 and that this 5 is most easily found by dividing 20 by 4. Since the 1 in the first ratio is multiplied by 5 to get the value of *n*, simply dividing 20 by 4 accomplishes the same result. Undoubtedly the able pupils will sense this, but keep their attention focused on multiplying both terms of the first ratio by the same number (5).

Discuss Problems D, E, and F in much the same way as you discussed Problems A, B, and C. Since there are no pictures for Problems D to F, the children, especially the slow learners, will benefit from using objects to illustrate the problem situations and find the answers.

Able pupils will undoubtedly see that the answer to Problem D can be found by directly multiplying 18 by 5. This short cut, although useful and traditional, does, however, obscure the meaning of the basic comparison in the problem, that Jim has 5 pennies to Don's 1 penny. Insist, therefore, that all pupils write an equation of two ratios for all "times as many" problems. The answers to Problems E and F may, of course, also be found without the help of ratios, but the short cut again hinders the understanding of the comparison situations.

Page 230: Continue in the same way with the first three problems. Point out in Problem J that the expression "twice as much" means the same as "2 times as much." Let the pupils give other examples of situations where they can say "twice as much."

Then assign the remaining problems as written work. Treat them in the same way as you did comparison and rate problems in the preceding lessons.

Providing for the able pupil

If your able pupils made up original problems using comparison situations (as suggested in "Providing for the able pupil" in the preceding lesson), let them now make up problems using the "times as many" situation. These problems can be put on cards and included with the collection described in Activity 14, page 362, or they can be illustrated, as suggested in the preceding lesson in the section for the able pupils (page 341).

Helping the slow learner

Let the slow learners have opportunities to work with objects to demonstrate the three types of "times as many" problem situations (as illustrated by Problems A, B, and C on pages 227-228).

For this activity have two boxes, each with a different kind of object in it. One box might have 50 corks in it, the other 65 buttons. Let pairs of children work together. They will probably find the type of situation illustrated by Problem B, on page 228 of the text ($3/1 = n/12$) the easiest to start with. You might say:

Jane, take 12 corks. Bob, you are to pick out 3 times as many buttons. What ratio is used to show 3 times as many? [*3/1*] *For every 1 cork of Jane's how many buttons should you draw?* [3]. [*Have the children do the drawing and matching.*] *Now, Bob, how many buttons have you? Henry, write the equation on the board and find the numeral to replace* n.

For another situation you can say:

Don, take 24 corks. Peggy, take 48 buttons. Peggy, you have how many times as many buttons as Don has corks?

Remind Peggy that she is to find how many buttons she has to each 1 of Don's corks. Have Don lay out his 24 corks. Then Peggy can divide her buttons equally into 24 groups to find the number in each group that matches each 1 of Don's. Also have the children solve this situation by use of an equation.

For the third situation, try the following:
Bill, draw 35 corks. If I say that you should have 5 times as many corks as John has buttons, how many buttons should John draw? What is the ratio that shows 5 times as many? [5/1]

Then tell Bill to arrange his corks so that he has 5 in a group. Then John can match 1 button to each group of 5 to find the number he can have. Let the children also solve the problem by an equation.

231 Keeping skillful

Lesson Briefs for this lesson ore on poges 208-209.

Expanded Notes are not considered necessary for this lesson.

CHARTING THE COURSE

Scale drawings and graphs

Scale drawings are one of the simplest of the applications of arithmetic and are also very frequently met. The ability to interpret scale drawings is far more important than the ability to make them. Pages 232 and 233 in *Seeing Through Arithmetic 5* give pupils experience in interpreting simple scale drawings and maps. The idea of a scale was introduced in *Seeing Through Arithmetic 4.*

The subdivisions on the scale of a scale drawing represent units of length, and, basically, scale drawings involve a rate situation in which lengths on the drawing (a map, for example) correspond to lengths on the object (which in the case of a map is usually a portion of the earth's surface). A ratio is involved, but when the scale is shown, it is not necessary to know what the ratio is. The scale may be used on the drawing to "measure" the lengths on the object indirectly.

Simple bar graphs and pictographs are now commonplace, and it has become customary to give some instruction in intrepreting them as early as the fifth grade. *Seeing Through Arithmetic 5* provides for such instruction on pages 234-237. It should be observed that these graphs provide another illustration of the use of scales. Experience with the construction of bar graphs and scale drawings is deferred until a later grade because concepts not as yet developed are involved.

232-233 Moving forward

Lesson Briefs for this lesson ore on poges 210-211.

Expanded Notes are not considered necessary for this lesson. See "Charting the Course" on this page.

234-237 Moving forward

Lesson Briefs for this lesson are on pages 212-215.

Expanded Notes are not considered necessary for this lesson. See "Charting the Course" on this page.

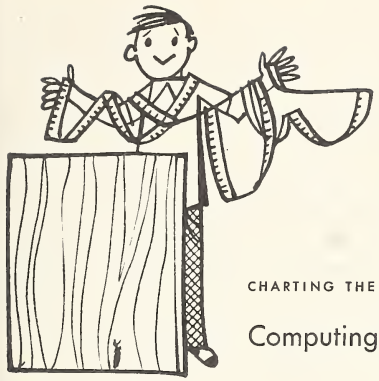
238-239 Thinking straight

Lesson Briefs for this lesson are on pages 215-217.

Expanded Notes other than suggestions for the able pupils are considered unnecessary for this lesson.

Providing for the able pupil

Supply (or have the children make) one-inch squares. Then tell them to use a given number of these squares (36, for example) to make as many different rectangles as they can. Make clear that they must use *all* the squares for each rectangle. Encourage the children to make a complete set of rectangles for the area you have specified. (For example, if the children are using 36 one-inch squares, they can make rectangles with these dimensions: 1″ x 36″; 2″ x 18″; 3″ x 12″; 4″ x 9″; 6″ x 6″.) When a rectangle has been completed, its dimensions and perimeter should be recorded. Now tell the children to examine their records and decide which dimensions give the longest perimeter and which give the shortest. By questions get them to observe that rectangles with the same area may have different perimeters and that the perimeter becomes shorter as the shape approaches a square. The 6″ x 6″ square has the shortest perimeter of all the rectangles that can be made with the 36 one-inch squares.



CHARTING THE COURSE

Computing area

The teaching of area has suffered in the past from two common faults. First, too little attention has been given to the *concept* of area. The tendency is to rush on to the computation of areas. Second, the computation of areas has depended too much on brief verbal rules which state a short-cut procedure. The fundamental reason the short-cut rules work is rarely made really clear. *Seeing Through Arithmetic 5* overcomes these weaknesses in prior instruction by providing a really new and distinctive introduction to area.

The idea or concept of area was developed in *Seeing Through Arithmetic 5* on pages 8-11. At that time the emphasis was upon the meaning, and areas were found by counting units of area and estimating. The computation of area was not introduced. Instructional materials for this important application of arithmetic are provided on pages 240-251.

The approach to area through the rectangle and square is customary, since the area of other polygons can be found by transforming them into rectangular shapes. However, the method of computing area developed in *Seeing Through Arithmetic 5* is not customary. It is an application of the concept of rate, which is basic in this situation. The familiar short-cut rule for the area of a rectangle, "Area equals length times width," produces the correct number, but the reason for this cannot be made clear without reference, explicitly or at least implicitly, to a rate.

In the case of the rectangle, the rate is the number of units of area that can be accommodated on one of the sides of the rectangle. For example, if the length of a rectangle is 5 inches, then a row of inch squares can be accommodated on the longer side, and there will be 5 of them. The rate, then, is 5 square inches per 1 row. The other dimension (the width) determines the number of rows of inch squares that can be placed in the rectangle. If the width is 3 inches, there will be 3 rows.

The total number of square inches is at first not known, but temporarily it may be represented by n. Then the rate may also be represented by the ratio $n/3$, and so $5/1 = n/3$. The ratio 5/1 is to be replaced by a ratio whose second term is 3, so both terms of 5/1 should be multiplied by 3. The result is 15/3, which shows that $n = 15$.

From this basic approach it follows that as a short cut the area can be found by multiplying the rate (the number of square units per row) by the number of rows. The number of square units per row (the rate) is the same as the length of the rectangle. Also, the number of rows is the same as the width, so by a further abbreviation one arrives at the brief verbal rule: "The area of a rectangle is equal to the length times the width."

Teachers who have not themselves learned to recognize and use rates may consider the above explanation difficult to understand. *Seeing Through Arithmetic 5* developed the rate concept earlier (pages 112-118) because it is basic to a great many applications of arithmetic. The computation of area is only one of these. Although pupils may quickly satisfy themselves that the short-cut rule for computing areas produces the right answer, it does not follow that they understand it. In conformity with their fundamental principles, the authors of *Seeing Through Arithmetic 5* have sought first to develop understanding, and have postponed the presentation of the short-cut rule to a later grade.

240-244 Exploring problems

Lesson Briefs for this lesson are on pages 217-220.

Overview

In this lesson the children learn to find the area of a rectangle by applying what they now know about rate. They learn to find area by means of an equation made of two ratios. One ratio expresses the rate as square units per row along the base, that is, the number of square units per *one* row. The other ratio expresses the rate as square units per all the rows.

Teaching the whole class

Page 240: Begin this lesson by asking a pupil to tell how the area of a rectangle was found in the early part of the book (pages 8 to 11). [By covering the surface with 1-inch squares and counting the total number of square inches] Then have the pupils look

at Picture A and read its accompanying text. They should decide that they cannot tell the area in square inches from the diagram as shown. Discuss what can be done. [Draw a 3″ x 5″ rectangle and cover the surface with 1-inch squares.] Discuss the fact that the diagrams in this lesson are scale drawings and that the 1-inch squares are also in the same scale.

Then have the pupils read the text for Picture B and look at Picture B. They should understand that the picture shows the beginning of the covering of the rectangle with inch squares. Point out that only one row of squares has been placed on the rectangle so far. Be sure the pupils understand why 5 one-inch squares are needed for one row. They should understand that this is true because the base of the rectangle measures 5 inches. Remind them that an inch square is equal to one square inch. Therefore, the surface covered in this one row is 5 square inches. Go on somewhat like this:

Could we find the total area by covering the entire surface of this rectangle with inch squares? Sometimes, however, we have to work with surfaces too large to do that conveniently, as, for example, in finding the area of this room. So it is to our advantage to discover a way to find the area of a rectangle by computation.

Look again at Picture B. We are going to cover the rectangle with inch squares at the rate of five per row. We can express this rate by a ratio as shown in the book. We will cover the whole rectangle in rows at the same rate as for the first row. What ratio expresses the rate for 1 row? [*5/1*] *Can we replace this ratio by an equal ratio that will express the rate for all the rows?*

Read the text beside the arrow pointing to Picture C. Our first problem is to find how many rows there will be. What is the altitude of this rectangle? The squares in each row must be how big? Then how many rows will there be? We want to replace the ratio 5/1 with an equivalent ratio for 3 rows. Where was the numeral showing the number of rows placed in the first ratio? [*lower term*] *What must the numeral in the lower term of the equal ratio express?* [*rows*] *Since we don't know the number of inch squares for 3 rows, we will use the letter* n. *Why do we use it as the upper term of the second ratio? How can we express the new ratio?* [n/*3*] *Find this ratio in the book.*

Page 241: Next have the pupils look at the equation. They must understand that the rate of inch squares for 3 rows is the same as the rate for 1 row and that therefore the ratios are equal and can be written as an equation.

Direct attention next to the work opposite Picture E. Then ask for suggestions as to how to find the numeral that replaces *n*. Remind the pupils that since 1 can be multiplied by 3 to get 3, the 5 also must be multiplied by 3. Point out that Picture E shows that at the rate of 5 one-inch squares per row, it takes 15 one-inch squares to cover the 3 rows or the entire surface of the rectangle. Ask the pupils to count the inch squares to verify this fact. Direct attention to the ratio with the numeral 15 replacing the *n*. Point out that the equation means that a rate of 5 one-inch squares per 1 row is the same as a rate of 15 one-inch squares per 3 rows. Be sure the pupils realize that the answer to the problem is 15 *square inches.*

The pupils may benefit by covering a 5″ x 3″ rectangle with inch squares. (3″ x 5″ cards may be used effectively, and if the inch squares that were used in connection with pages 8 to 11 were saved, they may now be used here.) Let the children measure the card and an inch square with a ruler. Then have them find the number of inch squares in one row along the base of the card. Discuss the fact that they have used inch squares at the rate of 5 per 1 row. Tell them to write this rate as a ratio. Then discuss the fact that they are to find an equal ratio showing the number of inch squares there will be in all the rows. Ask the pupils how they can use a few of their squares to find how many rows in all there will be. (They can lay 1-inch squares along the 3-inch dimension of the rectangle to find that there will be 3 rows.) Have the pupils now make an equal ratio, using the letter *n* to indicate the total number of inch squares for 3 rows. Next tell the pupils to complete covering the card at the same rate of 5 one-inch squares per 1 row. Then ask them to use the equation to find the answer by computation.

Page 242: The new problem is solved in exactly the same way as the first problem on page 240. Adapt the procedures used with page 240 to this page. Have the children answer the questions in the text, and avoid unnecessary explanations.

Page 243: For the work through Picture K, adapt the procedures suggested in the notes for page 241. For the square in Picture L, ask the children to *imagine* inch squares laid along the base. Then they will know the rate of inch squares per 1 row. They can also imagine inch squares laid along the altitude to find the number of rows of 5 one-inch squares. Discuss the equation in the way you have been doing. Be sure the children realize that the answer means *square inches.*

Page 244: Before you ask the children to find the areas of Rectangles A to H, discuss Rectangle A. Point out that one row along the base will require 8 one-

inch squares. Have a child demonstrate with inch squares, which he can lay out on his desk, that this is so. Have the pupils state the rate of 8 one-inch squares per 1 row as the ratio 8/1. Also have a pupil use the inch squares to show that there will be 5 rows. Then let the pupils express this rate of n inch squares per 5 rows as the ratio $n/5$. Be sure they understand that the two ratios are equal and know how to write the equation. For Rectangles B to H, tell the children to write an equation for each rectangle.

When the pupils have completed this work, supply answers and discuss the exercises that caused the most trouble.

Problems A to K should be used for written work. Tell the children to write an equation for each problem. Then discuss the problems that caused the most trouble.

Providing for the able pupil

Suggest that the pupils draw rectangles of varying lengths and widths (measured by ruler and involving only whole inches). Tell them to label the lengths of the sides, and beneath each rectangle to write the equation that is used to find the area. If you plan to have the papers displayed, you may suggest that the pupils color the rectangles.

Helping the slow learner

Interest in finding the area of rectangles will be increased for the slow learners if they are allowed to find the areas of rectangular objects in the classroom. Before this activity is started, locate objects in the classroom whose surface areas can be measured in square inches and whose measurements do not involve fractional parts of inches. (If not enough small objects can be found, draw and cut from construction paper rectangles of various sizes.) On cards write the descriptions or locations of these objects so that the child can find or recognize them. Let each child take a card and then find the object described and measure its dimensions. On a separate paper have him write the equation that is used in finding its area.

245-248 Moving forward

Lesson Briefs for this lesson are on pages 221-223.

Overview

The concept of area is extended to include the square foot and the square yard. The square foot is introduced in terms of its area in square inches (page 245), and the square yard is introduced in terms of its area in square feet (page 246). The computational reduction of square measure is also developed through equations of ratios.

Teaching the whole class

Page 245: So far the pupils have computed the areas of rectangles that involved square inches only. Now, however, the rectangle they are to deal with represents a one-foot square. Direct attention to this foot square (Picture A). Call on a pupil to read aloud the text immediately below the picture. Bring out that the square foot is also a unit of square measure and that sometimes it is more convenient to measure in square feet than in square inches. In discussing Exercise A, the children should notice that since 12 inches equal 1 foot, each dimension of the square in Diagram A can also be thought of as 12 inches. Have the children look at Picture B and observe that 1 row of inch squares along the base requires 12 square inches. If at all possible, have available a piece of cardboard that is 1 foot square and a number of 1-inch squares. Let a child place inch squares in a row along the base. Be sure the children understand that the number of inch squares in this row may be considered a rate (12 sq. in. per 1 row) and then call attention to the ratio in the book that expresses this rate.

Have the pupils look at Picture B again and discuss why 12 rows are necessary to cover the 1-foot square. Let a child use the cardboard model, and place 1-inch squares to indicate the 12 rows. Ask him what rate expresses the number of inch squares in 12 rows [n sq. in. per 12 rows]. Then have the pupils look at the ratio $n/12$ in the text and answer the questions asked about it.

Next discuss Exercise C in relation to Picture C. Get the children to explain why the ratios 12/1 and $n/12$ are equal and thus can be used to make an equation. When you work through the solution of the equation, be sure the children understand why 12 has to be multiplied by 12 to find the value of n. When the children have found the numeral that replaces n, point out that they have discovered that there are 144 square inches in 1 square foot. Some children may wish to count the squares in Diagram C to verify this fact.

Page 246: Point out that Picture D represents a square that measures 1 yard on a side. Have the children read the text immediately below Picture D and explain that the square yard is also a unit of square measure. Then point out that this area can also be expressed in square feet.

Get the class to recall the number of feet in 1 yard. Then have someone explain, in relation to Picture E, how we know that it takes 3 one-foot squares to cover 1 row. Continue to develop the relationship between the number of square feet and 1 square yard in much

the same way that you developed the relationship between the number of square inches and 1 square foot. Use the text and Pictures E and F to guide the children's thinking. Also, if possible, have available a yard square cardboard model and some foot squares so that the children can indicate the square feet in one row and the number of rows.

Direct attention next to Picture G on page 247 and have the children note its dimensions. Have someone read Exercise G on page 246. Let him write the equation $(6/1=n/5)$ that shows how to find the area of the rectangle in square yards.

Then point out that sometimes it is desirable to know how to express an area in terms of some other unit of square measure. (The piece of glass in Picture G might be sold at a certain price per square foot, for example.) Tell the pupils that to change 30 square yards to square feet, they must first think of the rate of square feet per 1 square yard. This rate is 9 square feet per 1 square yard, or 9 per 1, as the text tells them. When the children examine Picture H, get them to distinguish the squares that represent square yards from those that represent square feet. Let them count to verify that there are 30 square yards in the piece of glass and that each square yard contains 9 square feet. You might point out that the square yard marked off in square feet is like the square yard in Picture F, except that a smaller scale has been used. Then discuss how to express the rate as square feet per 1 square yard [9 per 1] and as square feet per 30 square yards [n per 30]. Ask some of the pupils to explain what the symbols in the equation $9/1=n/30$ stand for. Be sure the pupils understand that in this equation the 1 stands for 1 square yard and not 1 row. Then have a pupil explain how to find the numeral that replaces n. Emphasize the equality of 270 square feet and 30 square yards.

Page 247: Have the pupils write the equation for finding the area of the table top described in Exercise I. Ask someone to explain this equation. Then direct attention to Problem J. Some children will be helped if they draw a rough diagram of the table top, similar to Picture H. Warn them, however, to make the picture large since it is hard to represent the 144 square inches contained in 1 square foot. Have the pupils explain the equation and do the computation to find the numeral that replaces n. Be sure the children understand that the 1 in the ratio 144/1 stands for 1 square foot.

Exercise K provides another application of changing square yards to square feet. Discuss the problem and the equation with the class. Then ask the pupils to find the numeral that replaces n.

For Exercise L, have the pupils study Picture I. Let someone show at the chalkboard how to find the area in square feet.

Then go on to discuss changing 45 square feet to square yards. Point out that Diagram I shows that 9 square feet are used for 1 square yard, and 9 per 1 expresses the rate of square feet per 1 square yard. Discuss why 45 per n expresses the same rate when applied to the entire rectangle. Pay special attention to the position of n (n represents the number of square yards). Be sure the pupils see that now they are changing to a larger unit of square measure; so they can expect a smaller number of units. Have them find the numeral that replaces n.

For Exercise N, have the children first write and solve the equation for finding the area in square feet. Then discuss the equation that is used to change the square feet to square yards. Work for understanding of the difference between the situation shown in Picture H and that in Picture I. (Picture H shows the reduction of a larger to a smaller unit of square measure; Picture I shows the reduction to a larger unit.)

Page 248: Have the children find the area in square inches of the picture described in Exercise O. Then, following the suggestions in the book, discuss how to change this answer to square feet. Be sure the children see that 144 is used because square inches are used at the rate of 144 square inches per 1 square foot. Discuss why n is the lower term of the second ratio. [n represents the number of square feet.]

In discussing Rectangle A, make sure the pupils are aware that the dimensions are expressed in different units. Following the directions in Exercise P, ask the pupils to explain how to change 3 feet to inches. Then help them make the equation for finding the area in square inches. Write it on the board $[36/1=n/24]$.

Next refer to Exercise Q and have the children find the area in the two ways described. They should note that the easier way is first to change 24 inches to feet.

The children should work independently to find the areas of Rectangles B to E. Tell the children that when the dimensions of a rectangle are expressed in different units, they may select the unit that seems simpler to work with and change the other dimension.

Conclude the lesson by assigning Exercises A to J as independent written work.

Providing for the able pupil

Let the able pupils find the surface area of rectangular objects in the classroom. Before this activity is started, locate objects in the classroom whose surface areas can be measured in square feet or square

yards, and whose measurements do not involve fractions of a foot or yard (desk and table tops, doors, pictures, etc.). Write descriptions of these objects on cards. Some of the cards may show one dimension in inches (or feet) and the other dimension in feet (or yards). Let each child draw a card and then find the object and measure its dimensions. On a separate piece of paper have him write the equation he would use to find its area and then do the necessary computation to find the numeral that replaces *n*.

Helping the slow learner

The slow learners should also be allowed to measure objects in the room and find their surface area. These children should not be expected to work with as many areas as the abler children, and the dimensions they use should be in the same unit.

249-250 Using arithmetic

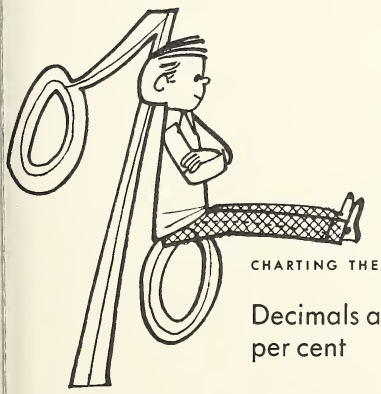
Lesson Briefs for this lesson ore on pages 224-225.

Expanded Notes are not considered necessary for this lesson.

250-251 Thinking straight

Lesson Briefs for this lesson ore on pages 226-227.

Expanded Notes are not considered necessary for this lesson.



CHARTING THE COURSE

Decimals and per cent

A brief introduction to decimal fractions and the idea of per cent is now given in many fifth-grade programs. There is good justification for this because decimal fractions and per cents are frequently found in various reading materials that the boys and girls may be using.

If pupils have had a good introduction to rates and ratios, the idea of per cent is a relatively easy extension, since per cent refers to any rate in which the second term of the ratio is arbitrarily taken as 100. The only new thing to be learned is that this is a convenient thing to do, and along with it goes the term *per cent* and the symbol %.

Similarly, pupils who have learned that the same fraction may be represented by many different fraction numerals should have little difficulty in seeing that $\frac{1}{5}=\frac{2}{10}=\frac{20}{100}$ and that other numerals for the same fraction, namely .2 and .20 can be adopted. The great advantage which decimal fraction numerals have over common fraction numerals cannot be appreciated until these numerals have been used in computation. The use of decimal fraction numerals in computation is now commonly deferred to later grades.

252-253 Moving forward

Lesson Briefs for this lesson ore on pages 227-229.

Expanded Notes other than suggestions for the able pupils are not considered necessary for this lesson.

Providing for the able pupil

Able pupils can collect newspaper and magazine clippings showing the use of per cents. They can then interpret the meaning of these per cents.

These pupils can also keep a record of progress in some subject (spelling, for example), that shows the per cent of words correctly spelled in relation to the number of words that were to have been learned.

For this activity you may have to provide scores in terms of per cents when the number of items involved do not make ratios that are easily reduced to per cents. If possible, at this point, include numbers of items that can be so reduced easily (5, 10, 20, 25, etc.).

254 Checking up

Lesson Briefs for this lesson ore on pages 229-230.

Expanded Notes are not considered necessary for this lesson.

255-256 Moving forward

Lesson Briefs for this lesson ore on pages 230-231.

Expanded Notes are not considered necessary for this lesson.

257-258 Using arithmetic

Lessan Briefs for this lessan are an pages 232-233.

Expanded Notes are not considered necessary for this lesson.

259 Thinking straight

Lessan Briefs for this lessan ore on pages 233-234.

Expanded Notes are not considered necessary for this lesson.

260-263 Looking back

Lesson Briefs far these lessons are an pages 234-237.

Expanded Notes are not considered necessary for these lessons.

264-267 Checking up

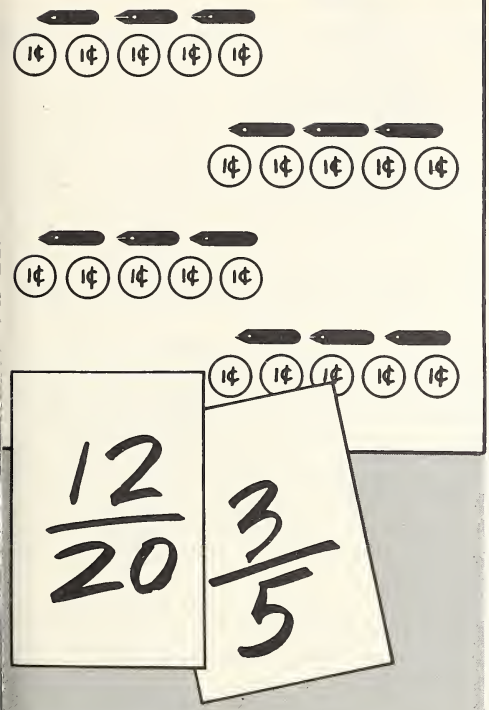
Lessan Briefs far these lessans are an pages 238-241.

Expanded Notes are not considered necessary for these lessons.

Your notes

Your notes

Activities

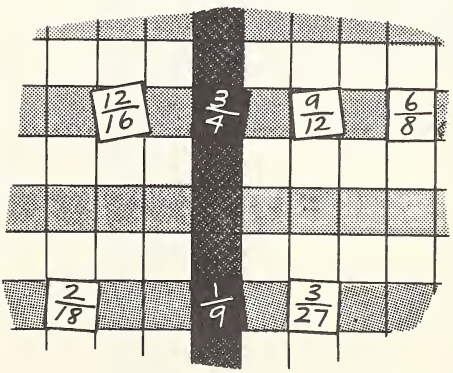


This section of the *Teaching Guide* contains suggested activities specially designed to extend the child's concepts and generalizations about arithmetic. Specific references are made to these activities in the Expanded Notes.

The activities are arranged in the order of their use in the Expanded Notes and are numbered to make reference to them easier.

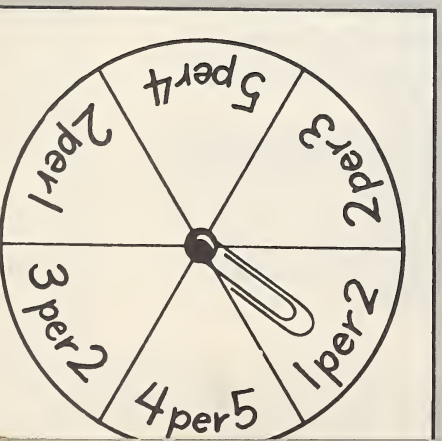
You will undoubtedly discover new ways of using these activities and ways of adapting them to other topics.

You will also be sure to think up other activities as good as or better than the ones described here.



Activities

Useful arithmetic activities are those that lead to good thinking and generalizations. They should not require time-consuming and mechanical work. They should be directly arithmetical.



Activity 1

Experimenting with points and lines

The six challenging experiments described below will help the pupils make generalizations about points and lines. Some typical answers are also given.

You will need to prepare the following materials: for Experiments A, B, C, and D: 6″ x 4″ cards on which the experiments have been written or typed for each child; for Experiment E, cutouts of polygons as described on page 358; for Experiment F, the chart illustrated on page 358 (reproduced on the chalkboard).

Experiment A: The children should work independently, but you may need to remind them that a polygon is any closed shape having three or more straight sides.

Experiment A

1. Make as many arrangements of 2 straight lines as you can. Are any of your arrangements polygons?

2. Make as many arrangements as you can using 3 straight lines. Which of your arrangements are polygons?

3. Now see what you can do with 4 straight lines. Which of your arrangements are polygons?

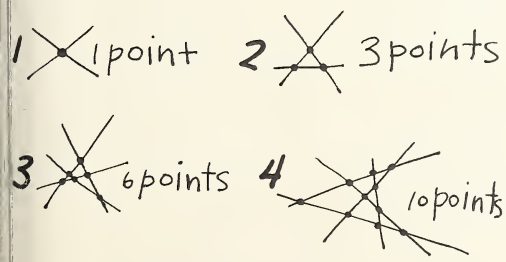
4. Make as many arrangements as you can using 5 straight lines. How many polygons can you make?

Some arrangements the children might make for Experiment A are shown above.

Experiment B: Hint: After the first line has been drawn, each succeeding line must cross all the lines already drawn, and not more than two lines should meet at any point. Answers to Experiment B are shown below the picture of the card.

Experiment B

1. What is the greatest number of points at which two straight lines can be made to cross?
2. What is the greatest number of points at which three straight lines can be made to cross?
3. What is the greatest number of points at which four straight lines can be made to cross?
4. What is the greatest number of points at which five straight lines can be made to cross?

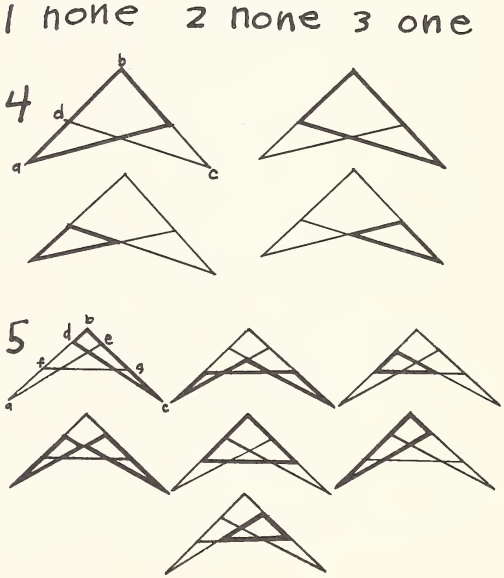


Pictures of the cards for Experiments C and D and answers for the exercises are shown in the next column and on page 358. Note that for Experiment C the triangles overlap.

Experiment D: Hint: Draw a line from each point to each of the other points. Tell the children to make

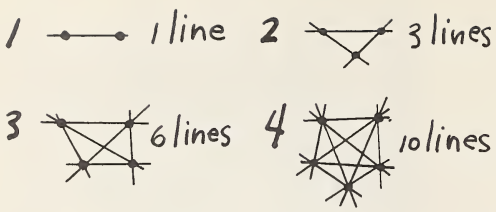
Experiment C

1. How many triangles can be made with 1 straight line?
2. How many triangles can be made with 2 straight lines?
3. How many triangles can be made with 3 straight lines?
4. How many triangles can be made with 4 straight lines? It is easy to make three. Can you make four?
5. Now see how many triangles you can make with 5 straight lines. (Start by making four, using 4 straight lines. Then experiment with the fifth line. Try to get as many overlapping triangles as possible. What is the best way to do this?) It is possible to make as many as 10 different triangles.



Experiment D

1. Draw as many straight lines as you can through 2 points.
2. Draw as many straight lines as you can through 3 points. Draw each line through 2 points.
3. Draw as many straight lines as you can through 4 points. Draw each line through 2 points.
4. Draw as many straight lines as you can through 5 points. Draw each line through 2 points.



their points far apart, so that the lines will not be crowded. Some of the more able children may observe the sequence 1, 3, 6, 10, and may be able to extend it without drawing and counting lines. The lines the children draw may be numbered as shown above.

Experiment E: Supply the children with sets of regular polygons—a set of triangles, a set of squares, a set of pentagons, a set of hexagons, etc. Each set should consist of 15 to 20 identical shapes cut out of oaktag or construction paper. Let the children experiment with each set to see if the polygons can be used to make designs that will fit together (as for a floor covering).

Experiment F: On the chalkboard make a chart like the one below. Then the children may either copy the chart on paper and fill it in, or they may take turns filling in the spaces on the board. Let them think of objects to write in the first column, decide which shape describes a surface of the object and write its name in the second column, and then check the term or terms that describe its dimensions.

Object	Shape	Base	Altitude	Diameter
book	Rectangle	✓	✓	
clock	Circle			✓

Activity 2

Polygon map

If possible, supply, for each child, printed outline maps of the United States showing the states (or of Canada showing the provinces).

Using straight lines only, the children should draw in red within each state (or province) the polygon that they think the shape of the state (or province) resembles. Have the children label each shape they draw with the name of the polygon if they know its name.

Activity 3

Working with polygons and circles

Provide large oaktag cutouts of parallelograms, triangles, and circles (a 20″ by 30″ rectangle, a triangle with a 27″ base and a 23″ altitude, etc.). To identify the cutouts, print a capital letter on each. Label the edges of the polygons with letters. Mark the centers of the circles. (If necessary, draw and label the line on which the altitude of the parallelogram or triangle can be measured.)

Divide the class into small groups and give each group one cutout. Write on the chalkboard instructions for the children to follow. These instructions may be: (1) Write on paper the identifying letter (A, B, etc.) of your cutout; (2) write the name of the cutout (circle, rectangle, etc.); (3) measure the sides and write the length of each side beside the letter for that side; (4) measure the altitude or diameter and record its length.

Activity 4

Estimating areas

A simple activity can be used to give the children experience in estimating the areas of squares. Prepare a worksheet on which five squares with sides of 1 inch, 2 inches, 3 inches, 4 inches, and 5 inches are shown. Label the squares A to E. Below the squares make a little chart for the children to fill in. Write the headings "Estimate" and "Squares Used" with the letters A to E in a column. Copies of the worksheet may be duplicated for all the pupils in your class. Each child will also need 25 one-inch paper squares.

Do not discuss with the children what the area of each square might be; rather, let them judge for themselves how many square inches will cover each square. Let them study the pictures, then opposite the letters A to E on the chart write the number of square inches they estimate the area of each square to be.

When this has been done, have each child cover the squares with his paper one-inch squares and then write on the chart the number of squares he used.

After the children have completed the work with the squares on the worksheet, hold up before the class 2-inch, 4-inch, 5-inch, and 6-inch squares. Again have the children estimate the areas in square inches and write their estimates on paper. Then have a child use paper one-inch squares to determine the area of each square so that the class can compare these areas with their estimates.

Another way for children to estimate the areas of squares is to have them draw, without using rulers, squares which they think have areas of 1 square inch, 4 square inches, 9 square inches, etc. Then let them use the paper one-inch squares to verify the areas of the squares they have drawn.

After the children have had experience in estimating the areas of squares, let them try to estimate the areas of other polygons. Use only simple polygons. Do not at this time have the children use the linear dimensions to verify the areas. Have them use their paper one-inch squares only.

Activity 5

Number "spelldown"

To give children practice in changing groupings (ones into tens, tens into hundreds, hundreds into thousands, etc.), and in thinking of numbers in terms of different groupings, let them have a spelldown as described below. The children should first "number off," with the "evens" on one team and the "odds" on the other.

To start the game, the leader of one team asks the leader of the other team to change 3 hundreds 14 tens 8 ones, for example, to the simpler numerical form [448]. If the opposing member does so correctly, it is his turn to make up an expression containing hundreds, tens, and ones (or thousands, hundreds, tens, and ones) and ask the second person on the first team to change it to the simpler numerical form. The children should understand that one of the groups in the number must contain more than 9. When a player misses, he must sit down and give the next member on the opposing team a chance to answer. The child who remains standing at the end is the winner.

Another way to play the game is to give a number —for example, 134—and ask that it be changed to hundreds, tens, and ones in such a way that one of the groups will contain more than 9. (For example, 134 may be thought of as 13 tens 4 ones or as 1 hundred 2 tens 14 ones.)

Activity 6

Number along

Three to nine children can play this game, which provides practice in regrouping for use in carrying.

To prepare for the game, cut 3″ x 5″ cards in half to make 3″ x 2½″ cards. Decide on a series of 100 numerals, such as 401 to 500, 1001 to 1100, 15,001 to 15,100, etc., and in the corner of each card write one of the 100 numerals. The purpose of the game is to get the ten cards for one of the decades (such as 401, 402, 403, 404, 405, 406, 407, 408, 409, 410 or 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010). The child who gets such a sequence of ten first lays down his hand, calls out "Number along," and wins.

The pack of 100 cards (face down) is placed in the middle of the table, and each child in turn takes a card until he has 10 cards. Each player then examines his hand to see if he has a start toward a sequence of numerals in any series of ten. If he does, he should save these cards and try to get the missing numerals in that decade.

The child at the right of the last player to pick up his tenth card starts the game. He must offer to trade, for an unknown card, a card that he does not want. He chooses from his hand a card he wants to trade and describes the numeral on it by saying it in the form of hundreds, tens, and ones, so arranged that there are more than 9 ones or more than 9 tens. For example, for the 142 card he might say "1 hundred, 3 tens, 12 ones."

If some of the players are saving "one hundred forties," they will want to trade and will call out "trade." The first one to call out may trade any card he does not want for the 142 card. If the player who wants to get rid of the 142 card calls the numeral by its usual name (one hundred forty-two) instead of by hundreds, tens, and ones, he loses his chance to get a new card. He must keep the old card for that round.

If a player has described his numeral correctly and no one wants to trade, he may discard the card by putting it face down at the bottom of the pile in the middle of the table. He then may draw a new card from the top of the pile.

A player may lay his hand of ten cards down and call "Number along" only when it is his turn to play.

Activity 7

Roman numeral chart

An easy Roman numeral chart can be outlined by the teacher and filled in by the slow learner.

On the left side of the sheet of paper write in a column the Roman numerals from one to ten. At the bottom of the chart and to the right of the numeral X, write the numeral for each decade to 100. Then draw lines separating the numerals for the decades,

making a chart like the one shown below. Show the children how to fill in each sequence by writing numerals in the columns. For example, for the first numeral in the first column (opposite I), they should write "XI."

Other charts can be made for the sequences between 100 and 500, or 100 and 1000, if you wish. However, if the child is able to understand how the Roman numerals from 1 to 100 are formed, he will be able to decipher larger numbers, since they follow the same pattern.

The chart may be drawn on the chalkboard to be copied and completed by the child, or it may be duplicated and given to the child to fill in.

I	XI								
II									
III									
IV									
V									
VI									
VII									
VIII									
IX									
X	XX	XXX	XL	L	LX	LXX	LXXX	XC	C

Activity 8

Basic fact lists

To give children a review of the multiplication and division basic facts in a different way, have them arrange the facts according to number groups. For this activity, instruct the children to write the numerals 40, 42, 45, 48, 49, 54, 56, etc., in rows across the top of their papers. Show them how to leave sufficient space between and below numerals so that they can write the complete equations in a column under each number. (See the sample below.)

40	42	45
$5 \times 8 = 40$	$6 \times 7 = 42$	$5 \times 9 = 45$
$8 \times 5 = 40$	$7 \times 6 = 42$	$9 \times 5 = 45$

This activity may be used for the division basic facts also. Instruct the children to put the dividends for the division groups across the top of their papers in the way explained above for multiplication. Then tell them to show in equation form all the basic facts that belong in each group.

For the 40, 42, and 45 groups, their work would look like this:

40	42	45
$40 \div 5 = 8$	$42 \div 6 = 7$	$45 \div 5 = 9$
$40 \div 8 = 5$	$42 \div 7 = 6$	$45 \div 9 = 5$

Activity 9

Working with number series

For each number series the children are to find the pattern used in making the series and to write numerals for enough additional numbers to complete at least one more sequence of the series.

Sample number series of suitable types are indicated below for use with various pages of the pupils' book.

For page 18:
8, 9, 11, 14, 18, 23, . . . (+1, +2, +3, +4, etc.)
28, 25, 22, 19, 16, . . . (−3 each time)
6, 9, 7, 10, 8, 11, . . . (+3, −2; +3, −2)
15, 14, 13, 12, 13, 12, 11, 10, 11, . . .
(−1, −1, −1, + 1; −1, −1, −1, +1)

For page 66:
4, 4, 8, 24, 96, . . . (×1, ×2, ×3, ×4, etc.)
1, 6, 30, 120, . . . (×6, ×5, ×4, etc.)
6, 8, 16, 18, 36, . . . (+2, ×2; +2, ×2)

For page 90:
172, 86, 43, 44, 22, 11, 12, . . .
(÷2, ÷2, +1; ÷2, ÷2, +1)
6, 24, 12, 48, 24, . . . (×4, ÷2; ×4, ÷2)

For page 111: Any of the above types

For page 197:
$\frac{1}{4}, \frac{3}{4}, 1\frac{1}{4}, \ldots$ ($+\frac{1}{2}$ each time)
$\frac{5}{6}, \frac{3}{4}, \frac{2}{3}, \frac{7}{12}, \ldots$ ($-\frac{1}{12}$ each time)
$1, 1\frac{1}{3}, 1\frac{1}{12}, 1\frac{5}{12}, 1\frac{1}{6}, \ldots$ ($+\frac{1}{3}, -\frac{1}{4}; +\frac{1}{3}, -\frac{1}{4}$)

For page 202: Any of the above types

Activity 10

Target toss

The game of "Target Toss" can furnish practice in column addition of one-figure numbers, two-figure numbers, and three-figure numbers.

Materials needed are a large square board (the end of an orange crate will do), 5 drapery angle screws or hooks, 5 or more jar rubbers, and a set of large cards, each with a different numeral on it (one-

figure, two-figure, or three-figure). Punch a hole in the top center of each card. Make the target by screwing the drapery hooks into the board, one hook in the center and two rows of two hooks each. Then hang a card on each hook.

The difficulty of the game may be controlled by the size of the numbers whose numerals you choose to hang up and by the number of rings that are thrown. Be sure to keep the difficulty within the range of the players.

Decide before the game begins how many "ringers" a player may have at each turn at play. For example, the game can be limited to two ringers at each turn, with a maximum of five jar rings to throw. When a player has made two ringers, he is through for the turn. He may throw any number of jar rings up to five to get two ringers. The game can be made more difficult by requiring three ringers in 5 throws. If the children find it hard to get two (or three) ringers, provide jar rings for more throws.

Let each player take a turn at throwing jar rings, one at a time, at the hooks from a distance of six feet. One child may be designated as scorekeeper to write the ringed numerals on the board under a player's name. Then let each player add up his own total score. The player with the highest total score wins. Ties may be broken in a toss-off. Abler pupils can try adding the numbers mentally, but the slower children should use paper and pencil.

Activity 11

Cork drop

"Cork Drop" is a game that will give children practice in adding numbers with two and three figures.

Materials needed are an empty coffee can (or other similar container) and a supply of medium-sized corks. Write or paint a two-figure or a three-figure numeral on the top of each cork. Let each child have up to five tries at dropping corks into the can. As soon as he has two in the can, he should add the numbers. The corks should all be dropped from the same height.

Able students may use mental calculation when adding the numbers shown on the corks dropped into the can. Slow pupils should be allowed to use pencil and paper in adding their numbers.

The player with the highest total wins.

Provide enough corks so that the children cannot become familiar with the number combinations that it is possible to make.

Activity 12

Multiplication and division chart

A chart for the multiplication and division basic facts may be prepared by the pupils.

Have the children mark off a 4½″ square into half-inch spaces, 9 across and 9 down. Beginning with the second from the left space at the top, have them write the numerals from 2 to 9 in the spaces going *across*. Then have them write the numerals from 2 to 9 in the spaces going *down* the left side, starting with the second from the top space. Tell the pupils to multiply the 2 at the top by the 2 at the left, and write the answer directly under the top 2 and opposite the side 2. Then tell them to multiply the 3 at the top by the 2 at the left, and write the answer beneath the 3 and opposite the 2. Have them continue multiplying the numbers across the top by 2 and writing in the answers until the second horizontal row is filled. Then have them multiply the numbers in the top row by the 3 at the left side and write the answers in the proper columns. Have them continue in this way until as many multiplication basic facts as the children know have been filled in.

The heavy line on the chart below shows how far the chart could be filled in when the children have reviewed the work on page 18. As other facts are reviewed, the proper numerals may be put on the chart. Once the chart has been completed correctly, the children can use it to verify any multiplication fact they are unsure of.

	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

This chart, of course, can also be used to review the division basic facts. When the children want to check division facts, they use the figures in the first column at the left as divisors. The figures in the rows to the right show numbers that are to be divided, and the figures in the top row are answers.

Activity 13

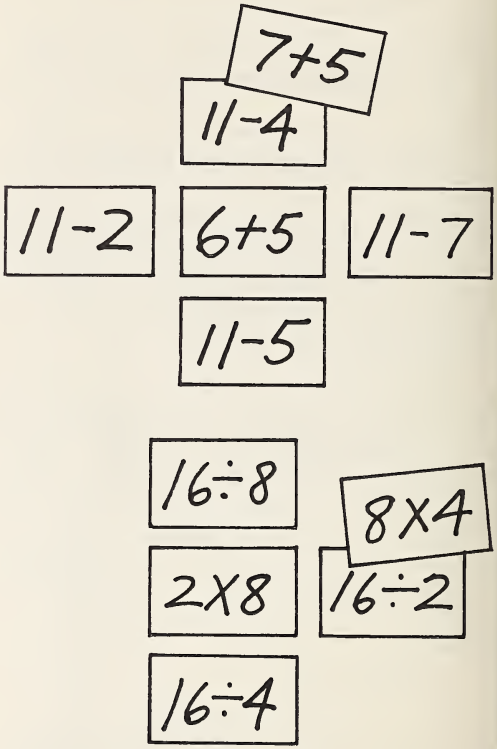
Basic fact changeover

This game reviews the basic facts. It can be arranged to give practice on addition and subtraction basic facts, or on division and multiplication basic facts. For drill on all the addition and subtraction facts, 162 cards (81 for each process) are needed. For multiplication and division, limit the facts on the cards to the number groups already reviewed. Three to five children may play.

Cut 3″ x 5″ cards into four pieces so that each small card will be 1½″ x 2½″. Write a basic fact on each card. Shuffle the cards thoroughly and let each player, in turn, draw a card until he has six cards. (The remaining cards are kept in a pile to draw from as the game proceeds.) The first child to draw a card with an *answer* of 10 or more must put that card face up in the center of the table as the master card and draw another card. The child who is last to draw the sixth card for his hand is the first one to play. If he has a card showing a basic fact that starts with the same number as the *answer* for the basic fact on the master card in the center of the table, he must lay it to one side of the master card. Then the next child draws a card and plays to the master card if he can. Play continues in this manner until 4 cards have been placed around the master card, as shown in the next column. (If multiplication and division basic facts are being used, it will not always be possible to place 4 cards around the master card. The maximum possible number may be 1, 2, 3, or 4 cards. For example if 2×8 is the master card, only the cards showing $16 \div 4$, $16 \div 2$, and $16 \div 8$ are available to be placed around the master card.)

Now the four cards around the master card (or 1, 2, 3, or 4 cards for multiplication and division) are new master cards. The children look for basic facts that begin with the same number as the answer on any one of the new master cards. When a player has such a card, he lays it on top of the proper card. The card just played then becomes a new master card in turn. The children continue to draw and play in turn until the pile is exhausted or until one player is out of cards.

The player who gets rid of all his cards first is the winner. If all the cards in the pile have been drawn before someone has played all his cards, the player with the fewest cards left is the winner.



Activity 14

Problem cards

Prepare 5″ x 3″ cards (lettered A, B, C, etc.) and write or paste one problem on each card. You may make up these problems or cut them from old textbooks. Be sure the problems are within the range of the processes taught thus far. Include cards for new problem types as they are taught.

Let each pupil pick out three cards, five cards, or as many as you specify. On his paper, opposite the letter of the card, he is to write the equation for the problem, with the letter n where the answer is to be inserted. If a problem requires computation, the child should show his computation.

Activity 15

Make up a problem

Have the children work in groups and take turns being "It," as described below. Let one child do the timing and experiment with times of 1 minute to 3 minutes to determine how much time should be allowed. (It will be helpful if you can provide the group with a three-minute egg timer, a stop watch, or a watch with a second hand.) The child who is "It" writes on the board an equation with the letter *n* for the missing number. Then he points to a child in the group and asks him to make up a problem to fit the equation. If the child can state a problem and give the answer before the time is up, he becomes "It." If he cannot think of a problem, the child who is "It" asks another child to make up a problem.

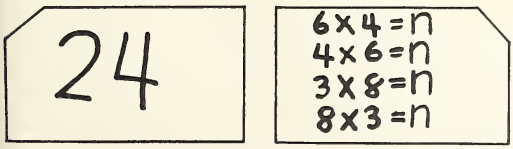
All the children in the group should verify the answers. Supervise the activity to make sure that the children do not go beyond the range of the processes studied thus far. Permit discussion of all disputed problems.

Activity 16

Multiplication and division basic fact practice cards

As a follow-up to Activity 8 or as a separate activity, have the children make practice cards for the multiplication and division basic facts. They may use the lists made for Activity 8 to make a card for each number group. Each card should have a group number on one side and all the basic facts for that group on the other side.

The practice cards should be made of oaktag. A sample card for the 24 group in multiplication is shown below.



For multiplication, arrange the cards with the equation side up, and cut a piece from the upper right corner, so that the cards can be assembled easily with the equation side up.

In using the cards, the children may work individually or in pairs, reading the equations and deciding which group number is the product. By reversing the card, they can verify their answer. New cards may be made as higher number groups are studied.

Similar cards can be made for the division basic facts. Arrange these with the group number up, and have the children give the division basic facts for that group.

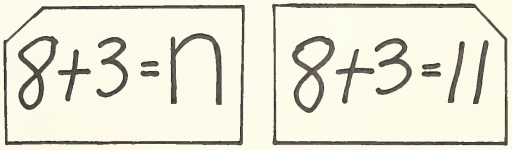
These practice cards can also be used by the pupils on their own to practice responses for the basic facts. The pupils can work in teams of two.

Activity 17

Trouble spots

Individual practice cards that contain only the basic facts the child finds difficult provide a useful activity. Let each child put the facts he finds troublesome on small pieces of drawing paper. Have him write the fact in equation form (without the answer) on one side of the card and the same fact in equation form with the answer on the other side. Lay the cards so that all the sides without answers are face up and cut off a small part of the upper left corners. The cut corners will help the child arrange the cards so that all the sides with answers or all the sides without answers are facing the same way.

Practice cards of this kind may be made for addition, subtraction, multiplication, and division basic facts. A pupil may use these cards in his spare time, working alone, or he may use them in practice sessions with a partner. As soon as a pupil is sure of a fact, he should remove its card from his pack. Encourage him to eliminate cards as soon as he can. In this way, he competes with himself.



Commercially prepared cards, for both individual and class use, are available (in computational form only).*

**Arithmetic Flash Cards for Individual Use* (2″ x 3½″ in envelope). Set 5: Addition and Subtraction (200 cards); Set 6: Multiplication and Division (190 cards). Scott, Foresman and Company.

Arithmetic Flash Cards for Class Use (4″ x 7″, boxed). Set 1: Addition (100 cards); Set 2: Subtraction (100 cards); Set 3: Multiplication (100 cards); Set 4: Division (90 cards). Scott, Foresman and Company.

Activity 18

Multiplication and division tables

Another way to review basic facts is to have the children make their own "tables." For multiplication, have them start with a number representing the size of the equal groups and show how the answers (products) change when this number is multiplied by successive numbers. The children should make one or more of these tables, but make them only as far as they have reviewed the basic facts. As new facts are reviewed, they can be included in the appropriate table. A sample table is given below.

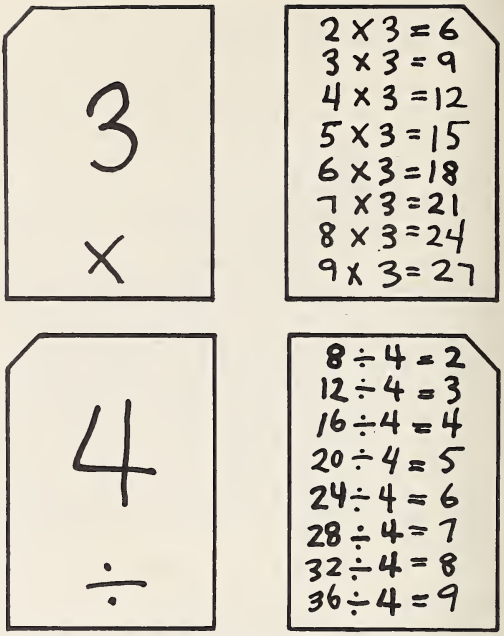
2 x 8 = 16	16 ÷ 8 = 2
3 x 8 = 24	24 ÷ 8 = 3
4 x 8 = 32	32 ÷ 8 = 4
5 x 8 = 40	40 ÷ 8 = 5
6 x 8 = 48	48 ÷ 8 = 6
7 x 8 = 56	56 ÷ 8 = 7
8 x 8 = 64	64 ÷ 8 = 8
9 x 8 = 72	72 ÷ 8 = 9

For division tables, have the children take a number representing the size of the equal groups (divisor) and show how the answers (quotients) change as increasingly larger numbers are divided by this number. Have them use number groups only as far as they have reviewed. See the sample table above.

Activity 19

Multiplication and division table cards

For a more permanent record, the tables made in Activity 18 may be transferred to small oaktag cards. For multiplication, have the children write the numeral representing the size of the equal groups (multiplicand) on one side of the card. The basic facts using that numeral should be arranged in table form on the other side of the card to show how answers (products) change when this number (multiplicand) is multiplied by successively larger numbers.



These cards may be used individually for oral or written work. The equations on the back provide the children with a means of verifying their work.

Division cards may also be made. The numeral representing the size of the equal groups (divisor) should be written on one side of the card. The basic facts should be arranged in table form on the other side of the card to show how the answers (quotients) change as increasingly larger numbers are divided by this number.

Activity 20

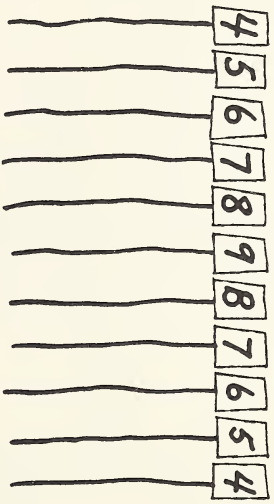
Number football

A "football game" can be improvised very easily in a small space in the classroom. The game as described uses the multiplication or division basic facts through the 36 group.

You will need 11 pieces of string, each roughly 4′ long, and 11 pieces of typing paper, six of which are numbered 4 to 9, and five of which are numbered 4 to 8. These numerals should be large enough to be seen easily. You will also need three cardboard ovals, numbered 2, 3, and 4, to represent footballs. (Or you may use boxes, similarly numbered.)

Place the strings about a foot apart and parallel to one another on the floor. They can be fastened down with cellophane tape at each end. (If you prefer, you can draw chalk lines on the floor.) Place the papers under the strings, or tape them over the chalk lines, in ascending numerical order from 4 to 9 and in descending numerical order from 9 to 4. See the picture below.

Divide the class into two teams. (There should be both able pupils and slow learners on each side.) Then select two children of similar learning ability, one from each team. (If you wish, several groups may be organized. Two teams of able pupils or two teams of



slow learners could then play against each other.) Give one of the two children a "football" and tell him to start at one end of the field and progress as far as he can. Explain that at each line he must multiply the number shown on his football (if the multiplication basic facts are being practiced) by the number shown on the line and give the answer. If he gives all the answers correctly, he gets 7 points for a touchdown.

Now the pupil from the opposing team carries the ball. He must progress down the field from the opposite end and give all the answers as he goes. If he gives them correctly, he, too, scores 7 for a touchdown. A turn is now given to another pupil from the first team. His football should be one with a numeral different from the one used by the first player.

If a player fails to give a correct answer on his way down the field, his team may send in a substitute. The new player will then try to complete the run to the goal line. If he fails, the ball goes to the opposing team. If this team has already scored with this particular ball, a new player should use a ball with a different numeral.

The game can be adapted to provide practice for basic facts higher than 36 by using footballs with numerals larger than 4 on them.

This game may be used to practice division basic facts by putting a sequence of dividends all divisible by the same number on the lines and changing the sequence for each football. (Some of the lines will be without numerals.) As the player proceeds down the field, the divisor is indicated by the numeral on his football, and the number to be divided is indicated by the numeral on the floor. The player must give the answer. The game can be adapted to division with remainders by controlling the numbers shown on the lines.

Activity 21

All-in-a-row

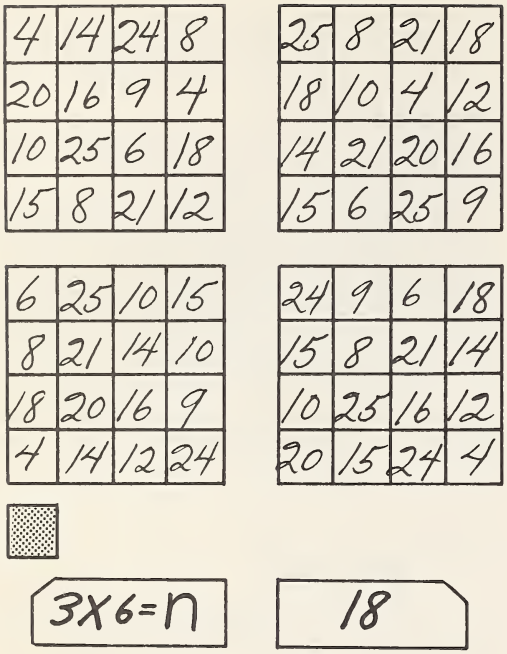
Practice on the multiplication and division basic facts is provided by this game.

In advance, prepare master cards about 5″ x 5″. Rule these cards into four horizontal and four vertical rows. For multiplication, write in each square a numeral that is a product (4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 24 and 25). If you wish to review the facts with larger products, select an assortment of 14 products from 27 to 81. No two cards should have the same layout, but two of the numerals will occur twice on each card. Be careful to vary the repeated numerals from card to card. Master cards for the basic facts through the 25 group are illustrated on page 366.

Also prepare a set of basic fact cards (2″ x 1″) including all the basic facts involved with the products on the master card. On one side of the card write, without the answer, a multiplication equation. On the other side, write the answer.

Provide each player with 16 "cover" cards. These are construction-paper squares, slightly smaller than the squares on the master cards (about 1″ x 1″), to use to cover the answers when a basic fact is called.

Any number of pupils may play the game, so long as a master card is available for each player. One child should be selected as the "caller" for each game. The "caller" draws a basic fact card and reads it without the answer. Each player looks to see if he has the answer for this basic fact on his master card. If a



player does have the answer, he covers it with one of his cover cards.

The "caller" should put down the basic fact card he has called with the answer side up. Later he will use this pile of basic fact cards to check the winning master card. The "caller" will continue to draw and read basic facts until a child has a winning master card.

The player wins who first gets a vertical row, a horizontal row, a diagonal row, or the four corners filled with cover cards.

For division, the game is played in the same way. Division basic fact cards will have to be made for the "caller" to use. The master cards will show the answers to division basic facts which include only the numbers 2 to 9. Each of these numerals will be used twice on each master card. Be careful to vary the arrangement from card to card. When a basic fact whose answer is 2 is called, each child will place a cover card over one of the 2's on his master card. The next time a basic fact whose answer is 2 is called, he will place a cover card over the other 2.

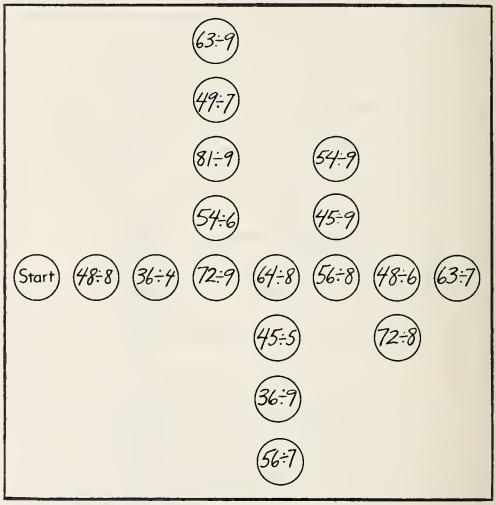
Pupils should be encouraged to make a record of the basic multiplication and division facts that caused them trouble and to pay especial attention to these facts.

Activity 22

Which one?

The following game can be played by as many children as can be accommodated around a table. As described here, the game provides practice on the division basic facts, but it can be adapted for practice on the multiplication basic facts by preparing boards and cards that show the multiplication facts.

Make a board (see the illustration below) for each player.



In each of the 17 circles on the board, write a division fact without the answer. Identical facts must appear on all the boards used in a game, but they should be arranged in different order. Do not use any fact more than once on a board. The facts shown on the board in the illustration are a good representative assortment, but any facts that you feel the children need to practice may be used. Two sets of boards can be made so that one set shows the harder facts and the other set shows easier facts.

You will also need some small cards that can be made by cutting 3″ x 5″ cards into three parts. On each card write one of the numerals from 4 to 9 (or whatever numerals are needed for the answers for the facts on the board). (If the game is adapted for multiplication, the numerals on the cards will be the answers for the multiplication facts.) Nine or ten cards should be made for each answer. Be sure to mix the cards thoroughly before using them. Each player also will need two markers. Flat beads, pieces of macaroni, buttons, or small corks are suitable.

To play the game, each child puts his board before him with one of his markers in the square marked "Start." The cards are placed face down in the center of the table. The first player draws a card and places it face up on the table so that everyone can see it. If the numeral indicates the answer to the division fact in the first circle on his board, he moves his marker to that circle and places the card face down in the discard pile. If the numeral on the card does not indicate the answer to his first division fact, he places the card face down in the discard pile and draws another card. If the second card shows the answer to his first division fact, he moves his marker and discards the card. If a player cannot move after drawing the second card, he does not discard it but places it face up on the pile for the next player. The second player may move if the card indicates the number he requires. If it does not, he discards it and draws another from the pile. If the first player has not left a card face up, the second player plays in the same way as the first, drawing a card and moving or discarding the first card and drawing another, then moving if the numeral on the second card shows the answer to his first division fact. He also leaves the second card for the next player if he cannot move. No player may move a marker more than once in any turn. If a player moves on the first draw, or on the card left by the previous player, he does not draw again in that turn.

When a player moves his first marker to the third circle in the horizontal row, he may have an opportunity to put his second marker into play. If he draws a card showing the answer to the division fact in the circle above the third circle, he puts his *second* marker in that circle. Thereafter he may move either marker, depending on the answer shown on the card he has drawn. If he draws a card that permits him to move either marker, he must decide which he prefers to move. He may not move both. If a player has not been able to use his second marker on the first sidetrack, he may do so on any of the other three sidetracks if he draws a card with the answer to the division fact in the first circle of a sidetrack at the time his first marker is in a circle of the horizontal row that is next to a sidetrack. Once his second marker is on the board, he also has the choice of moving his first marker down a sidetrack if he draws a card that permits him to do so.

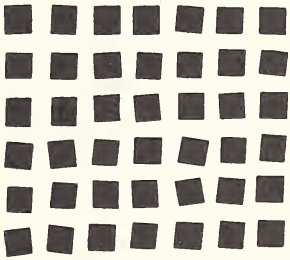
If all the cards have been drawn and put in the discard pile before the game is finished, the discard pile may be mixed and used again. The winner is the player who first gets one of his markers to the end of one of the five tracks.

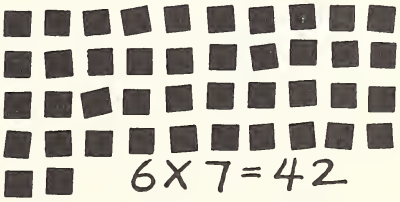
Activity 23

Multiplication builders

Some children need many opportunities to visualize the basic multiplication facts. Give such children markers (small paper squares will do) and let them illustrate the multiplication basic facts that are hard for them. To illustrate 6×7, for example, they should arrange their squares in 6 rows of 7 squares each. Then, to determine the product, tell them to reassemble the squares into tens and ones. For 6×7 this reassembling will result in 4 rows of 10 squares each and 2 single squares.

To carry out this activity, write the basic facts that need to be practiced on cards (put one fact on each card) and have each pupil draw as many cards as you think he can illustrate in the time available. Tell him to write each basic fact on his paper, illustrate it with his markers (or squares) and then write the answer for the basic fact he has just illustrated.





Activity 24

Remainders

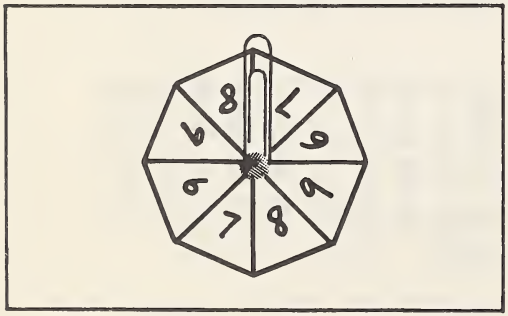
This game, which requires a specially prepared board and a spinner, gives children practice in dividing numbers that involve remainders. Only two children can play at a time. The players can be rotated by having the winner of each game challenge a new child.

The numerals written on the spinner determine the numerals to be put on the playing board. Here are four sets of corresponding numerals for the spinner and the board:

Spinner	*Board*
6-7-8-9	18 to 59
3-4-5-6	12 to 29
4-5-6-7	14 to 39
7-8-9	18 to 69

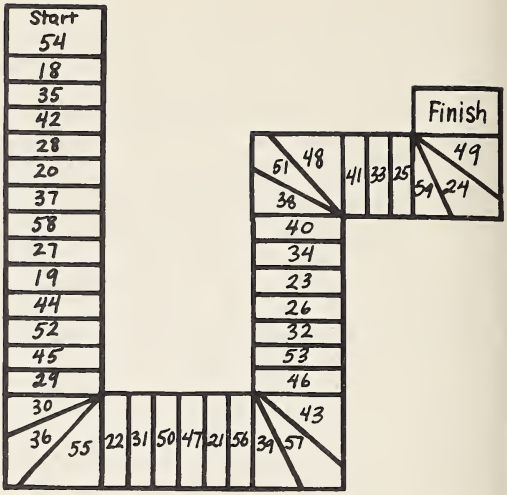
The spinner can be drawn on a piece of oaktag. Be careful to leave enough space around the spinner so that a child can hold the card without touching the spinner with his fingers. For the first three sets of spinner numerals given above, the spinner should be eight-sided. The last set of numerals will require only a six-sided spinner. If the spinners are made in this way, each numeral can be used twice on it. If an eight-sided spinner is used for 7, 8, and 9, use two of the numerals three times.

In making the spinner, make strong pencil or crayon lines to delineate each section or wedge. Fasten a paper clip to the center of the spinner by putting the double flanges of a brass brad through the paper clip and then through the spinner. Fold back the two ends of the brad on the under side of the card. (It is important to fasten the brad loosely, so that the clip can spin easily.) The illustration below shows a spinner with the first set of spinner numerals listed above. Note that the numerals 6 to 9 have been used twice on the spinner.



The playing board to accompany this spinner should look like the one above at the right. The path from "start" to "finish" could, of course, take any shape. There should be enough space on each side of the numerals so that markers will not cover them.

Each child should be given a distinctive marker. To start the game, each child should place his marker beside the numeral 54. One child spins. If the clip stops on a numeral showing a number that divides



54 (i.e., without a remainder), he must stay there. If it stops on a numeral showing a number which will leave a remainder when 54 is divided by it, he can move his marker forward as many spaces as the number in the remainder indicates. For example, if the spinner stops at 8, the answer to "54÷8" is "6 and a remainder of 6." The child can advance his marker 6 spaces (to the number 37). When he takes his next turn, the number he spins is divided into 37. The first one to reach "Finish" wins the game.

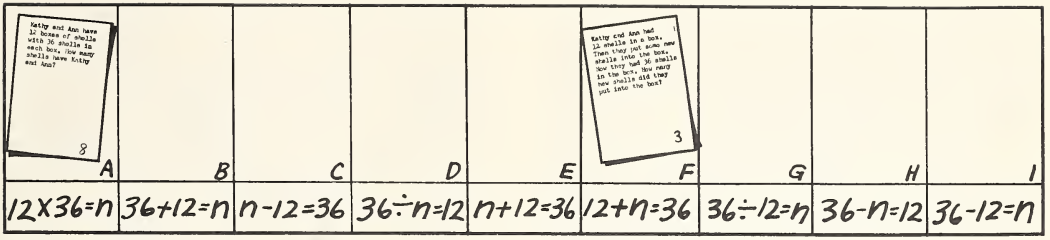
Activity 25

Solve it

The following activity will give the children practice in recognizing the proper equations for problem situations. The activity can be used for problems that can be solved in one step. This is an individual, not a group, activity.

The child will need a board and some problem cards. To make the board, mark off a piece of oaktag or cardboard 22½″ x 4½″ into spaces 2½″ x 4½″. Then draw a line about an inch above the lower edge of the board to make small spaces below the larger spaces. Letter each large space A to I, as shown in the illustration on page 369, and write an equation in each of the smaller spaces.

To carry out the activity, the child is given a board and the problem cards. (As explained below, he should have more than 9 of these problem cards.) He selects a problem card and looks for an equation that fits the problem. He then places the problem card



in the space above the equation. Then he solves the equation and writes on a piece of paper the letter of the space, the number of the problem, and the answer. For the first equation shown in the illustration above, he would write "A, 8, 432."

Since each board will show 9 equations, you should prepare more than 9 problem cards for each board. The problems should be written on 3″ x 2″ cards and should be numbered. A sample set of problems and equations is given above. Extra problems are made so that the child will always have to do some thinking in deciding which equation fits a given problem. Otherwise, no decision would be required after a few equations had been eliminated. Warn the children that they may have more than one card for some of the spaces on the board. The problems should differ only in wording. The objects, names of characters, and numbers used should be the same.

Several boards may be made with different equations and different sets of problems. Remember that in division problems you should use numbers that can be divided without remainders. The children can either verify their own answers or exchange boards and verify one another's work.

Here is a sample set of problems and equations:

1 Kathy had 36 shells in a box. Ann put 12 more shells into the same box. How many shells were in the box then? (36+12=n)
2 Kathy had some shells in a box. Ann put 12 more shells into the same box. Then Kathy had 36 shells in the box. How many shells were in the box to begin with? (n+12=36)
3 Kathy and Ann had 12 shells in a box. Then they put some new shells into the box. Now they had 36 shells in the box. How many new shells did they put into the box? (12+n=36)
4 Kathy had 36 shells in a box. She gave some of them to Ann. Then she had 12 shells left. How many shells did she give to Ann? (36−n=12)
5 Kathy had 36 shells in a box. Ann took 12 of them out of the box. How many shells were left in the box? (36−12=n)
6 Kathy had some shells in a box. She took out 12 of them. Then 36 shells were left in the box. How many shells did she have in the box to begin with? (n−12=36)
7 Kathy had 36 shells in a box. Ann helped her put them into 12 smaller boxes, with the same number in each box. How many shells did they put into each of the smaller boxes? (36÷n=12)
8 Kathy and Ann had 12 boxes of shells with 36 shells in each box. How many shells did Kathy and Ann have? (12×36=n)
9 Kathy had 36 shells in a box. Ann helped her put 12 shells into each of several small boxes. Into how many small boxes did Kathy and Ann put the shells? (36÷12=n)
10 Kathy had 36 shells in a box, and Ann had 12 shells in another box. The girls put all the shells into a larger box. How many shells were in the larger box? (36+12=n)
11 Ann had 12 shells in a box. Kathy put some more shells into the box. Then there were 36 shells in it. How many shells did Kathy put into the box? (12+n=36)
12 Ann had 36 shells in a box. She put them into 12 smaller boxes with the same number in each box. How many shells did she put into each of the smaller boxes? (36÷n=12)

Activity 26

Try for the number

Cut twelve 3″ x 5″ white cards in half to make three sets of 8 cards. Number the cards in each set from 2 to 9. Cut in half twelve pink (or any color except white) 3″ x 5″ cards to make three sets of 8 cards each. Label the cards in each set with the numerals 200, 300, 400, 500, 600, 700, 800, and 900. (If you want to use this game for slow learners, label these cards with the decade numbers from 20 to 90 instead of the "hundreds" numbers.) Four or fewer children may play this game.

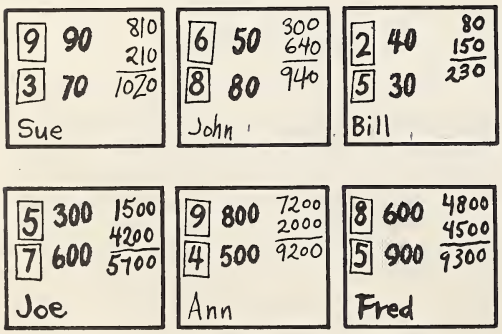
The three sets of white cards should be well shuffled. Also shuffle the three sets of pink cards. Place the pile of white cards face down on the table. Also place the pile of pink cards face down beside the pile of white cards. Each player should draw 2 white cards and 2 pink cards to begin the game.

The object of the game is to see who can manipulate his numbers to come nearest to a game score of 10,000. A player does this by multiplying the number shown on one of his pink cards by the number on one of his white cards, then multiplying the number shown on his other pink card by the number on his remaining white card and adding the two products together. He should do all of the figuring on paper so that it can be verified. It is unlikely that a child will get a score of exactly 10,000. If a player has a score that is no more than 1000 points over or under 10,000, he can lay down his cards. If other players also have scores within 1000 points of 10,000, they lay down their cards, too. Then the scores are compared. The player whose score is closest to 10,000 is the winner of the game.

If no one is able to lay his cards down, the scores of that hand are not counted, but the game continues with the same hand except that now the players may improve their hands by drawing one card and discarding one. A player may draw either a white card or a pink card, but he must discard a card of the same color. Discarded cards are placed at the bottom of their respective packs.

If the game is played with decade numbers instead of "hundreds" numbers shown on the pink cards, the method of play is the same except that, since the pink cards show smaller numbers, the total aimed for is 1000 instead of 10,000. To lay cards down, a player must have a score that is no more than 100 points over or under 1000.

Sample hands for both the slow learners and the able students, together with the computation kept by the players, are shown below.



In the first set of hands in the illustration, both Sue and John think themselves winners because they have scores within 100 points of 1000. Sue proves to be the winner because her score of 1020 is within 20 of 1000, while John's score of 940 is within 60 of 1000.

In the next set, Fred is the winner because he is closer than Ann to 10,000.

This game may be adapted for use with multiplication of any two-figure and three-figure numbers. If mental calculating is required from the able pupils, be sure to use such numerals as 21, 72, 309, 405, and 610.

Activity 27

Crossword puzzles

Crossword puzzles can be used to review many arithmetic ideas. Both able and slow pupils can work them, and they can use spare time profitably in this way.

It is a good idea to duplicate a supply of each crossword puzzle in advance. Three sample puzzles are given here. The two puzzles below are intended for the slow pupils only. You may have to assist these pupils in getting the idea of how to use the key and in being resourceful about clues obtained from the key.

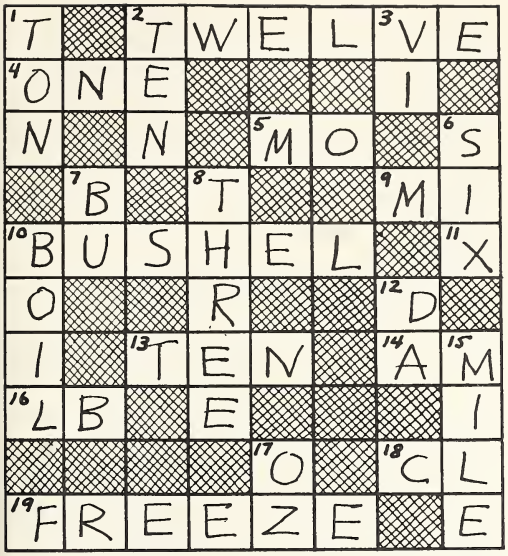
Able pupils may enjoy making crossword puzzles of this simple kind. Be sure they confine the content to arithmetic terms and facts.

Across
1. 21 ft. = --- yd.
4. 1 fewer than 1 doz.
5. Inch
7. Each one is divided into 12 and 52 parts

Down
1. --- of them equal an hour
2. Six
3. 4 equal 3 doz.
6. Day

Across
1. Water at 212° will ----
4. 8 qt. = ---
7. Part of a foot
8. 2 c. = 1 ---

Down
1. 4 pk. = 1 ---
2. Water at 32° turns into ---
3. 16 pt. = 1 of them
5. 8 fl. oz. = 1 of them
6. 5280 ft. = 1 of them



The clues given below should cause you to think of the correct word or abbreviation or to compute to find a numeral to which the word belongs. You may use Roman numerals and abbreviations if you need to. Use capital letters only.

Across

1. Ton
2. Unequal parts of a year
4. Miles you can walk in 20 minutes
5. Any one of 2 across
9. 1760 yd.
10. 32 qt.
11. Ten
13. Pints in 5 quarts
14. After midnight
16. 16 oz.
18. 150
19. 32^{0}

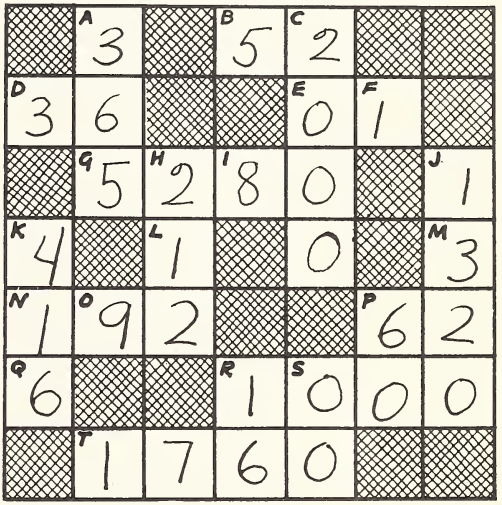
Down

1. 2000 lb.
2. Gallons in 80 pints
3. Six
6. Yards in 18 feet
7. 10 across
8. 36 months equals -- of them.
10. 212^{0}
12. Day
15. 1760 yd.
17. Ounce

Activity 28

Cross number puzzle

Cross number puzzles of the type shown at the right can be made on sheets of 9″ x 12″ paper. The puzzle illustrated uses measurement facts. The information shown in parentheses should not appear on the puzzles given to the children. If this particular puzzle is used, tell the children that the clue for each letter across or down should suggest a number. Sometimes several numbers will be suggested by a clue, but only one of these numbers will be usable. For example, the clue for A across is "yard." This clue suggests two numbers, 36 and 3, since there are 36 inches and also 3 feet in one yard. But as only one space is to be filled, the child knows that 3 must be the correct number. When he has written 3, he also knows that A down will be a number beginning with 3. As he finds each number, he should write, after each clue, the information he used to find the right number. This information is shown in parentheses in the sample puzzle.



Across

A Yard (3 ft. = 1 yd.)
B Year (52 wk. = 1 yr.)
D Yard (36 in. = 1 yd.)
E Cent, with something missing ($.01)
G Mile (5280 ft. = 1 mi.)
K Bushel (4 pk. = 1 bu.)
L 100¢ (1 dollar)
M Yard (3 ft. = 1 yd.)
N 6 bu. (6 bu. = 192 qt.)
P 1 hr. and 2 (1 hr. = 60 min.)
Q 2 × --- = 1 doz. (2 × 6 = 12)
R 2 × --- = 1 ton (2000 lb. = 1 T.)
T Mile (1760 yd. = 1 mi.)

Down

A Year (365 da. = 1 yr.)
B Nickel (5 cents = 1 nickel)
C Ton (2000 lb. = 1 T.)
D 4 × --- = 1 doz. (4 × 3 = 12)
F 12 × --- = 1 doz. (12 × 1 = 12)
H Water boils (212^{0} above 0)
I 1 gal. (1 gal. = 8 pt.)
J 4 × --- = 1 mi. (4 × 1320 ft. = 5280 ft. 5280 ft. = 1 mile)
K 13 bu. (1 bu. = 32 qt.) (13 bu. = 416 qt.)
O 4 × --- = 1 yd. (4 × 9 = 36; 36″ = 1 yd.)
P Minute (60 sec. = 1 min.)
R Lb. (16 oz. = 1 lb.)
S Part of 100

The very able pupils may wish to try constructing similar puzzles. Be sure they can justify the clues they provide.

Activity 29

Problem relay

This game can be played by the whole class, or by the slower pupils, if supervised. The game involves group solving of problems. As described below, the game gives practice in solving two-step problems, but it can be adapted to give practice in solving other types of problems.

You will need to provide some two-step problems (get them from old textbooks or make them up yourself). Paste, staple, or typewrite each one on a 3″ x 5″ card. Be sure the processes involved in the solutions do not go beyond the range of knowledge that the children have acquired at this point.

To play this game, separate the children into two teams. Appoint one child as captain for the A team and another child as captain for the B team. To start the game, Captain A selects a problem card from the pack. He reads it aloud and then consults with the members of his team to determine the proper "master" equation for its solution. He writes this equation on the board and explains it. If Team B accepts this equation as correct, Team A gets 1 point. An able pupil may keep track of points earned by writing the number of points on the board or on paper under the team names.

When it has been determined that the "master" equation is correct, Captain A calls on the next person in line on his team to find the number that the screens in the master equation stand for. If computation is necessary, this pupil should compute on the board. When he has finished, Team B decides whether or not to accept his work. If they accept it, Team A earns another point, and proceeds to the final step—using the newly found numbers to find the answer to the problem, the *n* in the equation. Captain A calls on the next pupil to find this number. If Team B does not challenge the answer, Team A earns another point. It is thus possible to earn a total of 3 points for a two-step problem.

After Team A has successfully solved the first problem, Team B draws a problem and proceeds to solve it step by step, just as Team A did. At any point along the way, the opposing team may challenge a step in the solution. If it can be shown that the thinking or computation is incorrect, that team gets a chance to take over and make points.

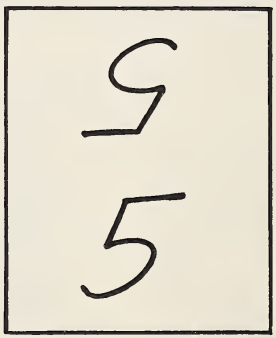
If you wish to adapt this game to one-step problems, you may use the cards described in Activity 14, page 362, if they are available. Only two points may be scored for the solution of each of these problems—one point for the correct equation and one point for the correct answer. If you make new cards for this game, they may be combined with those made for Activity 14.

Activity 30

Call the average

This game provides practice in estimating and computing averages. It also helps the children to make generalizations about how the average is affected by zero or by one or more numbers that are much larger or much smaller than the other numbers.

Prepare in advance sets of cards with numerals on them. (3″ x 5″ cards may be cut in half to make 3″ x 2½″ cards.) On each card write a numeral in two positions, so that it can be read from two positions. (See the drawing below.) You can control the difficulty of the game by the numerals used on the sets of cards. For example, if you are preparing a set



of cards for the slower pupils, make 5 cards each of the numerals 0 to 9 and about 5 cards more, each showing a two-figure numeral. No number larger than 25 should be represented.

For the able pupils, prepare a set of cards made up of two-figure and three-figure numerals and a few zeros. For example, you can prepare cards with any of the numerals from 20 to 50, five cards for larger numerals like 95, 105, 115, five cards for the numbers 1 to 5, and two or three cards with zero.

Five or more children may play the game at one time. Each child draws one card from the pack and places it face up in front of him. All of the children

look at the exposed cards, and each child estimates the average. Someone makes a list of the estimates. Then the children all compute to find the average. Remainders should be disregarded, both in estimating and in computing. The winner is the child whose estimate is nearest to the computed average. In case of a tie after a predetermined number of rounds have been played, subsequent rounds can be played to determine who came closest to the average the greatest number of times. Used cards should be placed to one side until the pack has been used up. Then the whole pack may be reshuffled, and play can continue as before.

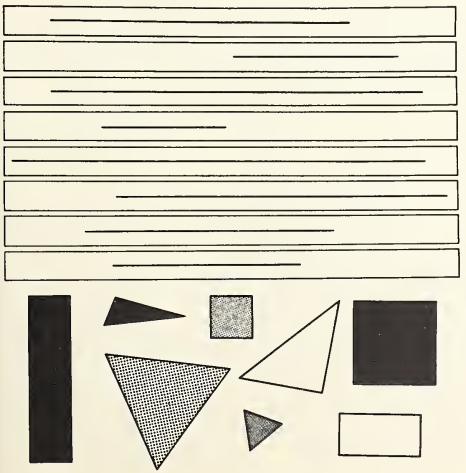
Activity 31

Judging lengths

The purpose of this activity is to give the children experience in judging perimeters and to impress on them the idea that a perimeter is the length of a closed line.

Cut from colored paper different-sized squares, rectangles, and triangles. The perimeters of these polygons may vary from 3 to 10 inches.

Then cut strips of oaktag, making each strip about 1 inch wide and 11 inches long. On each strip draw a line that is the same length as the perimeter of one of the polygons. Do not put any dimensions on the strip. Begin each line at a different distance from the left end of the strip. (This forces the children to judge the lengths entirely "by eye.") Include several extra strips with lines that do not match the perimeters of any of the polygons. See the picture of typical strips and polygons (reduced in size) below.



A child may work alone on this activity, or two children may work together with a set of the materials. Have them place the polygons and strips on a table, with the strips face up so that all the lines can be seen. Each child should pick up a polygon and decide which of the lines on the strips is the same length as the perimeter of the polygon he is holding. When he has decided, he places one side of the polygon on the line, puts a finger on the line to indicate the length of the side, turns the polygon and places the next side at that point on the line, and so on, thus verifying his judgment. If his judgment has been correct, he picks up another polygon and finds a line that he thinks is equal to its perimeter. If he has misjudged, he looks for another line that he thinks might be the right one.

The data below may be used for making the polygons and also the perimeter lines:

1-inch square; perimeter 4 inches
2-inch square; perimeter 8 inches
rectangle 1 inch x 4 inches; perimeter 10 inches
rectangle 1 inch x 2 inches; perimeter 6 inches
equilateral triangle with 1-inch sides; perimeter 3 inches
equilateral triangle with 3-inch sides; perimeter 9 inches
right triangle with sides of 1½ inches, 2 inches, and 2½ inches; perimeter 6 inches
scalene triangle with sides of 2 inches, 1¾ inches, ¾ inch; perimeter 4½ inches

Activity 32

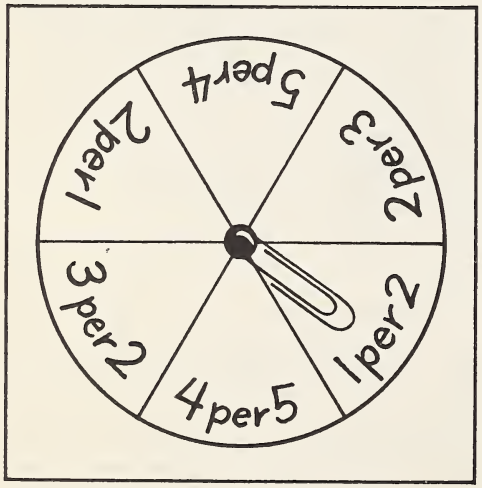
Spin-a-rate

"Spin-a-rate" will give pupils practice in recognizing different ratios that can be written for one rate.

Make Spin-a-rate cards 4½″ square. Divide each card into nine 1½″ squares. In each square write a ratio that expresses an equivalent for one of the six rates you choose to use for the game. Vary the ratios on the cards so that no two cards are exactly alike and so that no one card contains equivalent expressions for all six rates. Some of the ratios, of course, may be repeated on different cards. A sample card is shown on page 374. You may make as many cards as there are children in the class, since the game may be played either by a small number of children or by the whole group.

On a spinner, like the one pictured on page 374, express the six rates you have chosen and whose equivalents appear on the Spin-a-rate cards. This time, write the rates in the form "2 per 1," "3 per 2," etc.

9/18	10/8	21/14
6/3	40/32	16/24
4/8	15/10	45/36



The spinner may be made of a piece of oaktag or cardboard about 4″ square, a paper clip, and a brass brad. Draw a circle about 3″ in diameter and divide it into six sections for the six rates you are going to use. Then fasten the paper clip to the center of the card by putting the double flanges of the brad through the clip and then through the center of the circle. (It is important to fasten the brad loosely, so that the clip can spin easily.

You will also need some markers for the children to place on the Spin-a-rate cards. (Corn, buttons, or small pieces of paper may be used.) Each child playing will need nine or more markers.

When the game is played by small groups of children (two to six), have each child take a card and nine or more markers. Then have the players take turns spinning the spinner and playing. The rate the child uses is the one expressed in the section where the spinner stops. If there is a ratio on his card that expresses this rate, the child puts a marker on that space. Only one space can be filled at each spin. The child who fills his card first is the winner.

When the game is played by a large group or the whole class, each child is given a card and nine or more markers. Then one child spins the spinner and calls out the rate indicated. Each time he spins and calls a rate, any child who has on his card a ratio that expresses that rate places a marker on that space on his card. The winner is the player who covers all the spaces on his card first. The winner becomes the caller for the next game.

Below are given a set of six rates and some equivalent ratios. You may want to substitute others of your own choosing. The ratios should be distributed among the Spin-a-rate cards.

On spinner — *On cards*

1 per 2—3/6, 4/8, 5/10, 6/12, 7/14, 8/16, 9/18, 10/20

2 per 1—6/3, 8/4, 10/5, 12/6, 14/7, 16/8, 18/9, 20/10

2 per 3—4/6, 6/9, 8/12, 10/15, 12/18, 14/21, 16/24, 18/27

3 per 2—6/4, 9/6, 12/8, 15/10, 18/12, 21/14, 24/16, 27/18

4 per 5—8/10, 12/15, 16/20, 20/25, 24/30, 28/35, 32/40, 36/45

5 per 4—10/8, 15/12, 20/16, 25/20, 30/24, 35/28, 40/32, 45/36

This activity may be adapted to the work with comparisons by writing comparisons on the spinner in the form of "3 to 2," "2 to 1," etc.

Activity 33

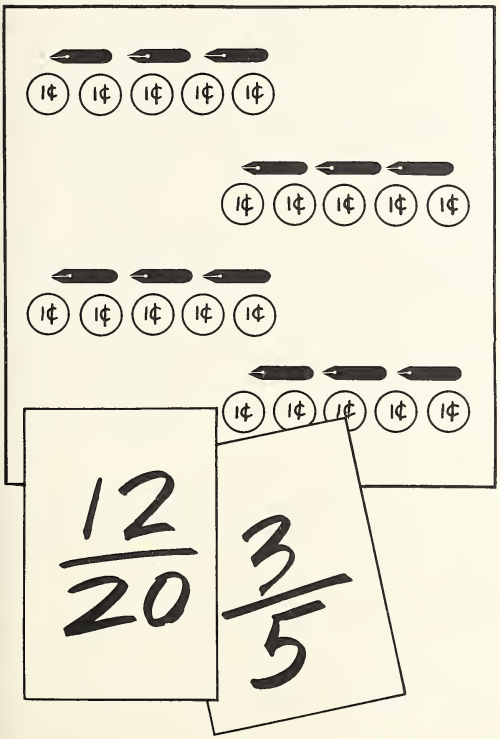
Ratio

"Ratio" is designed to give all the pupils in your class extra practice in recognizing ratios that express rates shown in pictures.

Make 12 picture cards, each 5″ square. On each card draw a picture similar to the pictures on page 117 of the pupils' book, showing objects and coins in equal groups. (This work may be done by children in the class who draw fairly well.) Select six rates (see the suggestions on page 375) and make two picture cards for each rate, or 12 cards altogether.

Make 32 ratio cards, 2″ x 3″. On each card write a ratio that expresses a rate illustrated by one of the picture cards. Although any number of ratios can be written for the six rates illustrated on the picture cards, use only 32 ratios and make one card for each.

Four children should play the game at one time. One child should be the dealer, who gives each player three picture cards and 8 ratio cards. Each player should lay his picture cards face up in front of him and hold the ratio cards in his hand. Each player should look through his ratio cards to find ratios that are illustrated by his picture cards. Each ratio card that matches a picture card should be placed beside that picture card. (See the illustration below.)



After the players have laid down all the ratio cards they can, the child to the right of the dealer draws one ratio card from those that the child to his right is still holding in his hand. If the ratio card drawn illustrates one of his picture cards, he lays it beside the correct picture card. If it does not match any of his picture cards, he puts it in his hand with the other ratio cards he is holding. The next player draws a card from the child on his right, and the play continues in this way.

The object of the game is to place all the ratio cards beside picture cards. The player who first gets rid of all the ratio cards in his hand is the winner.

For one set of picture cards, use the following as a guide in drawing the objects. Make two cards for each:

4 groups of 3 per 5
3 groups of 2 per 4
2 groups of 6 per 5
3 groups of 2 per 5
3 groups of 4 per 6
2 groups of 5 per 10

Picture cards may be made with either X's and O's or with different objects (apples and pennies, for example).

For the set of picture cards described above, the following ratios may be used on the ratio cards:
3/5, 5/3, 6/10, 10/6, 12/20, 20/12, 2/4, 4/2, 4/8, 8/4, 6/12, 12/6, 6/5, 5/6, 12/10, 10/12, 2/5, 5/2, 4/10, 10/4, 6/15, 15/6, 4/6, 6/4, 8/12, 12/8, 12/18, 18/12, 5/10, 10/5, 10/20, 20/10

For a second set of picture cards, use the following as a guide in drawing the objects:

3 groups of 3 per 4
2 groups of 2 per 10
2 groups of 5 per 8
4 groups of 2 per 3
3 groups of 4 per 5
2 groups of 8 per 15

Ratio cards for this second set may include the following ratios:
3/4, 4/3, 6/8, 8/6, 9/12, 12/9, 2/10, 10/2, 4/20, 20/4, 5/8, 8/5, 10/16, 16/10, 2/3, 3/2, 4/6, 6/4, 6/9, 9/6, 8/12, 12/8, 4/5, 5/4, 8/10, 10/8, 12/15, 15/12, 8/15, 15/8, 16/30, 30/16

Activity 34

Flannel board

A sturdy felt board or flannel board is a very helpful device, since it can be used many times throughout the year. It is a useful device for demonstrating various geometric shapes, and it also may be used to provide experiences with fractions.

Three fourths of a yard of 36″ flannel or felt, preferably a dark color, will be sufficient to cover a 24″ x 32″ piece of heavy cardboard or easelboard. Cutouts to be used with this board should be made of oaktag, felt, or colored construction paper. Small pieces of colored felt can be purchased in most hobby shops.

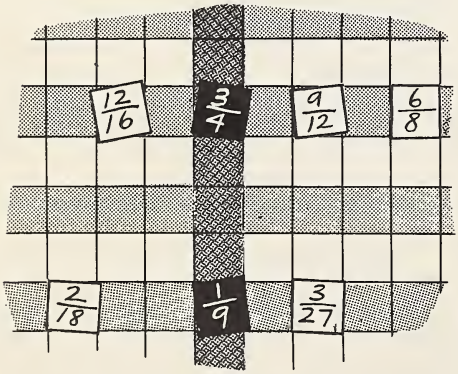
Able pupils can prepare cutouts of such shapes as squares, rectangles, triangles, and hexagons. The supply of cutouts should include a variety of sizes for each figure.

If you use oaktag or construction paper for cutouts, paste a small strip of coarse sandpaper on the back of each cutout. This will make the oaktag stick to the board when it is given a slight downward pressure. For fraction study, you can make equal circular disks and cut them into fractional parts (halves, thirds, fourths, etc.). Keep some disks whole to use in demonstrating mixed numbers. And if you make two or three sets of each fractional piece, equivalent improper fractions and mixed numbers can be demonstrated.

Activity 35

Fraction row

First make a playing board. Mark off a sheet of 8½″ x 14″ paper (oaktag or cardboard) into one-half inch squares. If 14″ paper is not available, one and one-half sheets of 8½″ x 11″ paper can be pasted or stapled together. You will need 27 horizontal rows of squares and 17 vertical rows of squares. The middle vertical row of squares should be colored red, and the top horizontal row and every other horizontal row should be colored green. You will have 14 green rows.



Cut from colored cardboard or paper 14 one-half inch squares. Write on one of the cards the first fraction (½, ⅓, etc.) from each of the groups listed in the next column. These are the master fraction cards. Cut from white cardboard or paper 54 more one-half inch squares. Write on each square one of the other fraction numerals (¾, ⅔, etc.) listed at the top of the next column. These are the equal-fraction cards.

½	¾	⅜	⅛	5/10	6/12	7/14	8/16	9/18	10/20	11/22	
⅓	⅔	⅓	4/12	5/15	6/18	7/21					
⅔	4/6	6/9	8/12	10/15	12/18	14/21		4/5	8/10	12/15	16/20
¼	⅜	3/12	4/16	5/20				⅙	2/12	3/18	4/24
¾	⅝	9/12	12/16	15/20				5/6	10/12	15/18	20/24
⅕	2/10	3/15	4/20					1/7	2/14	3/21	
⅖	4/10	6/15	8/20					⅛	2/16	3/24	
⅗	6/10	9/15	12/20					1/9	2/18	3/27	

To play the game, first shuffle all 68 fraction cards and place them face down in a pile before the players (two, three, or four children may play). Each child in turn draws five of the fraction cards and lays them face up in front of him. If he finds he has drawn a master fraction card, he should place it on any red square in a green row on the playing board.

After a master fraction card has been put on the playing board, equal-fraction cards can be put on green squares to the left or right of the red square. For example, after someone has played ⅖, then 4/10, 6/15, and 8/20 may be played beside it in either direction and in any order. However, no equal-fraction card can be played until the master fraction card has been put on the playing board.

At each turn, the player may place on the board as many fraction cards (master or equal) as he can. At the end of each turn, the child records his count for that turn (see below), and then draws as many new fraction cards as he has just played. Thus, if he lays four fraction cards on the board, he draws four new fraction cards. If a child cannot lay any fraction cards on the board, he may still draw one new fraction card. The children can draw as long as any fraction cards are left.

The count for each play is 5 points for each master fraction card played and 1 point for each equal-fraction card played. The winner is the child who has the largest score at the end of the game. A game is over when no fraction cards remain to be drawn and when no more plays can be made.

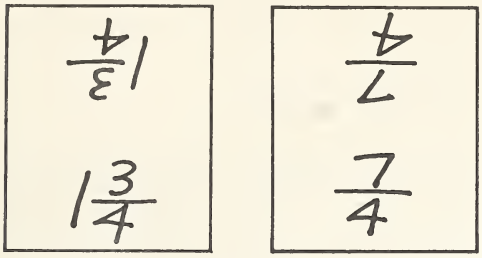
Activity 36

Fraction match

The game of "Fraction match" will provide practice in changing improper fractions to mixed numbers.

Provide one set of 36 white cards (2½″ x 3″) with improper fraction numerals on them with denominators through ninths. These are the fraction cards. Also provide one set of 36 colored cards (2½″ x 3″) with mixed numbers on them. These are the mixed number cards. For each improper fraction on a fraction card

there should be an equivalent mixed number on a mixed number card. The numerals should be written twice on each card so they can be read from either side. (See the illustration below.)



Three to six children can play the game. One child should take the set of 36 fraction cards and distribute 6 cards to each player. The remaining cards in this set, if any, are put aside. The set of mixed number cards is placed in a central position, face down, where all can reach it.

The object of the game is to see who can match all his fraction cards with corresponding mixed number cards. (You will find that some children will have to compute on paper to change the improper fractions to mixed numbers. Have scratch paper available for this.)

The children take turns drawing a mixed number card from the pile. If the mixed number card is equivalent to a fraction card in his hand, the child can match the two cards, and lay them down in front of him. If a child cannot match any cards, he must put the mixed number card in a discard pile. The child next to him continues the play.

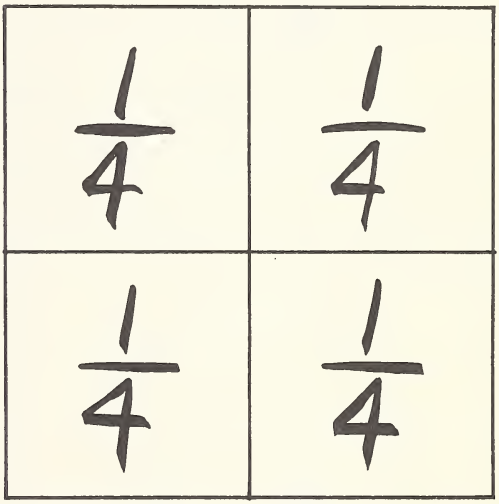
If a child lays down two cards that do not match, he should be challenged by another player. If possible, the challenger should match the mixed number card that was incorrectly laid down with a fraction card in his hand. If the mixed number card that was incorrectly laid down does not match any fraction card of the challenger, it should simply be discarded. The child who made the error must take back the fraction card he tried to play and wait for his next turn before he plays again. Consequently, all players will find that it pays to watch each time one of the other players lays down a pair of mixed number and fraction cards.

When the set of mixed number cards has been used up, the discard pile should be reshuffled and play continued as usual until one player has matched all of the fraction cards in his hand with mixed number cards. He is the winner.

Activity 37

Make one

This game requires a set of fraction cards. To make these cards, divide 11 equal-sized squares of paper or cardboard into fractional parts that show halves, thirds, fourths, fifths, sixths, eighths, ninths, tenths, twelfths, fourteenths, and sixteenths. Label each fractional part. See, in the picture below, how the square is cut into fourths and labeled. Cut the squares into their respective fractional parts.



Put all the cards into a container, and let the children take turns drawing one card at a time from the box. The cards drawn are to be rearranged into the squares from which the cards were cut. The purpose of the game is to see how many squares each child can complete.

Any card not usable at once may be kept by the child and used by him when he needs it, or it may be traded for a card another child has not used. Each player has the privilege of making such a trade instead of drawing a new card from the container. Any time a child trades a card, he must give one of his own unused cards in exchange.

The game ends when all the cards have been drawn from the box. The winner is the child who has completed the greatest number of squares.

For most pupils the game has the greatest value if all the cards are the same color. However, for the slow learner who has difficulty in understanding, you may use different colors or shades for the various

squares. For example, the square cut into fourths may be red and the one cut into sixths, blue.

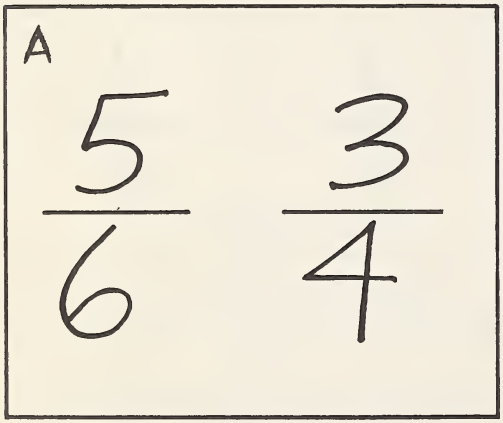
The game works out best when it is played by groups of two, three, or four children.

Activity 38

Finding the common denominator

This activity provides practice in finding a common denominator for fraction numerals that originally have unlike denominators.

The activity requires a number of small cards on which pairs of fraction numerals with unlike denominators (¾ and ⅔, ⅓ and ⅙, etc.) have been written. Each card should be labeled with a letter. Prepare 25 to 35 of these cards, depending upon the number of pupils participating and upon the number of cards you want for each pair of pupils. Control the fractions so that the common denominator to be found in each case does not exceed 36. The picture below shows a typical card.



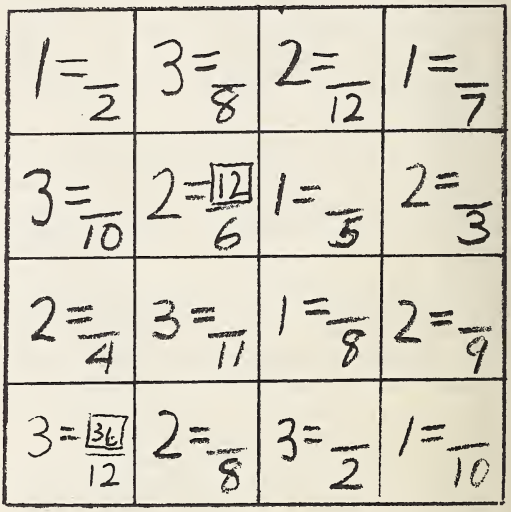
Let the pupils work in pairs. Each child should have his own sheet of paper and pencil. Give each pair of children five to ten cards. One child turns up a card. Both children, then, write first the identifying letter of the card. Next they write the fraction numerals as they appear on the card. Finally, they rewrite them with a common denominator. If you wish, you may also tell them to circle the numeral that shows the larger fraction. (Of course, they are to make the circles on their papers, not on the cards.) The child who finishes first turns up the next card, and the activity continues in the same way. To vary the activity, the children can be instructed to circle the numeral that shows the smaller fraction.

These cards may also be used to give practice in finding common denominators as preparation for adding and subtracting fractions. Suggestions for their use and adaptations of this activity will be found under "Providing for the able pupil" on page 320 of this *Teaching Guide.*

Activity 39

Fraction bingo

Use 7½″ squares of paper or cardboard for the bingo cards. Divide each of these squares into 16 equal rectangles by drawing three horizontal rules and three vertical rules. Each child who plays the game will need one of these cards. (A card, with two answers inserted, is shown below.)



Now make a list of 33 whole numbers and some improper fraction equivalents (2=⅛, for example) and from this list select the whole numbers and equivalents you need to fill in each bingo card. This selection should be made at random so no two bingo cards will be alike. (The card in the picture is selected from the 1, 2, 3 sequence.) When you fill in the cards, omit the numerators of the improper fraction equivalents. Next write, on small square cards, the numerators that are required in the improper fractions on the bingo cards. Make two sets of these answer cards.

Since a wide variety of fractions and whole numbers may be used for this game, choose those that you feel will be most helpful to your class. Whatever

fractions and whole numbers you choose, your list should never exceed 33, because if too many numbers are involved, the possibility of filling a bingo card and thereby having a winner is lessened. The game can be varied by using mixed numbers as well as whole numbers ($2\frac{1}{2}=\frac{5}{2}$).

The game may be played by a small group of children or by the whole class. When a small group of children play it, have them take turns drawing the answer cards. Whenever an answer completes a fraction on a child's bingo card, he places the answer card on that space on his bingo card. If he cannot use the answer card, he puts it back in the container from which it was drawn.

When the whole class plays the game, distribute a bingo card and some markers (corn, buttons, small pieces of paper, etc.) to each child and choose a caller. The caller draws one answer card at a time and reads the answer to the class. He then lays it face up on the table so that he can check the winner's answers at the end of the game. Each time the caller reads an answer, each child decides whether or not he can use the answer in one of the spaces on his card. If he can, he places a marker in the space where the answer fits. The children are to fill only one space on their cards at a time.

When a child has filled four spaces in a vertical row, in a horizontal row, or in a diagonal row, he calls "Bingo" and gives the answers he has used to fill the spaces on his card. The caller checks the answer cards to make sure that the answers the player gives have been called.

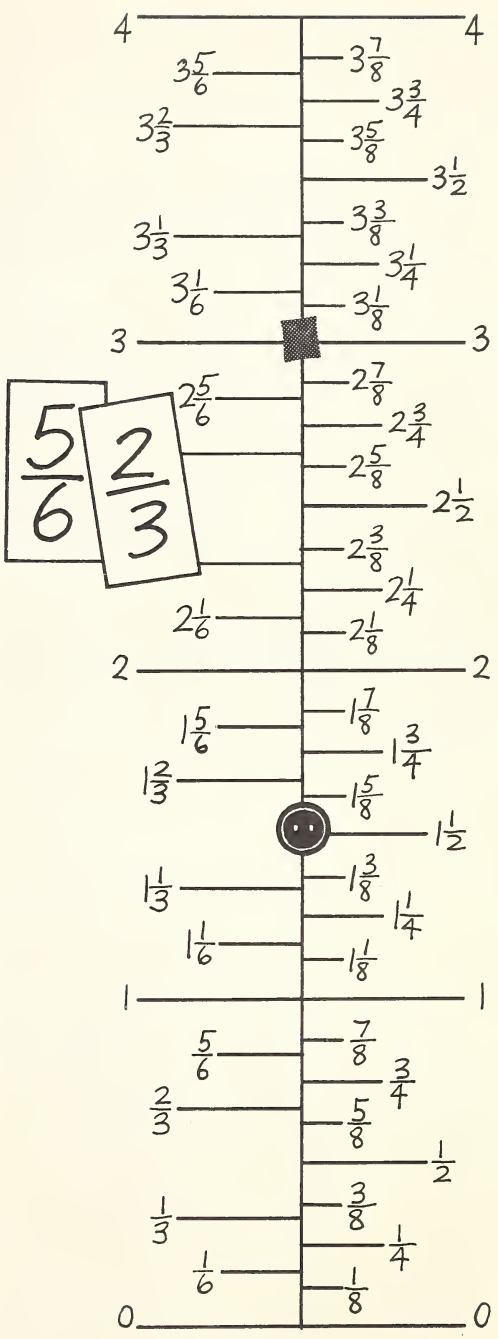
Activity 40

Fraction race

The game of "Fraction race" will provide practice in adding mixed numbers and fractions.

You will need to prepare a game board made of oaktag 8″ x 28″. (See the picture in the next column.) On the game board draw a vertical line 24″ long and divide it into four equal parts. Then draw a horizontal line at each end of the line and also at each division you have made and label these lines 0 to 4. Between the whole numbers (0-4) make and label subdivisions for halves, fourths, and eighths on the right side of the line and for thirds and sixths on the left side.

Also make a set of 82 fraction cards (1½″ x 3″) with the fraction numerals listed at the top of column 1, page 380, written on them. Make two cards for each numeral.



1/16	2/16	3/16	4/16	5/16	6/16	7/16	8/16
9/16	10/16	11/16	12/16	13/16	14/16	1/8	2/8
3/8	4/8	5/8	6/8	7/8	1/6	2/6	3/6
4/6	5/6	1/3	2/3	1/4	2/4	3/4	1/12
2/12	3/12	4/12	5/12	6/12	7/12	8/12	9/12
10/12							

Have a marker available for each player. Each marker should be of a different kind.

Two, three, or four players can play this game. The object is to see who can move his marker from 0 to 4 first. To begin the game, the pack of fraction cards should be shuffled, and seven cards should be dealt to each player. The remaining fraction cards are put face down in a pile in the center of the table. The first player to the right of the dealer then draws a card from the top of the pile, puts it with those in his hand, and examines the cards in his hand to see if he has two cards showing fractions whose sum is equal to one of the fractions or mixed numbers on the board. He should look for a combination of cards that will give him the largest possible mixed number and thus enable him to move his marker the farthest. He moves his marker to that place and lays the two cards he used face up in front of him. Scratch paper and pencils should be available for those who need it to compute the sums. Anyone who makes an error loses the play and cannot move his marker that round. Whether the player has moved or not, he then discards one card from his hand and puts it face down beside the pile in the center.

On the first play the player must add the fractions on two cards. On the following plays he draws one card, puts it in his hand, selects one fraction from the cards in his hand to add to the fraction at which his marker rests on the board, advances to another place on the board, and discards one card. A round ends when a player reaches 4. To reach 4, he must use a fraction that, when added to the number on which his marker rests on the board, makes exactly 4. A round also ends when a player has laid down all his cards. This will happen more often. However, this does not mean that the player who lays down all his cards first wins, because another player, who is still holding cards, may have advanced farther than he has. The winner is the one who has advanced to the highest number. Only the score of the winner is recorded for each round, and five rounds make a game. The winner is the pupil with the highest total score.

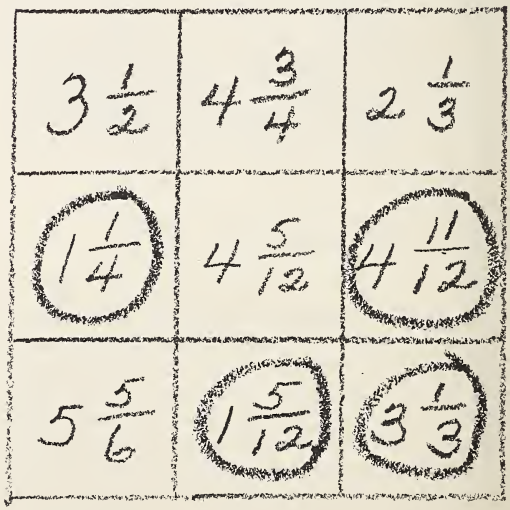
This game can be adapted for subtraction of mixed numbers by having the children start at 4 and, by subtracting the fraction on one card at a time, move down to 0. Complete details for this adaptation are given in "Providing for the able pupil" in connection with the lesson on subtraction of mixed numbers, page 330.

Activity 41

Mixed number magic squares

This activity uses addition and subtraction of mixed numbers. The objective is to fill in missing mixed numbers in rows so that the sums for the rows will be the same both vertically and horizontally.

To prepare for this activity, rule 4″ x 6″ cards into three rows of three rectangles each. Arrange mixed numbers, one in each rectangle, in such a way that, when added, the answers will be the same for each vertical and horizontal row. It will be necessary to fill all the squares in the top row with mixed numbers, because that is the only way the child can find out what the sum of each row should be. In each of the other horizontal rows omit two of the mixed numbers. A sample card is shown below. The circled numbers are the mixed numbers that should be omitted from the card made for the child. These are the mixed numbers he must find to complete the magic square. For the card shown, the sum of each row is $10\frac{7}{12}$.



Either give the child a master card (with some numerals omitted) or put a copy of it on the chalkboard. Tell the child to rule his paper into nine sections and copy the numerals in the spaces. Then explain to him that he is to find the mixed numbers for the other spaces that will make the sums the same for all verti-

cal and horizontal rows. If necessary, give him some hints on how to proceed, but do not give hints until the child asks for help. Hints may consist of discussion and questions to help him see that he knows the desired sum (from the full row) and that for all other rows he knows two of the mixed numbers used to get this sum.

Activity 42

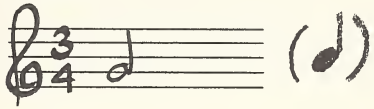
Musical fractions

In this activity, pupils who know how to read music add and subtract fractions that indicate values of musical notes to find the value of the missing note in a measure.

Prepare in advance some 4″ x 6″ cards, each card showing part of one measure. The child is to take a card and figure out the kind of note (half, quarter, eighth, sixteenth) that is needed to complete the measure.

Pictures of sample cards with one measure of music on each are shown at the right. For your convenience, the correct value of the missing note is shown in parentheses beside each picture. Since the children know the full value of the measure (¾, ¼, etc.), the problem is to find what value is needed to equal the total value of the measure. They should copy the measure and supply any note (or notes) that will complete the measure. No attempt need be made to produce a tune, but some of the children may try to do this.

Some of the children may like to make up incomplete measures. They can exchange cards with each other to see if their classmates can complete the measures.



Your notes

Reference materials

This section of the *Teaching Guide* provides you with charts and tables that will enable you to understand better both the entire program and the materials for this grade.

Bibliography

A list of books and articles that should add interest to and help in the teaching of arithmetic

Chart of problem types

An analysis of the types of verbal problems that the child learns to solve

Time schedule

The authors' estimate of the rate of progress you may expect to make during the year

Grade placement chart

A graphic means of discovering the grade placement of the important topics in arithmetic for Grades 1 to 6 in this program

Word list

A complete alphabetical list of all words used in the pupil's book

Mathematical content

A detailed topical list of the mathematical content of the book

Index

Entries are for the teaching aspects of arithmetic

Reference materials

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Problem types

non-comparison situations

type no.	problem description	sample problem	equation	computation	situation-process description	grade
N-1	Finding the resulting total when the original number and the number added are known	John had 9 apples. Tom gave him 3 apples. How many apples did John have then?	$9 + 3 = n$	$9 + 3$	additive-addition	3
N-2	Finding how many were added or how many more are needed when the original number and the resulting total or desired total are known	John had 9 apples. After Tom had given him some more apples, he had 12 apples. How many apples did Tom give him?	$9 + n = 12$	$12 - 9$	additive-subtraction	3
N-3	Finding how many there were to begin with when the number added and the resulting total are known	John had some apples. Tom gave him 3 apples. Then he had 12 apples. How many apples did John have to begin with?	$n + 3 = 12$	$12 - 3$	additive-subtraction	3
N-4	Finding the number left when the original number and the number gone are known	John had 12 apples. He lost 3 of them. How many apples did he have then?	$12 - 3 = n$	$12 - 3$	subtractive-subtraction	3
N-5	Finding the number gone when the original number and the number left are known	John had 12 apples. Then he lost some. He had 9 apples left. How many apples did he lose?	$12 - n = 9$	$12 - 9$	subtractive-subtraction	4
N-6	Finding the original number when the number gone and the number left are known	John had some apples. He lost 3 of them. Then he had 9 apples left. How many apples did he have to begin with?	$n - 3 = 9$	$9 + 3$	subtractive-addition	4
	rate situations					
R-1	Finding the total number when the number in each group and the number of equal groups are known	John had 4 boxes of 3 apples each. How many apples did he have?	$4 \times 3 = n$ $\frac{3}{1} = \frac{n}{4}$	4×3	multiplicative-multiplication rate-multiplication	3 5
R-2	Finding the number in each of the equal groups when the resulting total number and the number of groups are known	John put 4 equal groups of apples into a box. Then he saw that he had 12 apples in the box. How many apples were in each group that he put into the box?	$4 \times n = 12$ $\frac{n}{1} = \frac{12}{4}$	$12 \div 4$	multiplicative-division rate-division	5 5
R-3	Finding the number of equal groups when the resulting total number and the number in each of the equal groups are known	John put several groups of 3 apples each into a box. Then he saw that he had 12 apples in the box. How many groups of apples did he put into the box?	$n \times 3 = 12$ $\frac{3}{1} = \frac{12}{n}$	$12 \div 3$	multiplicative-division rate-division	5 5
R-4	Finding the number of equal groups when the original total and the number in each of the equal groups are known	John had 12 apples. He put a group of 3 apples into each of several boxes. How many boxes did he use?	$12 \div 3 = n$ $\frac{3}{1} = \frac{12}{n}$	$12 \div 3$	quotitive-division rate-division	3 5
R-5	Finding the number in each of the equal groups when the original total and the number of equal groups are known	John had 12 apples. He put equal groups of these apples into each of 4 boxes. How many apples did he put into each box?	$12 \div n = 4$ $\frac{n}{1} = \frac{12}{4}$	$12 \div 4$	partitive-division rate-division	3 5
R-6	Finding the original number when the number in each of the equal groups and the number of equal groups are known	John had some apples to put into 4 boxes. He put 3 apples into each box. How many apples did he have to put into the boxes?	$n \div 3 = 4$ $\frac{3}{1} = \frac{n}{4}$	4×3	divisive-multiplication rate-multiplication	5 5

comparison situations

type no.	problem description	sample problem	equation	computation	situation-process description	grade
C-1	Finding how {many more / much more} or how {many fewer / much less} one quantity is than another	John has 12 apples and 9 oranges. How many {more apples / fewer oranges} than {oranges / apples} has he?	$12 - 9 = n$	$12 - 9$	comparison-subtraction	3
C-2	Finding the smaller quantity when the larger quantity and the excess of the larger quantity or the deficiency of the smaller quantity are known	John has 12 apples. He has 3 {more apples than oranges. / fewer oranges than apples.} How many oranges has he?	$12 - n = 3$	$12 - 3$	comparison-subtraction	6
C-3	Finding the larger quantity when the smaller quantity and the deficiency of the smaller quantity or the excess of the larger quantity are known	John has 9 apples. He has 3 {more oranges than apples. / fewer apples than oranges.} How many aranges has he?	$n - 9 = 3$	$9 + 3$	comparison-addition	6
C-4	Finding the ratio of one quantity to another (situations involving "times as many as," "fraction as many as")	John has 12 apples and 4 oranges. He has how many times as many apples as oranges? He has what froction as many oranges as apples?	$\frac{n}{1} = \frac{12}{4}$ $\left(\frac{1}{n} = \frac{12}{4}\right)$	$12 \div 4$ $(4 \div 12)$	comparison-division	5 6
C-5	Finding a quantity when the other quantity and the ratio ("number times as many as" or "fraction as many as") of the first quantity to it are known	John hos 12 apples. He has {3 times as many / ⅓ as many} apples as oranges. How mony oranges has he?	$\frac{3}{1} = \frac{12}{n}$ $\left(\frac{⅓}{1} = \frac{12}{n}\right)$	$12 \div 3$ $(12 \div ⅓)$	comparison-division	5 6
C-6	Finding a quantity when the other quantity and its ratio ("number times as many as" or "fraction as many as") to the first quantity are known	John has 12 oranges. He has {3 times as many / ⅓ os many} apples os oranges. How many opples has he?	$\frac{3}{1} = \frac{n}{12}$ $\left(\frac{⅓}{1} = \frac{n}{12}\right)$	3×12 $(⅓ \times 12)$	comparison-multiplication	5 6

Time schedule

The authors do not believe it possible to provide a rigid schedule for teachers to follow in using *Seeing Through Arithmetic 5*. Differences in the backgrounds and abilities of children make a uniform schedule impractical. Nevertheless, many teachers and schools feel a need for some kind of schedule that will serve as a general guide in covering the year's work. The schedule at the right, given in six-week periods, is based upon the authors' judgment and experience with the materials and upon the experience of teachers now using Book 5. No attempt should be made to force a class to maintain the pace indicated or to retard a class to that pace.

period	pages of book 5	content
1	3-66	polygons and circles; concept of area; reteaching of multiplication of whole numbers
2	67-111	reteaching of divisian of whole numbers; multiple-step problems; finding perimeters
3	112-155	expressing rates, comparisons, and fractions; reduction of fraction numerals and ratias
4	156-197	addition and subtraction of proper fractions and mixed numbers
5	198-231	problems involving rates and comparisons (solved by use of ratios)
6	232-267	scale drawing; graphs; area af rectangles; per cents; decimals; review

topic	grade 1	grade 2	grade 3
groups	recognizing, combining, separating, and rearranging thraugh 10	recognizing, combining, separating, and rearranging thraugh 18	recagnitian through 36
number system	through 99; place value	through 999; place value	thraugh 9999; place value; preparation for carrying and borrowing
basic facts		through sums and minuends of 10; informally through products and dividends of 10	through sums and minuends of 1 through products and dividends of 36
money	coins through dime; related values; caunting	coins through quarter; related values; counting	equivalents of $1; writing; making change; additian and subtraction
measures	idea of standard unit; develapment of judgment and physical competence	idea of standard unit; concept of inch, foot, pint, quart; development of judgment	equivalents; dozen; calendar; time; weight; reduction
addition and subtraction of whole numbers			3-figure numbers; carrying and borrowing
multiplication of whole numbers			
division of whole numbers			
meaning of fractions		judgment in selecting halves and fourths	unit and non-unit through eighth reading and writing
addition and subtraction of fractions			
multiplication of fractions			
division of fractions			
ratio			preparation for reduction
graphs and scale drawing			
geometry			identification of circle, square, triangle
decimals			
percentage			
problem solving (see chart an pages 388-389)		pictorial representations; development of visualizations; intraduction af action ideas	types N-1, N-2, N-3, N-4, R-1, R-4, R-5, C-1
meaning of the processes		associating combining and separating actians with problems	order af addends and factoi inverse nature of processes
topic	grade 1	grade 2	grade 3

Grade placement of topics

Seeing through arithmetic

Scott, Foresman

grade 4	grade 5	grade 6	topic
			groups
through 999,999,999; place value	reading and understanding decimals through hundredths	development of decimal notation as a system	number system
through products and dividends of 81	used in processes	used in processes	basic facts
multiplication and division			money
scale; weight; liquid (fl. oz.); dry; mile; abbreviations	square measures; acre	cubic measures; meter; gram; liter; kilogram; kilaliter	measures
retought; extended to larger numbers	extended to larger numbers	maintained	addition and subtraction of whole numbers
1-figure and 2-figure multiplier; carrying	extended to larger numbers	maintained	multiplication of whole numbers
-figure and 2-figure divisors re divisors optional); remainders	2-figure divisors; remainders	maintained	division of whole numbers
uivalent; improper fractions and mixed numbers; use in measurement	reduction; comparing		meaning of fractions
	proper fractions and mixed numbers	maintained	addition and subtraction of fractions
	fractional part of a whole number ($\frac{3}{4}$ of 16; $\frac{1}{3}$ of 51)	proper fractions and mixed numbers	multiplication of fractions
		proper fractions and mixed numbers	division of fractions
	using ratios to compare two quantities; reduction	more complex applications in comparing quantities	ratio
meaning of linear scale on map	reading scales and scale drawing; interpreting bar graphs and pictographs	line graphs; making drawings to scale	graphs and scale drawing
parallel lines; right angle; rectangle	concept of area; perimeters; area of rectangles	identification of solids; concept of volume; volume of rectangular solids	geometry
	reading and understanding through hundredths	addition; subtraction; multiplication; division	decimals
	understanding; casual use of terms	meaning; simple uses	percentage
s N-5, N-6, C-4 (limited to "how many times as many")	types N-8, N-9, N-12, C-4	types R-2, R-3, R-6, C-4	problem solving (see chart on pages 388-389)
	extended to include fractions	extended to include decimals	meaning of the processes

Word list

This list contains the 979 different words that appear in *Seeing Through Arithmetic 5*. Of these 979 words, 658 were used in the preceding book, *Seeing Through Arithmetic 4*. These 658 words are in italics. Children who have completed The New Basic Reading Program of the Curriculum Foundation Series, through the new *More Times and Places* (4^2), will be familiar with 210 of the 321 words not used in *Seeing Through Arithmetic 4*. Words in this list that are used in The New Basic Reading Program, through the new *More Times and Places* (4^2), are indicated by asterisks (*). For children who have used The New Basic Reading Program, only 111 words will be new in this book. Of these 111 words, 40 are used in the new *Days and Deeds* (5^1) and in *More Days and Deeds* (5^2). Children using The New Basic Reading Program will have to cope with only 71 additional words.

The numeral at the right of each word indicates the page of first occurrence. For a list of words introduced in each lesson in the pupils' book, see the Lesson Briefs for the lesson in this *Teaching Guide* (pages 20 to 241).

The following forms of known words are not counted as new: inflected forms made by adding or dropping the endings *-s*, *-es*, *-ed*, *-ing*, *-n*, *-en*, and *-er*, *-est* of comparison (including those forms made by changing *y* to *i* or *f* to *v*, by dropping the final *e*, or by doubling the final consonant in the root word); possessives; derivatives formed by adding the prefixes *dis-*, *fore-*, *im-*, *re-*, or *un-*, and the suffixes *-en*, *-er*, *-ful*, *-ish*, *-less*, *-ly*, *-ment*, *-ness*, *-or*, *-ship*, *-ward*, or *-y*, and *-teen*, *-th*, or *-ty* of numerals; compounds made up of known words; contractions; nonsense words or syllables that represent sounds. Also, homographs are not counted as separate words; for example, if *pen* meaning "a writing instrument" has been introduced, *pen* meaning "an enclosure" is not counted as new.

*a, 8
abbreviation, 249
**able*, 22
**about*, 3
**above*, 93
absences, 235
*absent, 235
*acre, 249
**add*, 17
addition, 22
*addressing, 48
**after*, 22
**afternoon*, 47
**again*, 114
*age, 89
*ago, 257
*aid, 155
**airplanes*, 237
album, 134
*alike, 16
**all*, 16
allowance, 134
*allowed, 124
*along, 232
*already, 232
**also*, 21
altitude, 7
altogether, 26
**always*, 52
*among, 85
amount, 19
**and*, 3
*Andy's, 132
angle, 36
**animals*, 31
**Ann*, 19
**another*, 22
**answer*, 22
**any*, 37
**apart*, 233
apiece, 24
**apples*, 28
*applesauce, 124
approximate, 23
April, 235
*aprons, 85
Arabic, 17
**are*, 11
area, 8
*arithmetic, 20
**around*, 107
*arranged, 131
*arrived, 226
*arrowheads, 37
**as*, 17
aside, 49
**asked*, 25
**at*, 16
**ate*, 42
*automobile, 201
average, 102
**away*, 22
**baby*, 45
**back*, 41
*bacon, 154
*baggage, 258
**bags*, 50
**baked*, 26
**balloons*, 74
**balls*, 75
bananas, 154
*band, 226
**bank*, 42
**bars*, 26
**base*, 7
**basket*, 67
**bats*, 29
**be*, 20
**beach*, 263
beads, 61
**beans*, 26
**because*, 17
*become, 170
**been*, 75
**before*, 22
**began*, 47
**begin*, 41
*begun, 47
**bell*, 42
**belonged*, 89
**below*, 7
besides, 85
**best*, 105
**better*, 67
**Betty*, 45
*between, 11
**bicycle*, 100
**big*, 103
**bill*, 19
**birds*, 31
**birthday*, 134
**black*, 46
**blocks*, 22
**blue*, 46
**boards*, 26
**boat*, 76
**Bob*, 46
**boils*, 93
**books*, 20
Boston, 257
**both*, 17
bottles, 26
*bottom, 236
**bought*, 20
*bowls, 25
**box*, 19
**boy*, 19
*bread, 84
**breakfast*, 195
**bridge*, 93
**bring*, 21
**broke*, 133
**brother*, 155
**brought*, 19
**Brownies*, 22
**brush*, 201
budget, 124
*built, 65
**bulbs*, 25
**bunches*, 70
**bundles*, 55
buns, 138
**Burns*, 53
*bus, 197
**bushels*, 84
**but*, 46
**butter*, 147
**butterflies*, 230
**buttons*, 35
**buy*, 22
**by*, 7
**cage*, 42
**cakes*, 88
calendar, 93
*California, 163

*called, 17
*came, 45
*camera, 87
*camping, 154
*can, 11
*candy, 26
canteen, 155
*cards, 32
*carefully, 42
Carol, 42
*carrots, 42
*carry, 26
*cars, 23
*cartons, 26
*cases, 25
*Castletown, 106
caterpillars, 89
*cats, 27
*caught, 98
*cent, 88
*certain, 170
*change, 19
*charge, 147
charms, 32
*check, 12
*cheese, 179
*chickens, 125
*children, 22
*chocolate, 100
*choose, 206
cider, 52
*circle, 7
*city, 226
*class, 20
*cleaners, 45
clerk, 19
*clocks, 198
*cloth, 65
*clothing, 124
club, 61
clubhouse, 104
*coach, 258
cocoa, 154
coffee, 163
*coins, 35
*cold, 184
*collars, 46
*collected, 46
*collections, 66
*colored, 64
column, 17
combined, 165
*come, 65
*common, 161
*compare, 119
comparison, 119
compasses, 90
compute, 43
*contains, 168
*cookies, 84
*cooking, 22
*corners, 36
cornet, 110
*correct, 136
*cost, 22
*could, 26
*counted, 47
*county, 109
*covering, 157
*cows, 163
crates, 65
crayons, 19
crepe, 75
*cross, 193
*Cub, 88
*cupboard, 174
*cups, 52
custard, 220
*cut, 26
daisies, 110
*dancing, 47
*David, 42
*day, 19
*daylight, 199
*deciding, 106
decimal, 255
decorated, 75
decorations, 209
*deep, 238
*deliver, 102
*den, 88
denominator, 126
depth, 238
*describe, 238
*developed, 134
diagrams, 238
diameter, 7
*diamond, 109
*Dick, 23
*did, 19
*died, 42
differ, 257
*difference, 207
*different, 3
dimensions, 238
dining, 258
*dinner, 100
*dishes, 26
disk, 148
*distance, 100
divide, 24
division, 26
divisor, 205
*do, 23
*does, 17
*dog, 109
*dollar, 19
*dolls, 45
*Don, 22
*done, 106
*doorway, 232
dotted, 160
doughnuts, 50
*down, 183
*dozen, 67
*drank, 100
*draw, 159
drawers, 66
*dressed, 46
*drink, 26
*drive, 22
*drove, 22
drug, 233
*dry, 93
dues, 64
*dug, 100
*during, 21
*each, 7
*earned, 88
*east, 198
*easy, 23
*eat, 125
*edge, 232
*eggs, 105
*eighty, 13
*either, 155
*eleven, 150
*Ellen, 22
*empty, 26
*ending, 77
*engines, 122
*enough, 26
envelopes, 48
*equally, 24
equations, 22
equipment, 154
erasers, 123
*evening, 100
*every, 22
*exact, 23
example, 77
*except, 146
exercises, 133
*expect, 84
expensive, 134
*explain, 67
express, 112
*extra, 199
*facts, 235
*fair, 68
*family, 22
*far, 42
*farmer, 94
*farther, 208
*faster, 237
*father, 22
favors, 65
*feathers, 46
*fed, 124
*feeds, 42
*feet, 26
*fence, 109
*fewer, 22
*field, 109
*fifth, 19
*figure, 17
*fill, 52
*film, 88
*find, 19
*finish, 24
*first, 19
*Fisher, 24
*fit, 157
*five, 13
*flashes, 226
*flat, 11
*floor, 247
*flour, 147
*flowers, 19
fluid, 93
*folded, 133
*following, 133
*food, 42
*foot, 93
*for, 16
foreign, 45
forks, 25
*form, 146
Fort Laramie, 163
*forty, 260
*found, 42
*four, 12
fractions, 126
*freezer, 26
freight, 257
*fresh, 42
*Friday, 104
*friends, 44
*from, 16
*frosting, 183
*fruit, 66
fudge, 45
*Fuller, 76
*gallons, 52
*garden, 106
gasoline, 214
*gathered, 100
*gave, 42
*George, 44
*get, 26
*girl, 19
*given, 19
*glasses, 100
*going, 24
*goldfish, 42
*gone, 20
*good, 15
*got, 26
*grab, 68
grade, 19
*grandfather, 134
grapefruit, 67
graphs, 234
*great, 257
*green, 129
*grew, 124
*grocery, 26
*group, 24
*grown, 104
growth, 104
gumdrops, 63
gunpowder, 163
gym, 19
*had, 19
*half, 133
*hall, 220
*handkerchiefs, 26
*happens, 17
*hard, 156
*harvested, 124
*has, 17
*hats, 30
*have, 15
*he, 19
*head, 136
health, 22
*heavy, 197
height, 7
*held, 84
*help, 12
*her, 19
*here, 48
*higher, 204
hiked, 133
*hill, 183
*him, 46
*his, 26
*hobby, 257
*holds, 81
*home, 22
*hooks, 96
*hope, 75
*horns, 27
*horses, 76
*hot, 154
*hours, 85
*how, 3
*hundreds, 12
*I, 46
*ice cream, 26

*if, 24
Illinois, 236
*imagine, 11
*important, 36
*in, 3
*inch, 8
*Indiana, 236
*Indians, 46
*instead, 51
*invitations, 48
*ironing, 26
*is, 7
*it, 11
*itself, 137
*Jack, 47
*Jane, 19
*jars, 84
*Jim, 45
*John, 22
*joined, 47
*juice, 106
*June, 76
Jupiter, 89
*just, 52
Kathy, 24
*keep, 105
*kept, 45
*kettle, 84
*kind, 8
kit, 110
knapsack, 155
*knew, 26
*know, 11
*laid, 125
*lake, 46
*land, 249
lard, 192
*large, 17
*last, 106
*later, 110
*lead, 163
*learn, 3
*least, 134
*leaves, 88
*left, 19
lemons, 117
*length, 7
*less, 67
*lesson, 3
*let, 65
*letters, 16
*library, 22
*lids, 84
*like, 61
limit, 93
*line, 7
liquid, 93
*list, 236
*little, 75
*lives, 22
*load, 93
*loaf, 84
loaned, 101
locomotive, 257
lollipops, 26
*long, 26
*look, 16
loss, 147
*lost, 46
*lots, 238
*lowest, 93
*lunch, 19
*machine, 225
*made, 16
magazine, 244
magnets, 90
mail, 100
*make, 22
*man, 163
*many, 19
*map, 198
*marbles, 117
*marched, 47
*marked, 36
*market, 106
*Mary, 47
masks, 27
*match, 21
*materials, 42
mats, 64
*May's, 133
*means, 17
*meant, 7
*measure, 3
*meat, 133
*meet, 65
*member, 89
*men, 163
*mended, 75
Mercury's, 89
*mice, 40
*midnight, 199
*might, 67
*miles, 22
*milk, 67
millions, 13
*mine, 132
Minnesota, 236
minus, 156
*minutes, 26
*missing, 141
*mixed, 149
*model, 119
*Monday, 104
*money, 19
*month, 64
*more, 3
*morning, 22
*most, 23
*mother, 24
*motorboats, 122
*movie, 86
*Mr., 66
*Mrs., 25
*much, 19
mules, 163
multiplication, 26
multiply, 27
*museum, 90
*music, 219
*must, 19
*my, 67
*name, 67
*Nancy, 20
*narrower, 257
*nearest, 23
*necklaces, 61
*needed, 19
*neighbor, 201
*new, 8
*New York, 198
*next, 45
*night, 90
*ninety, 12
*no, 54
*noisemakers, 29
*not, 17
*notebook, 110
*noticed, 89
*now, 7
*numbers, 12
numerals, 12
numerator, 126
*objects, 93
*o'clock, 75
*odd, 136
*of, 8
*off, 23
*office, 88
*often, 38
*Ohio, 236
*old, 49
*on, 12
*ones, 12
*only, 16
*opened, 138
*opposite, 107
*or, 7
*orange, 26
*order, 169
*other, 11
ounces, 42
*our, 17
*over, 37
*oxen, 163
Pacific, 198
*packages, 26
*page, 43
pageant, 46
*paid, 19
*paint, 46
*pair, 112
*paper, 46
parakeet, 42
parallel, 36
*Parker, 156
*part, 22
*party, 48
*passenger, 23
pasted, 89
Patsy, 22
Paul, 66
*peaches, 154
*peanut, 45
*peck, 93
Peggy, 19
*pen, 265
pencil, 19
*pennies, 78
*people, 26
per, 64
perimeter, 107
*persons, 65
*pet, 44
petals, 65
*picked, 67
*picnic, 193
pictograph, 234
*picture, 7
*pie, 133
*pieces, 26
*pigs, 124
*piles, 48
Pilgrims, 46
pint, 67
*pinwheels, 129
*pipe, 45
*pitcher, 139
*place, 15
*planes, 119
planet, 89
*planned, 65
*plant, 25
*plates, 52
*player, 22
*playgrounds, 42
plus, 156
*pocket, 110
*point, 52
polish, 24
polygons, 107
*popcorn, 75
position, 17
*possible, 165
*postcards, 88
*potatoes, 94
*potholders, 43
*pound, 52
*poured, 139
*practicing, 110
*presents, 88
*press, 44
*price, 26
*printed, 44
*problems, 22
profit, 147
project, 85
*promised, 50
*proper, 148
*pumpkins, 28
*pupils, 19
*put, 25
*puzzle, 244
*quart, 26
*questions, 36
*rabbits, 42
*race, 226
*radio, 110
*rails, 257
*rain, 255
*raised, 124
raisin, 124
*raking, 88
*rate, 112
ratio, 112
*reached, 164
*reading, 12
*ready, 125
*real, 232
*received, 220
recipe, 176
record, 22
rectangle, 11
*red, 45
remainder, 37
*remember, 23
represent, 232
*rest, 45
*ribbon, 26
rickrack, 85
*ride, 183
*right, 17
*ring, 89
*road, 100
*rocks, 93
*rode, 100
*rolls, 26
Roman, 16
*rounding, 23
*rows, 27

*ruler, 19
Ruth, 183
*safes, 66
*said, 19
*sailboats, 122
*sale, 105
*Sally, 67
*salted, 52
*same, 17
*sandwiches, 65
*Saturday, 42
*saved, 22
*saw, 59
*say, 11
scales, 232
*school, 19
science, 22
*Scouts, 88
*scrapbooks, 21
*second, 76
section, 250
*see, 43
*seed, 42
*sell, 45
*sending, 106
sensible, 213
*sent, 59
sentences, 263
*separate, 65
September, 89
*set, 125
*seven, 13
*several, 99
*sewing, 22
*shapes, 3
*share, 24
*she, 19
*sheep, 106
sheets, 117
*shells, 33
*shelves, 26
*shirt, 19
*shoes, 19
*shopping, 26
*shorter, 22
*should, 7
*shown, 11
*sides, 36
*sign, 52
*simplest, 146
*since, 125
*single, 54
*sister, 45
*six, 13
*size, 106
*skated, 226
skirt, 85
*sleeping, 155
*slept, 154
*slower, 265
*small, 26
*snowfall, 265
*so, 23
*soap, 216
*sold, 19
*solve, 117
*some, 19
soup, 182
*space, 260
*speed, 118
*spelled, 253
*spend, 19
spent, 19
*spoiled, 44
*spools, 57
*spoons, 24
*spring, 125
sprouted, 252
*square, 8
*squirrels, 189
*stamps, 33
standard, 198
*stars, 89
*started, 45
*statements, 105
*stayed, 22
*steamboats, 122
Steve, 226
*sticks, 42
*still, 22
*stones, 34
*store, 19
*storeroom, 59
*story, 22
*straight, 107
*strawberries, 26
*strings, 160
*strips, 64
*studied, 22
subtract, 20
subtraction, 22
*such, 145
Sue, 19
*sugar, 100
sum, 41
*summer, 21
Sunday, 163
*supper, 133
*supplies, 100
*suppose, 89
*sure, 168
*surprise, 68
*Susan, 76
*swim, 22
*syrup, 100
system, 15
*tables, 75
tablets, 59
*take, 26
*talked, 100
*tall, 22
*taught, 65
*teach, 65
*team, 184
*teaspoons, 24
*telephone, 198
*tell, 17
*temperature, 93
*tens, 12
*tents, 154
*test, 18
*than, 21
*that, 11
*the, 7
*their, 24
*them, 3
*then, 20
*there, 19
thermometers, 93
*these, 17
*they, 16
*things, 19
*think, 22
*thirteen, 12
*this, 3
*those, 230
*though, 106
*thought, 114
*thousands, 12
*three, 13
*through, 163
*throw, 44
*Thursday, 104
*tickets, 44
*time, 24
*to, 3
*together, 19
*told, 84
*Tom, 19
*tomatoes, 26
*ton, 93
*Tony, 22
*too, 19
*took, 26
*top, 244
total, 19
towels, 26
*town, 22
*toys, 57
*tracks, 257
*train, 163
*traveled, 154
*tree, 89
triangles, 107
*trim, 85
*trips, 22
troop, 22
*truck, 93
*trunk, 89
*try, 17
*Tuesday, 19
*turn, 43
*twelve, 22
*twenty, 13
*twice, 230
*two, 13
*understand, 7
*United States, 23
*until, 132
*up, 12
*us, 48
*used, 16
*usually, 118
*vegetables, 42
*very, 67
*wagons, 75
*walk, 22
*wall, 65
*wanted, 22
*was, 22
*washed, 164
*water, 93
watermelon, 193
*way, 16
*we, 17
*wear, 183
Wednesday, 19
*week, 42
weighed, 22
*weight, 42
*went, 19
*were, 19
*west, 197
*what, 7
*wheels, 75
*when, 22
*where, 22
*whether, 133
*which, 7
*while, 225
*white, 40
*who, 67
*whole, 89
*whose, 250
*why, 17
*wide, 75
width, 7
*wigwams, 46
*will, 3
*window, 244
*with, 13
*without, 38
*women, 163
*wonders, 84
*wood, 147
*words, 42
*work, 24
*world, 198
*worms, 100
*worth, 26
*would, 65
*wrapped, 76
*writing, 12
*wrong, 106
*wrote, 23
*yard, 65
*year, 22
*yellow, 46
*you, 3
*young, 65
*your, 67
zero, 204
zones, 198
*zoo, 213

Mathematical content

Numerals and the number system
- Reading and writing numerals through millions (reteaching)
- Place value through millions (reteaching)
- Regrouping as required in carrying and borrowing (reteaching)
- Roman numerals (reteaching and extension)
- Rounding off numbers (reteaching and extension)
- Fraction numerals (reteaching and extension)
- Ratios
- Decimal fraction numerals—tenths, hundredths

Addition
- Addition of whole numbers (diagnosis)
- Addition of numbers representing money (diagnosis)
- Mental calculation (reteaching)
- Addition of proper fractions and mixed numbers

Subtraction
- Subtraction of whole numbers (diagnosis)
- Subtraction of numbers representing money (diagnosis)
- Mental calculation (reteaching)
- Subtraction of proper fractions and mixed numbers

Multiplication of whole numbers
- Basic facts, products of 36 and less (diagnosis)
- Basic facts, products 40 to 81 (reteaching)
- Multiplication of two-figure and three-figure numbers by one-figure multiplier (reteaching)
- Multiplication of two-figure and three-figure numbers by two-figure multiplier (reteaching)
- Multiplication by three-figure multiplier
- Mental calculation involving two-figure multiplicand and one-figure multiplier
- Multiplication of numbers representing money (reteaching)

Division of whole numbers
- Basic facts, dividends of 36 and less (diagnosis)
- Basic facts, dividends 40 to 81 (reteaching)
- Division by one-figure divisor (reteaching)
- Division by two-figure divisor
- Division by three-figure divisor
- Division involving numbers representing money
- Remainders in division (reteaching and extension)
- Estimating partial quotients

Measurement
- Measurement equivalents (reteaching)
- Reduction of measures (reteaching)
- Measures of area

Money
- Addition and subtraction (diagnosis)
- Multiplication and division

Graphs and scale drawings
- Pictographs and bar graphs
- Interpreting linear scale on scale drawings

Fractions
- Concept of fractions (reteaching and extension)
- Improper fractions and mixed numbers (reteaching and extension)
- Addition of proper fractions and mixed numbers
- Subtraction of proper fractions and mixed numbers
- Reduction
- Comparison of fractions
- Using the remainder in division to make a fraction

Ratios
- Ratios as expressions of rate
- Ratios as expressions of comparison
- Reduction of ratios
- Per cents

Geometry
- Parallel lines and right angles (reteaching)
- Recognition of polygons and circle (reteaching and extension)
- Using dimensions to describe polygons and circles
- Scale drawing
- Concept of area
- Measurement of area—rectangles
- Measurement of perimeter—polygons

Problem solving
- Review and reteaching of all previously taught problem types
- Problems involving division of numbers representing dollars and cents
- Multiple-step problems
- Finding the average (reteaching and extension)
- Treating the remainder resulting from division
- Multiplicative problems in which the number of equal groups is unknown
- Multiplicative problems in which the size of the equal groups is unknown
- Using fractions in problem situations involving addition and subtraction
- Using fractions in comparison situations
- Using ratios to solve problems involving rate
- Using ratios to solve problems involving comparison
- Using comparison expressed as "times as many" in problem solving

Index

he purpose of this index is to help you locate material this *Teaching Guide* that pertains to the *teaching* of various ithmetical concepts. For location of subject matter the pupils' text, see the index to the text.

bbreviations (measurement terms), 94, 96
ble pupils, providing for. (Each block of Expanded Notes, pages 243 to 350, contains suggestions for special work for able pupils.)
chievement, measurement of, 108, 258-259
cre, 224-225
ction principle, 10-11, 34, 35, 290
ctivities, 355-381 (*see also* Able pupils and Slow learners)
ddends, order of, 35
ddition
additive situations defined, 10-11
imagined action in, 35, 261, 271
inventory tests on, 33-34, 258-259
mental computation in, 147-148, 183-184, 315-316
mixed numbers, 166-169, 170-171, 321, 326-327, 379-380
one-, two-, and three-figure numbers, 360, 361
other methods, 259
proper fractions, 162-165, 170-171, 321, 323-326
dditive-addition, 35, 36, 41, 170, 261
dditive-subtraction, 10-11, 260, 261-262
ngles, 21, 23, 48, 107, 247, 249
nswer key, equation in, 35, 98
pproximating, *see* Rounding off numbers
rea, *see also* Square measures
computing area of a rectangle in square inches by ratio, 217-220, 345-347
computing area of a rectangle in square feet and square yards by ratio, 221-223, 347-349
concept of, 24-27, 149, 215-217, 226-227, 251-253, 344, 358
of triangle (informally), 149, 227, 252
reduction of square measures, 221-223, 224-225, 347, 348
Average, finding, 101-103, 290, 292-294, 372-373
Bibliography, 385-387
Borrowing with mixed numbers, 171-172, 175-177, 329-330
Carrying, *see* Multiplication
"Chain" problems, 13, 56-57
Charting the course, 244-245, 253-254, 260-261, 264-265, 270, 273-274, 279-280, 288, 289-290, 294, 296-298, 305-306, 306, 311, 316, 321, 334, 335, 336-337, 344, 349
"Checking up," purpose and administration of, 33, 108, 258-259
Circle, 20-24, 24-27, 246, 248, 250, 358
Common denominator, 152, 154, 157-159, 162, 163, 164, 166, 168, 169, 171, 172, 173, 174, 176, 177, 316, 318-320, 322, 323, 324, 326, 328, 329, 378
Commutative law, 66
Comparison
by ratio, 116-119, 202-205, 205-208, 297, 300-302, 340-342, 342-344
ideas of more, less, fewer, and difference in problem solving, 34, 37, 191-192, 260, 262
of fractions, 150-151, 152-154, 160-161, 316, 317, 318-320
of rates expressed by per cents, 227-229, 349
Decimal fraction numerals, 230-231, 349
Diagnostic tests, *see* Tests, end-of-block
Divisibility, tests for, 129-131, 304-305
Division
arrangement of work, reasons for, 74-75, 279
basic facts, 44-47, 50, 259, 264-265, 266-268, 360, 361, 362, 363, 364, 365, 366
computational form in, 74-75, 279, 280
estimating quotient, 189-191, 279, 280, 287-288, 334
finding the number of equal groups, 39, 40, 44, 88-90, 260, 279, 284-285, 288
finding the size of equal groups, 39, 40, 88-90, 260, 285, 286, 290, 292-294
of dividend and divisor by same number, 129-131, 188-189, 189-191, 280
of money, 88-90, 284-287
one-figure divisor, two-figure quotient, 74-79, 279-282
partial quotients in, 74-75, 81-82, 93, 189-191, 279, 280, 287-288, 334
partitive, 39, 40, 88, 90, 101-103, 132, 194, 195, 260-261, 285, 286, 290, 292-294
process of, 14, 74-75, 279-284
quotitive, 39, 40, 88, 132, 194, 195, 260, 279, 284-285, 288
remainders in, 49-50, 104-105, 194-196, 197, 268-269, 280, 290, 335, 367
two-figure divisor, 82-86, 282-284
three-figure divisor, 187
Divisive situations, 39, 40, 88-90, 94, 101, 196, 260, 284-287, 288, 290
Ellipses, 21, 22, 245, 246-247
Equal groups
finding number of, 39, 40, 44, 88-90, 267, 284-285, 288

finding size of, 39, 40, 88-90, 101, 285, 290, 292-294
in multiplication, 41
Equation form, use of in practice exercises, 33
Equations, 11-12, 35, 98, 101, 260-261, 290, 368-369
Equivalents, *see* Fractions, Measurement, and Ratio
Evaluation procedures, 33
Expanded notes, 243-350
Four-step teaching method, 14, 59
Fractions and fraction numerals
addition of mixed numbers, 166-169, 321, 326-327, 379, 380
addition of proper, 162-165, 321, 323-326, 379
changing improper fractions to mixed numbers or whole numbers, 143-144, 146, 311, 313-315
changing mixed numbers to equivalent forms, 171-172, 378-379
combining and separating, as preparation for adding and subtracting, 157-159, 321-323
common denominator, 152, 154, 157-159, 162, 163, 164, 166, 168, 169, 171, 172, 173, 174, 176, 177, 316, 318-320, 323, 324, 326, 328, 329, 378
comparison of, by size of denominator, 160-161
comparison of, by number line, 150-151, 316, 317
comparison problem situations, 191-192
distinction between, 8, 121, 302
distinguished from rates and comparisons, 8-10, 125-127, 296-298, 304
equivalent, 136-138, 146, 306, 308-311, 327, 376
expressed as decimals, 230-231
meaning of improper, 140-142, 311-313
meaning of proper, 121-125, 302-304
mixed numbers, 140, 142, 311-313
problem solving involving addition of, 170-171
problem solving involving subtraction of, 179-180, 331
reading and writing, 121-122, 302-304
reduction of, 136-138, 146, 165, 306, 308-311, 327
short cuts in adding, 170, 171
simplest form, 136, 138, 146, 162, 165, 310
subtraction of mixed numbers, 175-178, 321, 329-331
subtraction of proper, 172-175, 328-329, 381
used as remainder in division, 194-196, 197, 335
Geometric figures, 20-24, 48, 106-108, 149, 244-251, 294-296, 356-358
Geometry
dimensions of polygons and circles, 20-24, 244, 246, 247, 248, 249, 250, 358
labels, 23, 24, 249, 250, 358
parallel lines, 48, 107, 249, 295
right angles, 23, 48, 107, 249
Grade placement chart, 390-391
Graphs, 212-215, 344
Grouping, *see also* Number system by tens, 42, 43, 44
Groups
in division, 39, 40, 44, 88, 101, 267, 268, 288, 292
in measures, 94, 288-289
in multiplication, 41, 288
Hexagon, 21, 106, 227, 246, 247
Improper fractions, 140-142, 143-144, 311-313, 313-315
Individual differences, providing for, 59. (Each block of Expanded Notes, pages 243 to 350, contains detailed suggestions for handling individual differences, labeled "Providing for the able pupil" and "Helping the slow learner.")
Insight and arithmetic, 6-17, 321
Introduction, 4-18
Lesson briefs, 20-241
Linear measures, *see* Measurement
Long division, 74-79, 82-86, 187 (*see also* Division)
Map, meaning of scale on, 210-211
Measurement
altitude, 20, 21, 23, 245, 248, 250
base, 20, 21, 23, 248, 249, 250
diameter, 20, 21, 23, 248, 250
equivalents, 94-96, 347-349, 371
linear, 21, 23, 94, 106-108, 1 215-217
reduction of, 94, 95, 97, 221-2 288-289, 347, 348
square measures, 24-27, 149, 2 217-220, 221-223, 224-225, 2 253, 344, 345-347, 347-349
standard time and time zones, 1 183, 332-333
Measurement division, *see* Quoti division
Measuring units vs. counting u 72-74
Mental computation, 147-148, 1 184, 315-316, 333
Mixed numbers
addition of, 166-169, 321, 326-3 380
changing to equivalent form, 1 172, 175, 177, 329-330
comparison problem situations w 191-192
meaning of, 140, 142, 311-313
problem solving involving addi of, 170-171
problem solving involving subt tion of, 179-180, 331
subtraction of, 175-178, 321, 3 331, 380
Money
division of, 88-90, 284-287
multiplication of, 277
Multiplication
and money, 277
and problem solving, 39, 40, 2 263-264, 288
and the number system, 58, 66 264, 273
basic facts, 41-44, 50, 264-266, 3 361, 362, 363, 364, 365, 366,
by two-figure numbers ending one zero and by three-figure n bers ending in two zeros, 66 278
carrying in, 13-14, 59, 67, 274
distinguished from addition, 41
four-figure multiplicand, 155-15
meaning of, 41
mental calculation, 183-184, 33
of numbers ending in one or n zeros, 58, 273-274, 369-370

one-figure multiplier, 59-62, 62-66, 273, 274-275
other methods, 278
process of, 59
three-figure multiplicand, 62-66, 275
three-figure multiplier, 67, 81-82
two-figure multiplicand, 59-62, 274-275, 276-278
two-figure multiplier, 67-70, 273, 276-278
ultiplicative situations, 39, 40, 41, 94, 131-133, 261, 263-264, 305-306
use of in equations, 12, 34, 35, 261
umber line, 150-151, 231, 316, 317
umbers, *see also* Number system
arithmetical progressions, 259, 360
differentiated from numerals, 254
even and odd, 129, 130
prime, 305
umber series, 259, 360
umber system
and multiplication, 58, 66-67, 264, 273
decimal notation through tenths and hundredths, 230-231, 349
extended to fractions, 17, 317
grouping and regrouping in, 17, 27-30, 253-254, 254-255, 255-257, 359
place value in, 27-30, 254, 255, 256, 258
reading numerals, 28-30, 58, 231, 255-256
review of tens', hundreds', thousands', and millions' places, 27-30, 254-256, 257-258
zero in, 29, 255
:tagon, 21, 247
'ganization of content, simplified, 13-14
rallel lines, 48, 107, 295
rallelogram, 249, 295
rtial quotients, 74-75, 81-82, 93, 189-191, 279, 280, 287-288, 334
rtitive division, *see* Division
ntagon, 21, 106
r cent, 227-229, 349
rimeter, 106-108, 149, 215-217, 294-296, 344, 373
ace value, *see* Number system
Polygons, 20-24, 106-108, 149, 226-227, 245-250, 295-296, 356-357, 358
Practice
explanation of block arrangement of practice material and referral to, in answer key, 28
purpose of, 18
Problem types chart, 388-389
Problems, *see also* Problem solving
"chain," 13, 56-57
multiple step, 13, 98-101, 101-103, 289-290, 290-291, 372
procedures for administering sets of problems, 39, 87
Problem solving
addition in, 10-11, 34-36, 261
analysis of problem situation, 10-13, 34, 35, 260-261, 270, 284-285, 288, 289-290, 305-306
choosing the correct process in, 13, 260
division in, 39-40, 88-90, 101-103, 260-261, 284-287, 288-289, 290, 292-294, 305
finding how many are gone when the number to begin with and the number remaining are known, 54-55, 270, 271-272
finding how many items there were at first when the number gone and the number left are known, 52-53, 170, 270-271
finding how many more or fewer there are in one group than another, 34, 35, 37, 191-192, 262
finding how many more were added or are needed, 34, 35, 36, 260, 261, 262
finding the average, 101-103, 290, 292-294, 372
finding the difference, 191-192
finding the number of equal groups when the total number and the size of the equal groups are known, 39, 40, 88-90, 260, 263, 284-285, 288
finding the number to begin with when the total and the number added are known, 34, 35, 260, 261-262, 262-263
finding the remainder when part of a group is removed, 34, 35, 260, 261
finding the size of equal groups when the total number and the number of equal groups are known, 39, 40, 260-261, 263, 285, 286, 288, 290, 292-294
finding the total when one group is combined with another, 34-36, 170, 171, 260, 261
finding the total when the size and number of equal groups are known, 39, 40, 263-264, 288
imagined action in, 35, 36, 37, 261, 271
interpreting remainders in division, 196-197, 335
multiplicative problems in which the number of equal groups or the size of the equal groups is unknown, 131-133, 305-306
"times as many" situations, 205-208, 337, 342-344
using addition of fractions and mixed numbers in, 170-171
using ratios for expressing rates in, 198-201, 336-337, 337-339
using ratios for making comparisons in, 202-205, 337, 340-342
using subtraction of fractions and mixed numbers in, 179-180, 331
Profit and loss, 139-140, 233-234
Quotitive division, *see* Division
Rate, *see also* Ratio
concept of, 111-114, 296-300
differentiated from fraction and comparison, 8-10, 125-127, 296-298, 304
expressed by per cent, 227-229, 349
expressed by ratio, 111-114, 115, 193, 217-220, 221-222, 227-229, 297-300, 349, 373-374
used in problem solving, 198-201, 217-220, 336-337, 337-339, 345-347, 347-349
used to find area, 217-220, 221-223, 345-347, 347-349
Ratio
and per cent, 227-229, 349
comparison expressed as, 116-119, 193, 205-208, 297, 300-302

comparison problems solved by using, 202-205, 205-208, 337, 340-342, 342-344
computing area of rectangle by, 217-220, 221-222, 345-347, 347-349
equivalent, 111, 113, 114, 297, 299
numeral 1 in, 115, 205-208, 336-337, 342-344
rate expressed as, 111-114, 115, 193, 217-220, 221-222, 227-229, 296-300, 345-347, 347-349, 373-374
rate problems using, 198-201, 217-220, 336-337, 337-339, 345-347, 347-349
"times as many" problems using, 205-208, 342-344
Rectangle, 20, 21, 23, 24, 106, 107, 215-217, 217-220, 221-222, 246, 248, 249, 295, 344
Reduction
of divisor and dividend, 129-131, 188-189, 189-191, 304-305, 334
of fraction numerals, 136-138, 146, 306, 308-311, 327, 376
of improper fractions to mixed numbers or whole numbers, 143-144, 146, 311, 313-315
of measures, 94, 95, 97, 221-223, 224-225, 288-289, 347-348
of ratios, 133-135, 146, 193, 306-308, 347, 348
Remainders in division, 49-50, 104-105, 194-196, 197, 268-269, 280, 290, 335, 367-368
Remedial procedures, 33, 50, 146, 180, 229
Reteaching chart, 33, 109, 185, 234, 238
Right angles, 23, 48, 107, 149, 247, 249, 250
Roman numerals, 31-32, 258, 359
Rounding off numbers, 38, 147, 148, 188-189, 189-191, 280, 334
Scale, 210-211, 212-215, 344
Screen, use of in equations, 12, 13, 33, 35, 36, 98, 289
Slow learners, providing for. (Each block of Expanded Notes pages 243 to 350, contains suggestions for slow learners.)
Square, 20, 21, 23, 24, 106-108, 149, 216, 217, 226, 247, 249, 250, 295, 344
Square measures, 24-27, 149, 216, 217-220, 221-222, 224-225, 251-253, 344, 345-347, 347-349
Subtraction
finding the difference, 191-192
imagined action in, 261, 262, 272
inventory tests on, 33-34, 259
mental computation, 147-148, 183-184, 315-316
of mixed numbers, 175-178, 321, 329-331
of proper fractions, 172-175, 321, 328-329
Subtractive-addition, 52-53, 270-271
Subtractive situations, 11, 34, 35, 52-53, 54-56, 261, 270-271, 271-272
Symbols, meaning of, 12
Teaching devices
colored type, 16
dimming off, 16, 272
dotted background, 53, 68
four-step teaching method, 14, 274-275
ring, 42, 45, 49
screen, 35, 98
wavy line, 58
Tests
achievement, 108-110, 185-186, 241
administration of, 33, 108, 185, 238, 258
end-of-block (diagnostic), 50-51, 70-71, 91-92, 180-181, 229-230, 258-259
problem-solving, 108-110, 238-240
"Times as many," 205-208, 342-344
Time schedule, 389
Triangle, 20, 21, 23, 107, 149, 247, 248, 249, 251, 357
Visual aids, 14, 16
Vocabulary, 17, 20, 245
Wavy line, use of, 58
Wholes, learning by, 14, 59, 75, 289, 334
"Word clues," 12
Word list, 392-395
Zero
in division, 188-189, 279
and number line, 317
and place value, 29, 255

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 66 65 64 63 62 61 60 59

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